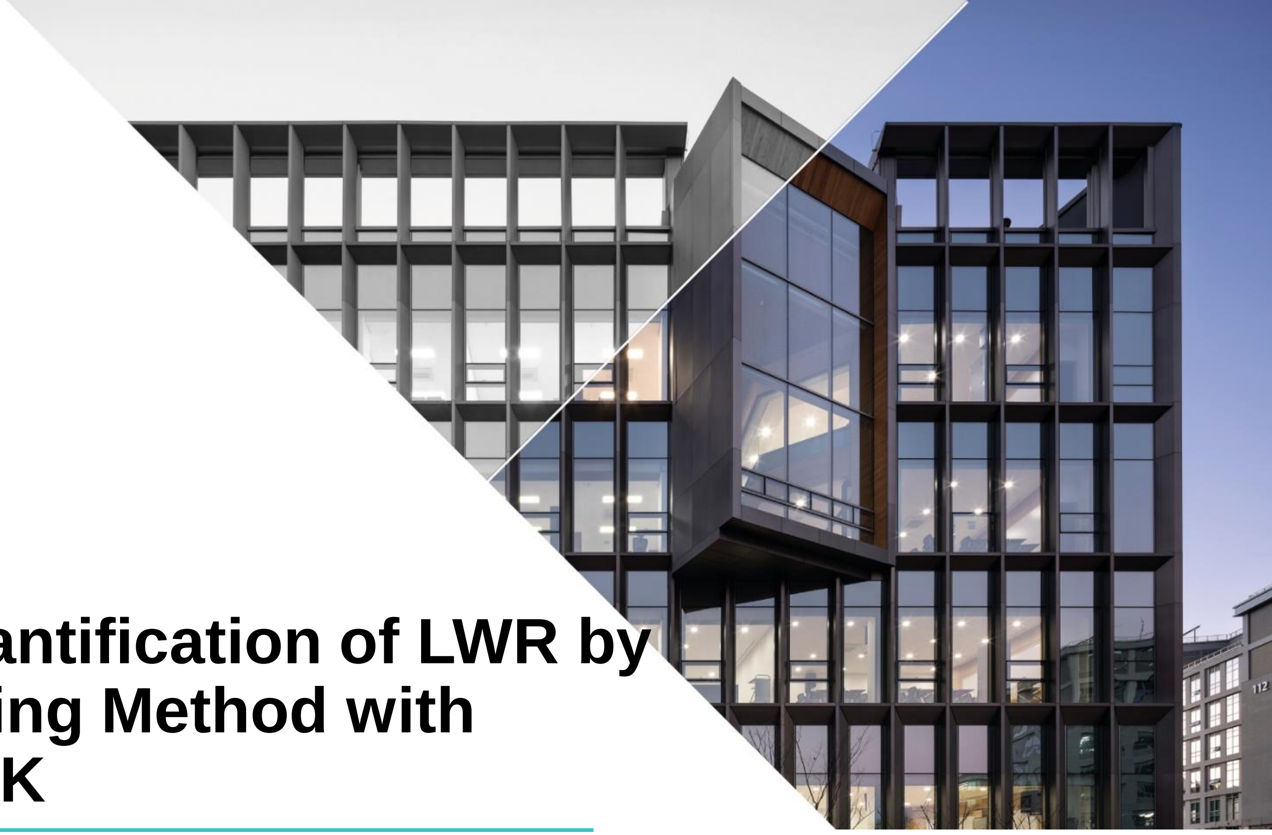

Uncertainty Quantification of LWR by Random Sampling Method with STREAM/RAST-K

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Content

- **Introduction**
- **Random sampling method**
- **Commercial core calculation**
- **Conclusion and Further work**

Introduction

- **The random sampling(RS) method for cross section perturbs cross section data by standard normal distribution.**

- **The relations between each group data are considered by using covariance data**

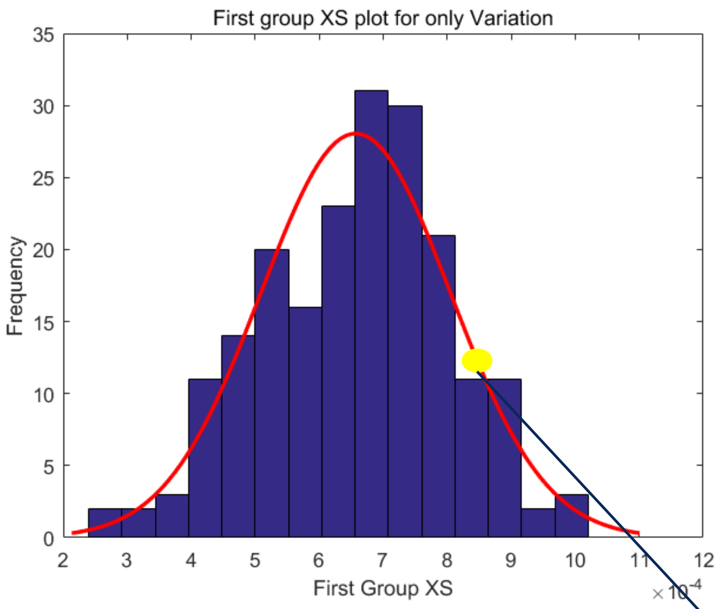
- **The relations between each nuclear data are also considered by using covariance data.**

- **This paper uses RS method with STREAM/RAST-K core analysis and uses ENDF-VII.1 nuclear data and covariance data to calculate uncertainty of many kinds of core property.**

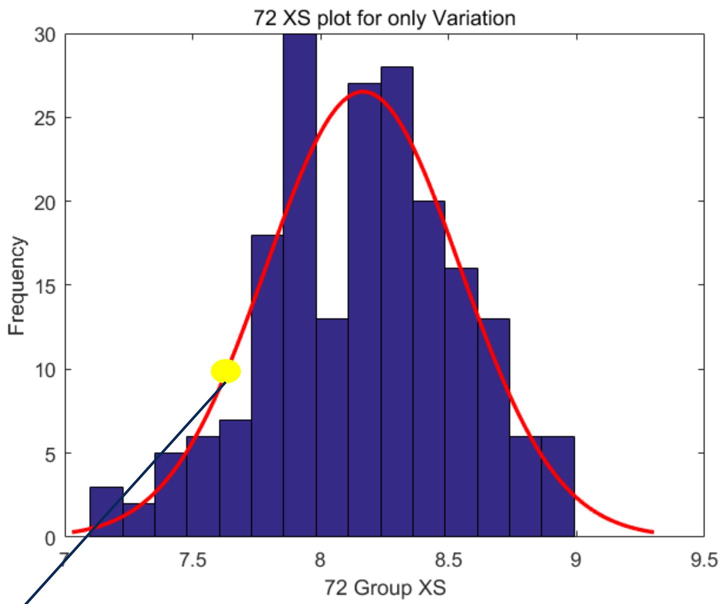
Random Sampling Method

Random Sampling Method

- **Perturbation by standard normal distribution**
  - **NJOY gives each group XS uncertainty**
  - **Perturbated XS by using its uncertainty**



72 Group XS



Perturbed library set

Random Sampling Method

- **Normal distribution**

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad \mu : \text{average} \quad \sigma^2 : \text{variation} \qquad \cdots (1)$$

- **Standard normal distribution, z**

Σ

$$P(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad \mu = 0 \quad \sigma^2 = 1 \qquad z = \frac{x-\mu}{\sigma} \qquad \cdots (2)$$

- **Basic Transformation for Standard Normal Distribution, z**

- **The Normal Distribution that have average and variation as μ and as σ^2 for one variation.**

$$x = \sigma \times z + \mu \qquad \cdots (3)$$

Random Sampling Method

- **Setting x for variative matrix, A.**

$$\mathbf{x} = \mathbf{Az} + \boldsymbol{\mu} \quad \cdots (4)$$

- **Expectation**

$$\mathbf{E[x] = E[Az + \boldsymbol{\mu}] = AE[z] + E[\boldsymbol{\mu}] = A \bullet 0 + \boldsymbol{\mu} = \boldsymbol{\mu}} \quad \cdots (5)$$

- **Variation, with covariance matrix C**

$$\begin{aligned}
\mathbf{V[x]} &= \mathbf{E[(x - E[x])(x - E[x])^T]} \\
&= \mathbf{E[(Az + \boldsymbol{\mu} - \boldsymbol{\mu})(Az + \boldsymbol{\mu} - \boldsymbol{\mu})^T]} \\
&= \mathbf{E[(Az)(Az)^T]} \\
&= \mathbf{AE[zz^T]A^T = AV[z]A^T = AA^T} \\
&\therefore \mathbf{V[X] = C = AA^T} \quad \cdots (6)
\end{aligned}$$

Akio Yamamoto, Kuniharu Kinoshita, Tomoaki Watanabe, Tomohiro Endo, Yasuhiro Kodama, Yasunori Ohoka, Tadashi Ushio and Hiroaki Nagano, "Uncertainty Quantification of LWR Core Characteristics Using Random Sampling Method", Nuclear Science and Engineering, 181:2, 160-174 (2015)

Random Sampling Method

- **Singular value decomposition**

$$\mathbf{C} = \mathbf{U\Sigma V^T} \quad \cdots (7)$$

where,

$\mathbf{U}$ = orthonormal eigenvector of $\mathbf{CC^T}$,

$\mathbf{\Sigma}$ = eigenvalues of $\mathbf{CC^T}(\mathbf{C^TC})$,

$\mathbf{V}$ = orthonormal eigenvector of $\mathbf{C^TC}$.

- **Symmetric covariance matrix, C**

$$\mathbf{C} = \mathbf{U\Sigma U^T} = (\mathbf{U}\sqrt{\mathbf{\Sigma}})(\sqrt{\mathbf{\Sigma}}\mathbf{U^T}) = (\mathbf{U}\sqrt{\mathbf{\Sigma}})(\mathbf{U}\sqrt{\mathbf{\Sigma}})^{\mathbf{T}}$$

$$\therefore \mathbf{A} = (\mathbf{U}\sqrt{\mathbf{\Sigma}}) \quad \cdots (8)$$

Random Sampling Method

- **Covariance matrix**

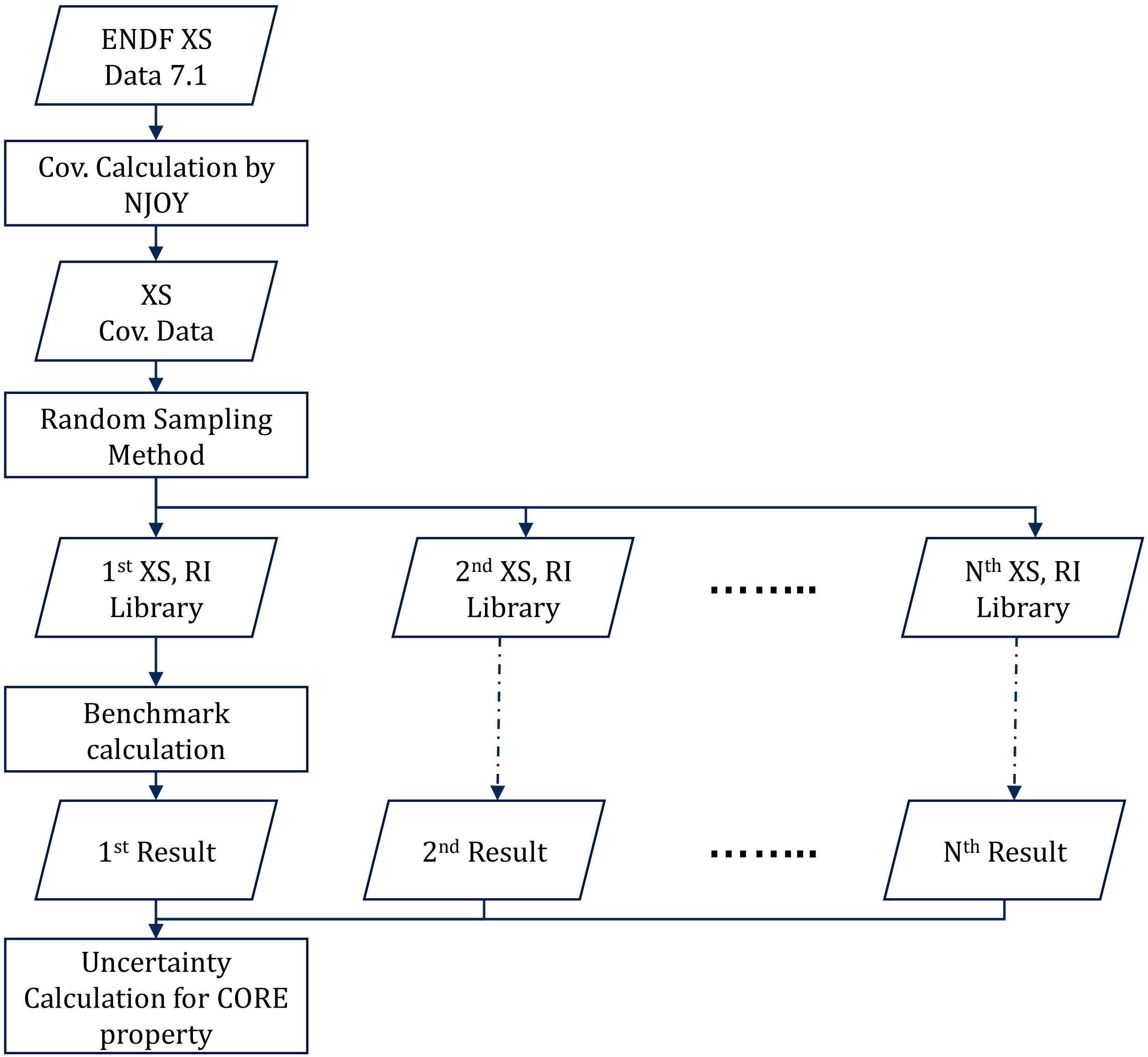
$$\begin{bmatrix} \mathbf{C}_{a,a} & \mathbf{C}_{a,b} & \cdots & \mathbf{C}_{a,z} \\ \mathbf{C}_{b,a} & \mathbf{C}_{b,b} & & \\ \vdots & & \ddots & \vdots \\ \mathbf{C}_{z,a} & & \cdots & \mathbf{C}_{z,z} \end{bmatrix} \quad \cdots (9)$$

where,

$\mathbf{C}_{i,j}$ = covariance matrix between nuclear data i and j.

Random Sampling Method

▪ Uncertainty calculation flow chart



Random Sampling Method

- **Multi-group cross section library**
  - **NJOY, ENDF/B-VII.1**
  - **423 isotopes**
  - **72 energy groups (1E-5 eV ~ 20 MeV)**
  - $\sigma_t, \sigma_a, \sigma_f$, σ_s, ν, …
  - **72 energy groups Covariance data(NJOY, ENDF/B-VII.1)**
- **Resonance integral library**
  - **NJOY, ENDF/B-VII.1**
  - **77 isotopes**
  - **39 energy groups (0.3 eV ~ 20 MeV)**
  - **10~19 background cross sections**
  - $\sigma_a(\sigma_b), \sigma_f(\sigma_b), \sigma_s(\sigma_b)$, …
  - **The covariance data for different dilution is not given.**
    - **RI group covariance data of XS are used to perturb RI.**
    - **For each dilution data, it is perturbed separately.**
    - **To use this RI perturbation, Equivalence theory is used for STREAM/SS**

Commercial Core Calculation

Commercial Core Calculation

▪ Covariance data

- ENDF VII.1 data are used.
- 28 nuclides that are important for LWR calculation are considered.

H-1	B-10	B-11	O-16	Zr-91
ZR-96	Rh-103	Xe-135	Sm-149	Gd-155
Gd-157	U-234	U-235	U-236	U-237
U-238	Np-237	Np-239	Pu-238	Pu-240
Pu-241	Pu-242	Am-241	Am-242*	Am-243
Cm-242	Cm-244	Cm-245		

- The Nu data is perturbed for prompt and delayed data separately.
- The chi data is not perturbed.

Commercial Core Calculation

- **NJOY**
  - **Cross section processing code**
  - **It generates multi-group library from ENDF VII.1**

- **STREAM**
  - **Neutron transport code**
  - **It generates effective multi-group cross section data.**

- **RAST-K**
  - **Diffusion code**
  - **It solves 2 group diffusion analysis.**

Commercial Core Calculation

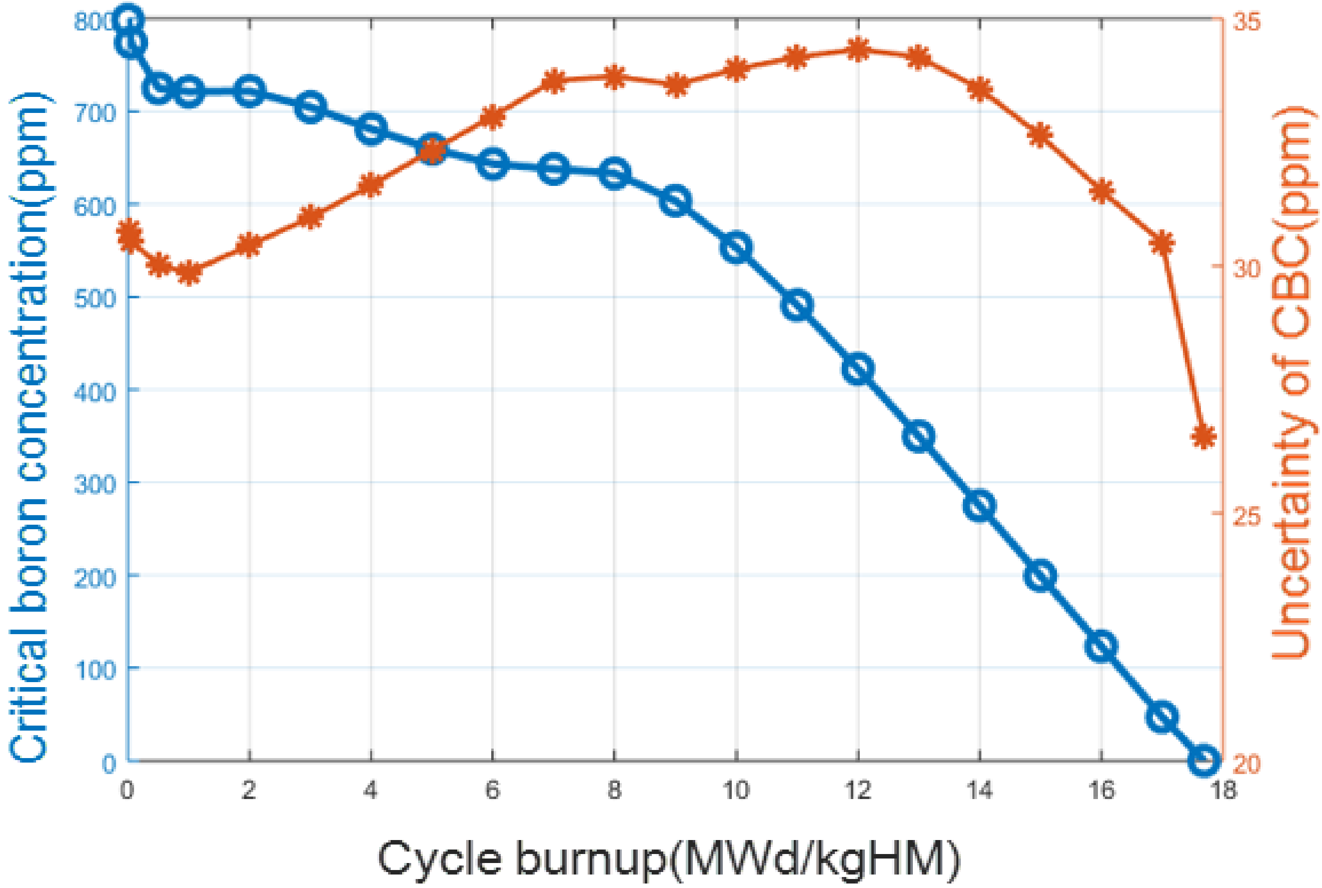
- **Core model**
  - **Commercial core design**
  - **Reactor model : APR-1400**
  - **Fuel model : PLUS7**

	J	K	L	M	N	P	R	S	L
9	A	A	C	A	B	A	B	C	B
10	A	B	A	B	A	B	A	B	C
11	C	A	C	A	C	A	C	B	B
12	A	B	A	B	A	B	A	B	C
13	B	A	C	A	C	A	B	C	
14	A	B	A	B	A	B	C	C	
15	B	A	C	A	B	C	C		
16	C	B	B	B	C	C			
17	B	C	B	C					

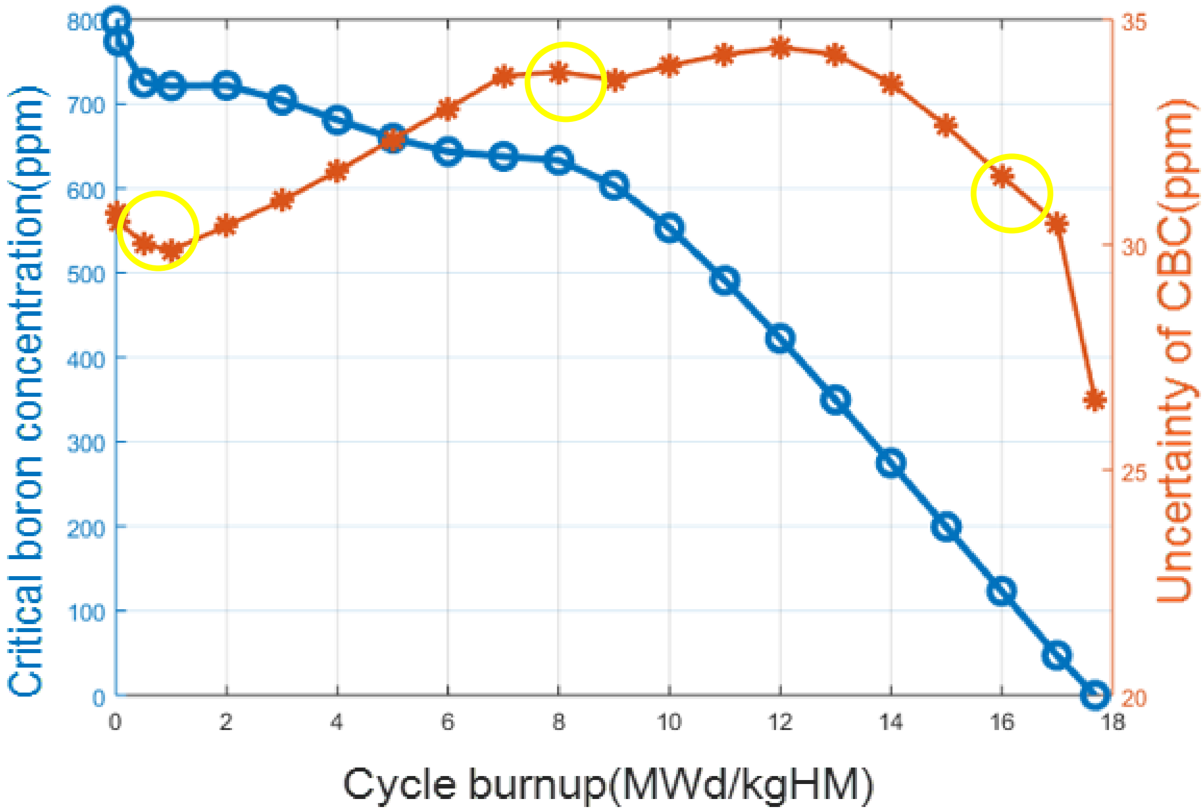
- **Calculation core property**
  - **Critical boron concentration**
  - **Axial power distribution**
  - **radial power distribution**
  - **Burnup distribution**
  - **Rod worth**

Commercial Core Calculation

- **Critical boron concentration**
  - **The average value and its uncertainty(ppm) is calculated.**



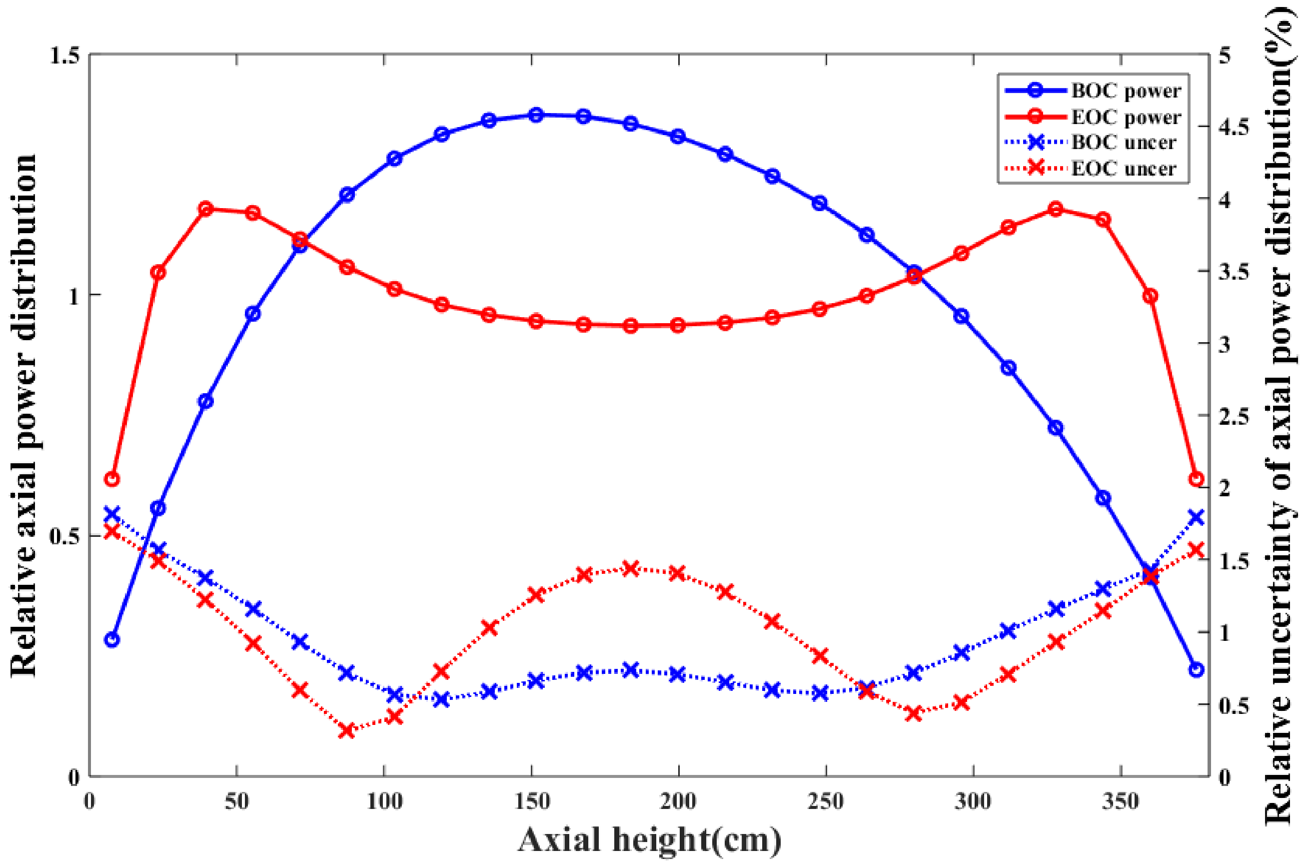
Commercial Core Calculation



- **There is initial decrease of uncertainty by U-235 decrease.**
- **Fission product generation increase uncertainty.**
- **There are rapidly decrease at the end of cycle because of decrease of whole nuclides**

Commercial Core Calculation

- **Relative axial power distribution**
  - **The uncertainty is calculated with relative form.**
  - **The uncertainty show lower value in middle of BOC than that of EOC.**
  - **That is because of fission product generation**



Commercial Core Calculation

- **Radial distribution**
  - **The relative radial power and burn-up distribution are calculated**
  - **The uncertainty are calculated with relative form**
  - **The BOC and EOC result will be shown**

Commercial Core Calculation

▪ Radial distribution at BOC

- Relative radial power(left) and Burn-up distribution(right)
- 0.05 MWd/kgHM burnup result.

	J	K	L	M	N	P	R	S	L
9	0.94								
	1.54								
10	0.9	1							
	1.45	1.31							
11	1.08	0.9	1.11						
	1.16	1.22	0.99						
12	0.92	1.04	0.92	1.04					
	1.05	0.97	0.93	0.72					
13	1.15	0.96	1.13	0.93	1.14				
	0.66	0.73	0.57	0.52	0.17				
14	0.98	1.17	0.96	1.06	0.94	1.02			
	0.44	0.32	0.33	0.14	0.09	0.5			
15	1.06	0.96	1.13	0.95	1.12	1.05	0.81		
	0.13	0.05	0.2	0.28	0.66	0.9	0.81		
16	1.11	1.03	1.06	0.99	1.07	0.78			
	0.66	0.64	0.7	0.74	0.9	0.87			
17	0.99	1.05	0.92	0.74					
	0.93	0.96	0.93	0.77					

FA POWER
Rel.Uncer(%)

	J	K	L	M	N	P	R	S	L
9	0.05								
	1.5								
10	0.05	0.05							
	1.41	1.27							
11	0.05	0.05	0.06						
	1.13	1.19	0.96						
12	0.05	0.05	0.05	0.05					
	1.03	0.95	0.91	0.71					
13	0.06	0.05	0.06	0.05	0.06				
	0.65	0.72	0.56	0.52	0.17				
14	0.05	0.06	0.05	0.05	0.05	0.05			
	0.438	0.31	0.33	0.15	0.09	0.48			
15	0.05	0.05	0.06	0.05	0.06	0.05	0.04		
	0.13	0.05	0.19	0.27	0.65	0.88	0.79		
16	0.06	0.05	0.05	0.05	0.05	0.04			
	0.65	0.62	0.69	0.73	0.89	0.86			
17	0.05	0.05	0.05	0.04					
	0.92	0.95	0.92	0.76					

FA BU
Rel.Uncer(%)

Commercial Core Calculation

▪ Radial distribution at EOC

- Relative radial power(left) and Burn-up distribution(right)
- 17 MWd/kgHM burnup result.

	J	K	L	M	N	P	R	S	L
9	16.22								
	0.53								
10	16.46	19.51							
	0.46	0.36							
11	20.74	17.04	21.33						
	0.31	0.40	0.27						
12	17.12	20.09	17.25	20.18					
	0.40	0.31	0.36	0.26					
13	20.59	17.29	21.13	17.20	21.22				
	0.27	0.35	0.22	0.27	0.08				
14	17.33	20.70	17.25	19.89	16.85	19.02			
	0.28	0.17	0.22	0.09	0.07	0.28			
15	20.22	17.31	20.79	16.55	19.04	17.88	12.91		
	0.06	0.10	0.10	0.08	0.30	0.49	0.43		
16	20.44	19.19	18.79	16.80	16.10	11.76			
	0.34	0.32	0.35	0.35	0.39	0.43			
17	15.07	15.74	13.86	11.37					
	0.36	0.39	0.40	0.40					

FA BU
Rel.Uncer(%)

	J	K	L	M	N	P	R	S	L
9	0.89								
	0.22								
10	0.92	1.10							
	0.25	0.32							
11	1.16	0.95	1.18						
	0.31	0.24	0.27						
12	0.94	1.12	0.96	1.13					
	0.19	0.21	0.18	0.13					
13	1.10	0.94	1.17	0.97	1.20				
	0.15	0.18	0.13	0.18	0.08				
14	0.94	1.12	0.96	1.14	0.97	1.14			
	0.23	0.16	0.21	0.10	0.19	0.13			
15	1.15	0.97	1.19	0.95	1.10	1.07	0.77		
	0.13	0.21	0.10	0.18	0.12	0.24	0.38		
16	1.18	1.11	1.09	0.99	0.91	0.69			
	0.11	0.10	0.11	0.16	0.26	0.41			
17	0.83	0.86	0.78	0.66					
	0.20	0.22	0.26	0.43					

FA POWER
Rel.Uncer(%)

Commercial Core Calculation

▪ Rod worth

- **Group worth and its relative uncertainty are calculated.**
- **The BOC and EOC result.**

Group	BOC HFP, Eq. Xe	
	Group worth± 1σ (pcm)	Relative Uncertainty (%)
5	300±3.72	1.24
4	405±2.80	0.69
3	760±9.77	1.28
2	977±7.66	0.78
1	1436±26.51	1.84
A	4640±27.68	0.59
B	5195±22.29	0.42

Group	EOC HFP, Eq. Xe	
	Group worth± 1σ (pcm)	Relative Uncertainty (%)
5	352±2.55	0.72
4	459±2.82	0.61
3	774±4.86	0.62
2	1019±5.69	0.55
1	1425±13.36	0.93
A	4174±35.60	0.85
B	4549±59.62	1.3

Conclusion and Further work

- **This paper try to calculate many kinds of LWR core property's uncertainty by using Random sampling method with STREAM/RAST-K core analysis tools.**

- **The core properties' uncertainty is mainly because of heavy metal and fission product that have high uncertainty during core operation.**

- **χ covariance data that largely affect to core uncertainty should be considered.**

- **There are different covariance data by library.(ENDF/B-VII.1, ENDF/B-VIII.0, SCALE, JENDL). By using each library covariance data, uncertainty will be calculated.**

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