Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 17-18, 2018

Sensitivity and Uncertainty Analysis on TMI-1 PWR Pin Cell with STREAM

Woonghee Lee, Sooyoung Choi, Jiankai Yu and Deokjung Lee*
Department of Nuclear Engineering, Ulsan National Institute of Science and Technology
50 UNIST-gil, Ulsan, 44919, Republic of Korea
**Corresponding author: deokjung@unist.ac.kr*

1. Introduction

The sensitivity and uncertainty (S&U) analysis is becoming a promising area in nuclear engineering. Recently, the eigenvalue and nuclear data uncertainty analysis capability based on the generalized perturbation theory (GPT) has been implemented in the deterministic code STREAM (Steady state and Transient REactor Analysis code with Method of Characteristics) developed at UNIST.

In this paper, we present the sensitivity and uncertainty calculation of TMI-1 PWR pin cell in LWR UAM benchmark[1] in STREAM. The results of STREAM calculation are compared to TSUNAMI-2D calculation results in SCALE6.2.

2. Method and Result

The generalized perturbation theory which is applied in STREAM and the implicit effect of the multigroup perturbation theory is introduced in section 2.1 and section 2.2, respectively. The sensitivity and uncertainty result of STREAM and TSUNAMI-2D calculation is shown in the section 2.3.

2.1. Generalized Perturbation Theory

The Boltzmann transport equation can be expressed as Eq. (1).

$$(A-\lambda B)\Phi = 0\,, \tag{1}$$

where

Φ = neutron flux,

λ = eigenvalue of the equation,

A = operator that represents all of the transport equation except for fission source operator,

and

B = fission source operator.

The general response R is expressed as Eq. (2).

$$R = \frac{<H_N,\psi>}{<H_D,\psi>}, \tag{2}$$

where

ψ = angular flux,

H_N = the response of interest in numerator of response R,

H_D = the response of interest in denominator of response R,

and

$<\ ,\ >$ = integration over volume, energy and direction.

Eq. (1) is called a forward transport equation. To obtain the sensitivity coefficient of response R, another form of Eq. (1) which is defined as Eq. (3) is solved.

$$(A^* - \lambda B^*)\Gamma^* = \frac{H_N}{<H_N,\psi>} - \frac{H_D}{<H_D,\psi>}, \tag{3}$$

where

A^* = adjoint of operator A,

B^* = adjoint of operator B,

and

Γ^* = generalized adjoint flux.

Eq. (3) is called a generalized adjoint equation. The solution of Eq. (3), the generalized adjoint flux, is needed to calculate the sensitivity coefficient of response R to cross-section data X. This is expressed as in Eq. (4).

$$S_{R,X} = X\frac{<\frac{\delta H_N}{\delta X},\psi>}{<H_N,\psi>} - X\frac{<\frac{\delta H_D}{\delta X},\psi>}{<H_D,\psi>} - X<\Gamma^*,(\frac{\delta A}{\delta X} - \lambda\frac{\delta B}{\delta X})\psi>. \tag{4}$$

Finally, the total uncertainty of response R can be obtained using Eq. (5).

$$u_R^2 = \sum_{X_1}\sum_{X_2} S_{R,X_1} C_{X_1,X_2} S_{R,X_2}\ . \tag{5}$$

2.2. Implicit Sensitivity Coefficient

In order to obtain the sensitivity coefficients for the energy range in which resonance self-shielding is important, we have to consider the “implicit effect” of perturbations[2]. In case of k-effective, the complete sensitivity coefficient can be computed from Eq. (6) using chain rule.

$$S_{K_{eff},X_I} = (S_{K_{eff},X_I})_{GPT} + \sum_y \sum_j [(S_{k_{eff},Y_J})_{GPT} \cdot S_{Y_J,X_I}]$$
$$= (S_{K_{eff},X_I})_{GPT} + \sum_y \sum_j [(S_{k_{eff},Y_J})_{GPT} \cdot S_{Y_J,N_I} \cdot S_{N_I,T_I} \cdot S_{T_I,X_I}]$$
$$= (S_{K_{eff},X_I})_{GPT} + \sum_y \sum_j [(S_{k_{eff},Y_J})_{GPT} \cdot S_{Y_J,N_I} \cdot S_{T_I,X_I}], \tag{6}$$

where

Y_J =cross-section data of isotope J which is sensitive to perturbations in X_I,

T_I = total macroscopic cross-section of isotope I, and

N_I = material number density of isotope I.

The second term on the right hand side of Eq. (6) is called an implicit sensitivity. S_{T_I,X_I} term can be computed analytically and S_{Y_J,N_I} term can be computed by the direct perturbation of material number density of isotope I.

2.3. Numerical Results

Table I shows the forward transport calculation result for k_{eff} on TMI-1 PWR pin cell at hot zero power condition.

Table I: k_{eff} in TMI-1 pin cell at hot zero power

Code	TSUNAMI-2D	STREAM	Diff. (pcm)
k_{eff}	1.43089	1.43216	-127

Both calculations were performed using the ENDF/B-VII.1 cross-section library. The number of energy group for SCALE6.2 and STREAM is 252 and 72, respectively. The k-effective difference between the two codes is 127 pcm.

Table II: Sensitivity coefficients of k_{eff}

Isotope - reaction pair	TSUNAMI -2D (GPT)	STREAM (GPT)	STREAM (Direct Perturbation*)
U-235 ($\bar{\nu}$)	9.399E-01	9.403E-01	9.402E-01
U-235 (fission)	2.528E-01	2.550E-01	2.529E-01
U-238 (n, γ)	-2.101E-01	-2.612E-01	-2.605E-01
H-1 (n, n)	1.805E-01	1.939E-01	1.915E-01
U-235 (n, γ)	-1.538E-01	-1.542E-01	-1.540E-01
U-238 ($\bar{\nu}$)	6.013E-02	5.965E-02	5.978E-02
H-1 (n, γ)	-3.902E-02	-3.859E-02	-3.857E-02
U-238 (fission)	2.842E-02	2.818E-02	2.830E-02

* +1% direct cross-section perturbation after resonance treatment

Table II shows the energy, region and mixture integrated sensitivity coefficients of k_{eff} in the TMI-1 pin cell calculation.

Table III shows the total uncertainty of k_{eff} and the five most significant contributions to uncertainty. The uncertainty calculation in STREAM is computed using 72-group covariance matrices generated from ENDF/B-VII.1 with NJOY.

Table III: Uncertainty information of k_{eff}

Isotope -Reaction pair	TSUNAMI -2D (% Δk/k)	STREAM (% Δk/k)	Diff. (% Δk/k)
U-235($\bar{\nu}$) - U-235($\bar{\nu}$)	0.3411	0.6068	-0.2657
U-238(n, γ) - U-238(n, γ)	0.2755	0.2898	-0.0143
U-235(n, γ) - U-235(n, γ)	0.1963	0.1964	-0.0001
U-235(χ) - U-235(χ)	0.1525	0.1515	-0.0010
U-238(n, n') - U-238(n, n')	0.1157	0.1156	0.0001
Total uncertainty	0.5429	0.7467	-0.2038

The contributions of U-235($\bar{\nu}$) - U-235($\bar{\nu}$) are quite different between the two codes. This is because the covariance matrices of nu-bar for Uranium-235 are different. The covariance data from SCALE6.2 is assembled from a variety of sources including JENDL-4.0 [3].

Figures 1 and 2 shows the covariance matrix of U-235($\bar{\nu}$) - U-235($\bar{\nu}$) which is used in STREAM and SCALE calculations, respectively.

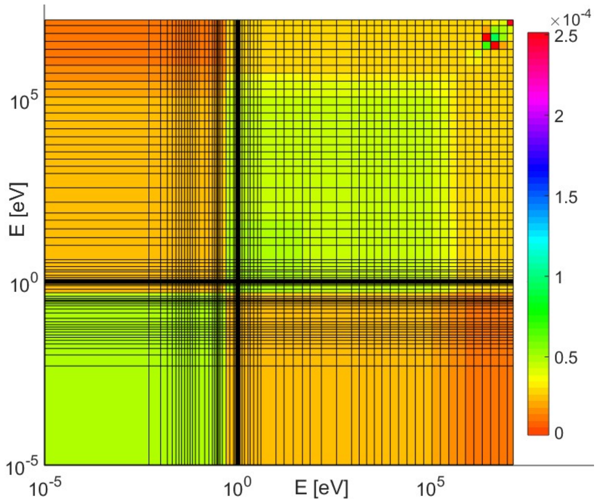

Fig. 1. Covariance matrix for U-235($\bar{\nu}$) - U-235($\bar{\nu}$) by ENDF/B-VII.1/NJOY

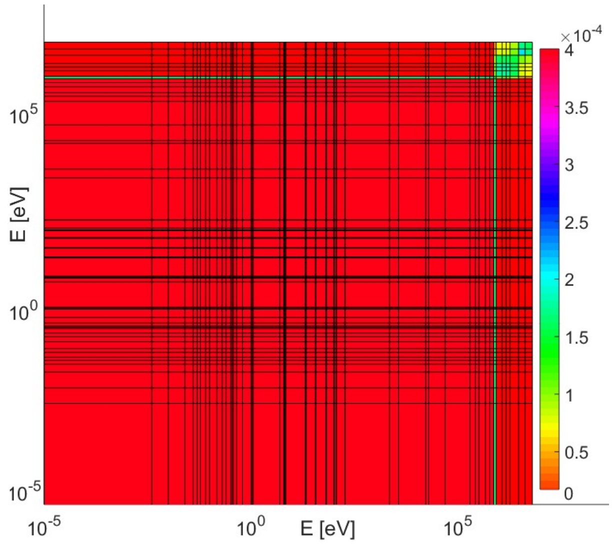

Fig. 2. Covariance matrix for U-235($\bar{\nu}$) - U-235($\bar{\nu}$) by SCALE6.2

We can also compute the uncertainty of general response R using GPT. Table IV shows the forward transport calculation result for response $R = \frac{<\Sigma_f^{235U}, \psi>}{<\Sigma_f^{238U}, \psi>}$ on TMI-1 PWR pin cell at hot zero power condition.

Table IV: Response R in TMI-1 pin cell at hot zero power condition

Code	TSUNAMI-2D	STREAM	Diff.
R	17.855	17.997	-0.142

The relative difference of response R between the two codes is -0.795%.

Table V: Sensitivity coefficients of response R

Isotope - reaction pair	TSUNAMI -2D (GPT)	STREAM (GPT)	STREAM (Direct Perturbation)
U-238 (fission)	-9.729E-01	-9.722E-01	-9.664E-01
H-1 (n, n)	7.178E-01	7.569E-01	7.504E-01
U-235 (fission)	2.720E-01	2.741E-01	2.750E-01
U-238 (n, γ)	-2.209E-01	-2.752E-01	-2.746E-01
U-238 (n, n')	2.219E-01	2.176E-01	2.154E-01
U-235 (n, γ)	-1.635E-01	-1.639E-01	-1.638E-01
O-16 (n, n)	8.145E-02	8.480E-02	8.064E-02
H-1 (n, γ)	-4.152E-01	-4.104E-02	-4.105E-02

Table V shows the energy, region and mixture integrated sensitivity coefficients of response R in the TMI-1 pin cell calculation. Table VI shows the total uncertainty of response R and the five most significant contributions to uncertainty.

Table VI: Uncertainty information of response R

Isotope - Reaction pair	TSUNAMI -2D (% ΔR/R)	STREAM (% ΔR/R)	Diff. (%ΔR/R)
U-235(χ) - U-235(χ)	4.3154	4.2306	0.0848
U-238(n, n') - U-238(n, n')	4.2120	4.1714	0.0406
U-238(χ) - U-238(χ)	0.6700	0.6519	0.0181
U-238(fiss.) - U-238(fiss.)	0.5075	0.5077	-0.0002
H-1(n, n) - H-1(n, n)	0.4327	0.9865	-0.5538
Total uncertainty	6.1127	6.0922	-0.0205

The large difference of H-1(n, n) - H-1(n, n) contribution between the two codes comes from the covariance data difference mentioned in a previous paragraph.

The implicit sensitivity coefficients for k_{eff} is also computed in STREAM. STREAM performs N resonance treatment calculations (N = number of isotopes) and perturbs the material number density of 1 isotope for 1 calculation. Table VII shows the sensitivity coefficient of k_{eff} to the total macroscopic cross-section of U-238 with two types of direct perturbation and GPT/DP implicit effect.

Table VII: Sensitivity coefficient of k_{eff} to one-group total macroscopic cross-section of U-238

Method	Sensitivity
Direct perturbation (explicit)*	-2.3664E-01
Direct perturbation (complete)**	-1.6924E-01
GPT (explicit)	-2.3597E-01
DP (Implicit)	6.7201E-02
GPT(explicit) + DP(implicit)	-1.6879E-01

* +1% direct cross-section perturbation after resonance treatment
** +1% direct perturbation of input number density

The relative sensitivity difference between direct perturbation (explicit) and direct perturbation (complete) is 28.48%. The portion of the implicit effect is quite large. Thus, the implicit sensitivity should be considered in this case. The relative sensitivity difference between direct perturbation (complete) and GPT/DP is only -0.267%.

3. Conclusion

Sensitivity analysis and uncertainty quantification capability based on generalized perturbation theory has been implemented in the deterministic code STREAM developed at UNIST. STREAM gives quite reasonable uncertainties compared to the SCALE6.2/TSUNAMI-2D for TMI-1 pin cell result. The only discrepancy observed is due to differences in the covariance data.

In order to consider implicit effect of resonance self-shielding for multigroup cross-sections, implicit sensitivity coefficient of K_{eff} is computed in STREAM. The method of resonance treatment used is the equivalence theory during the calculation of implicit sensitivity coefficients. For future work, it is necessary to calculate implicit sensitivities of general response R with advanced resonance treatment methods.

Acknowlegment

This research was supported by the project (L17S018000) by Korea Hydro & Nuclear Power Co. Ltd.

REFERENCES

[1] K. Ivanov, M. Avramova, S. Kamerow, I. Kodeli, E. Sartori, E. Ivanov, O. Cabellos, "Benchmarks for Uncertainty Analysis in Modelling (UAM) for the Design, Operation and Safety Analysis of LWRs. Volume I: Specification and Support Data for Neutronics Cases (Phase I)", NEA/NSC/DOC(2013)7.

[2] M. L. Williams, B. L. Broadhead, and C. V. Parks, “Eigenvalue Sensitivity Theory for Resonance Shielded Cross Sections,” Nucl. Sci. Eng. 138, 17–191 (2001).

[3] B. T. Rearden and M.A. Jessee, Eds., “SCALE Code System”, ORNL/TM-2005/39, Version 6.2.2, Oak Ridge National Laboratory, Oak Ridge, Tennessee (2017).