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Research Paper

NON-EXCITING WAKEFIELD STRUCTURED BUNCHES IN A ONE-DIMENSIONAL PLASMA MODEL

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Abstract

A model of one-dimensional (1D) cold plasma with an external train of rigidly structured bunches with diverse charges has been introduced. In this model, a solution is presented that cancels the wakefield after the train is found. The density of such bunches can be much greater than the density of the plasma, and a high amplitude electrical field arising inside the train can be used for charged-particle acceleration. In addition, analytical and numerical simulations have been performed.

Keywords: one-dimensional plasma model, wakefield, charged bunches.

1. Introduction

Using plasma as a medium to slow the propagation of electromagnetic waves, when it is possible to accelerate charged particles in this wave, has existed as a method for some time [1, 2]. However, such waves are often nonlinear and unstable [3–5].

These waves can be stabilized by binding them to the leading intense beam of charged particles. Waves behind such beams are termed wakefield and acceleration in such wake waves has been considered in other research (see [6, 7]). Plasma waves can be excited using charged-particle beams (beam-driven plasma wakefield acceleration [PWFA]), as well as by laser radiation (laser wakefield acceleration) [8]. Moreover, used charged particles can vary (electrons, positron, or proton beams [9–11]). The behavior of wake waves strongly depends on the relationship between the beam and plasma densities. When $n_b>n_0/2$, it is termed an underdense regime, and when $n_b<n_0/2$, it is considered an overdense regime [12] (density of the plasma n_0 compared with the doubled density of beam n_b).

The effect of the breakdown wake waves is noted when the density of the driving beam, n_b, exceeds half of the plasma density, and the plasma dynamic becomes highly nonlinear.

For justifiable experimental realization (see notes in Sec. Perturbation of the bunch density), the use of proton beams (Proton Driven PWFA [PD-PWFA]) is suitable. In 2016, In 2016, a proof-of-principle Advanced Proton Driven Plasma Wakefield Acceleration Experiment (AWAKE) was initiated at CERN aimed at investigating the use of plasma wakefields driven by a proton bunch to accelerate charged particles [13]. This acceleration technique could lead to future colliders of high energy, though of a much-reduced length when compared with the proposed linear accelerators. Recently, the first experimental works of this collaboration have appeared [14, 15].

Often, for complex physical systems, it is important to look for the exactly solvable models. The

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cold, uniform, and collision-less one-dimensional (1D) plasma model with fixed ions [16–18] was proposed as such a model in which an exact analytical solution of excitation of nonlinear oscillations in plasma using a finite electron beam was obtained (see [19, 20]). In such an 1D model, the wakefield breakdown effect is observed with the underdense plasma limit mentioned above. Moreover, this breakdown effect occurs at a certain distance from the driver bunch.

In this paper, we consider the structure of bunches with different densities, electrical charges, and lengths. A breakdown does not occur inside such a structure, and beyond the structure, the wakefield cancels at certain ratios between bunches lengths and densities. In this case, the bunch densities can be much higher than the plasma density (underdense mode), and the corresponding fields can be much larger than the breakdown limit for a separate bunch.

2. One-dimensional plasma model

Below, we derivate the main equations for cold, uniform, and collision-less 1D plasma models with fixed ions and rigid external beams. We assume that the beam propagates along the model axis, denoted as *z*.

Poisson's equation (1D model magnetic field components and electric field transversal components are omitted):

$$\frac{\partial^2\varphi}{\partial z^2} = -4\pi e(n_e + \frac{q}{e}n_b - n_0)\,, \tag{1}$$

the continuity equation for plasma electrons:

$$\frac{\partial n_e}{c\partial t} + \frac{\partial(\beta_e n_e)}{\partial z} = 0\,, \tag{2}$$

and the plasma electrons motion equation:

$$\frac{\partial(\beta_e\gamma_e)}{c\partial t} + \beta_e\frac{\partial(\beta_e\gamma_e)}{\partial z} = -\frac{e}{mc^2}\frac{\partial\varphi}{\partial z}\,, \tag{3}$$

Where n_e is the density of plasma electrons, n_b is the density of bunch particles with charge q, n_0 is the density of immovable plasma ions, φ is the potential of the electric field produced by all charged components, $\beta_e c$ is the velocity of the plasma electrons ($\gamma_e = (1-\beta_e^2)^{-1/2}$), and m_e is the electron mass. *c* indicates the speed of light, *e* is the electron charge (in contrast to [21], where the charge of the electron was noted as –*e*, so that *e* means the absolute [positive] value of the electron charge).

We look for steady-state solutions of Eqs. (1–3), which means that we assume that a bunch does not change its velocity $\beta_0 c$ (accordingly, we introduce $\gamma_0 = (1-\beta_0^2)^{-1/2}$) during this propagation in the plasma. This means that variables β_e, n_e, and φ are functions of a single variable

$$\hat{z} = z - \beta_0 ct\,. \tag{4}$$

Introducing (4) into Eqs. (1–3) we obtain:

$$\frac{\partial^2\varphi}{\partial\hat{z}^2} = -4\pi e(n_e + \frac{q}{e}n_b - n_0), \tag{5}$$

$$-\beta_0\frac{\partial n_e}{\partial\hat{z}} + \frac{\partial(\beta_e n_e)}{\partial\hat{z}} = 0, \tag{6}$$

$$(\beta_e - \beta_0)\frac{\partial(\beta_e\gamma_e)}{\partial\hat{z}} = -\frac{e}{mc^2}\frac{\partial\varphi}{\partial\hat{z}}. \tag{7}$$

We assume that the bunch is placed on the negative half axis of $\hat{z}$, and the front of the bunch corresponds to $\hat{z} = 0$.

Equation (6) integrates in form:

$$n_e(\beta_0 - \beta_e) = n_0\beta_0. \tag{8}$$

The integration constant is chosen to fulfill condition $n_e = n_0$, if $\beta_e = 0$.

It should be noted that, in determining n_0 as the density of immovable ions of plasma, such as the density of plasma atoms, we suppose that plasma is hydrogen-like. This restriction can be eliminated if, for multi-charged ions, the density n_0 is understood as the density of atoms multiplied by the number of electrons ionized from the atom.

Using relation $\partial\gamma_e/\partial\hat{z} = \beta_e\gamma_e^3\partial\beta_e/\partial\hat{z}$ one can integrate (7) in the following form:

$$\frac{1-\beta_0\beta_e}{\sqrt{1-\beta_e^2}} = 1 - \frac{e}{mc^2}\varphi \equiv \Phi\,. \tag{9}$$

The integration constant is chosen to fulfill $\varphi = 0$ if $\beta_e = 0$.

From (8), we have a restriction on the values of plasma electron's velocity

$$\beta_e < \beta_0 \tag{10}$$

and correspondingly,

$$\gamma_0^{-1} < \Phi \tag{11}$$

at the condition $\Phi > 1$ corresponding to the negative values of β_e.

Equation (9) can be rewritten as:

$$\beta_e = \frac{\beta_0 - \Phi\sqrt{\Phi^2 - \gamma_0^{-2}}}{\Phi^2 + \beta_0^2}. \quad (12)$$

Taking (8) into account, one can also find that

$$n_e = -n_0\left(\beta_0^2\gamma_0^2 - \frac{\beta_0\gamma_0^2\Phi}{\sqrt{\Phi^2 - \gamma_0^{-2}}}\right). \quad (13)$$

One can see that at a limit value of $\Phi = \gamma_0^{-1}$, which is the plasma electron density trend to infinity.

Using (12, 13), we finally obtain only one second order equation on potential Φ:

$$\frac{\partial^2\Phi}{\partial\xi^2} = \alpha - \gamma_0^2 + \frac{\beta_0\gamma_0^2\Phi}{\sqrt{\Phi^2 - \gamma_0^{-2}}}. \quad (14)$$

Where we enter dimensionless variables

$\xi = \hat{z}\omega_p / c, \ \alpha = (q/e)(n_b/n_0),$

where $\omega_p = \sqrt{4\pi e^2 n_0 / m}$.

The basic equation (14) of cold, uniform, and collision-less 1D plasma model coincides with the same equation derived in [20] (Appendix 1).

In the common case α, it depends on the parameter ξ. In case $\alpha = const$, (14) has energy integral

$$\varepsilon = \Phi'^2/2 + \gamma_0^2\left(\Phi - \beta_0\sqrt{\Phi^2 - \gamma_0^{-2}}\right) - \alpha\Phi \quad (15)$$

(in the following prime, we denote differentiation on ξ).

Here, we can separate the potential portion of energy:

$$U_\alpha = \gamma_0^2\left(\Phi - \beta_0\sqrt{\Phi^2 - \gamma_0^{-2}}\right) - \alpha\Phi \quad (16)$$

Thus, (14) describes the motion of the particle with coordinate Φ and velocity Φ' in potential with U_α. In Figure 1, profiles of U_α for different values of α are presented where negative α corresponds to proton/positron bunches.

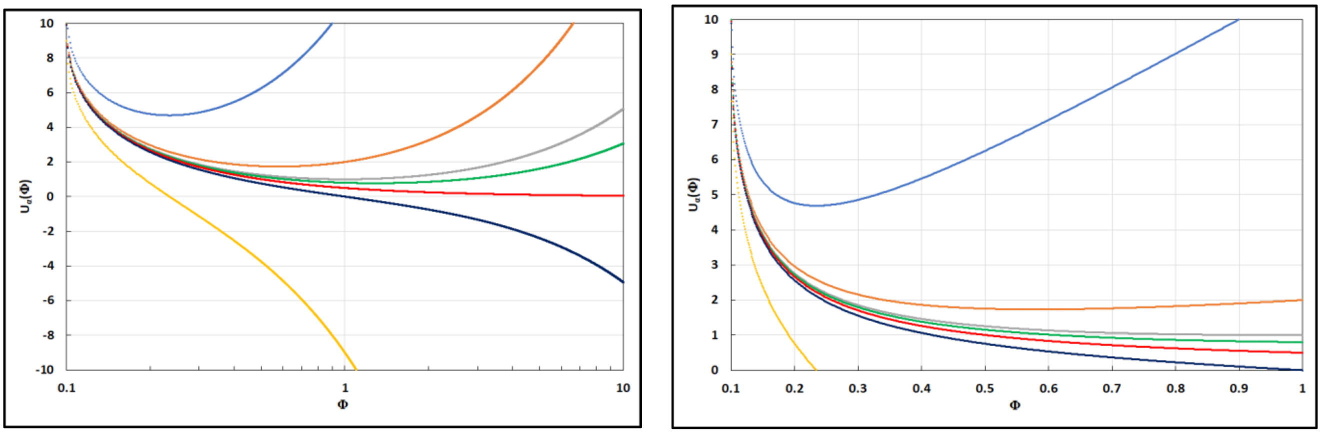

Fig. 1. Potential wells, U_α, for $\gamma_0 = 10$ and different values of α in different scales of the axis Φ are presented. Curves from top to bottom correspond to the following values of parameter α: −10, −1, 0, 0.2, $1/(1+\beta_0) = 0.501$, 1, and 10. The curve of the critical value $\alpha = 1/(1+\beta_0)$ separates the type of curves depicted in red.

One can see that curves U_α have the shape of the real well if

$$\alpha < 1/(1+\beta_0). \quad (17)$$

The front of the bunch, as we noted, should be used as initial conditions $\Phi = 1, \Phi' = 0$. These values correspond to the potential energy where $U_\alpha(1) = 1 - \alpha$. The potential energy at the limit value of $\Phi = \gamma_0^{-1}$ is equal to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$.

It is interesting to note the minimal potential energy (in the case of [17]):

$$U_\alpha^{\min} = \sqrt{(1-\alpha)^2 - \alpha^2\beta_0^2}. \quad (18)$$

Which reaches at

$$\Phi^* = \frac{1 - \alpha\gamma_0^{-2}}{\sqrt{(1-\alpha)^2 - \alpha^2\beta_0^2}}. \quad (19)$$

As an example, let us investigate the motion of the phase point for the value of γ_0=10 in two cases of a parameter α=0.45 (Figure 2) and α=4 (Figure 3) for different values of energy. In Figure 2, we point out the energy level 9.955, as it corresponds to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$. At lower energies, this value phase point can infinitely move well in potential, forming the structure of periodic wake waves (we use this term to emphasize the periodical character of wakefields). At the values above, the phase point at some time (parameter ξ) will obligatorily reach the value γ_0^{-1} below, where the model becomes nonphysical. The corresponding curve in (Φ, Φ') we call separatrix.

Interestingly, α=0, thereby indicating plasma without bunch (see Figure 4).

One can see that there are wake wave solutions only for energies $\varepsilon < 10$, and energy $\varepsilon = 1$ corresponds to the bottom of a potential well ($\Phi^* = 1$). These exact values are what we use as initials for the point of penetration of the bunch into the plasma.

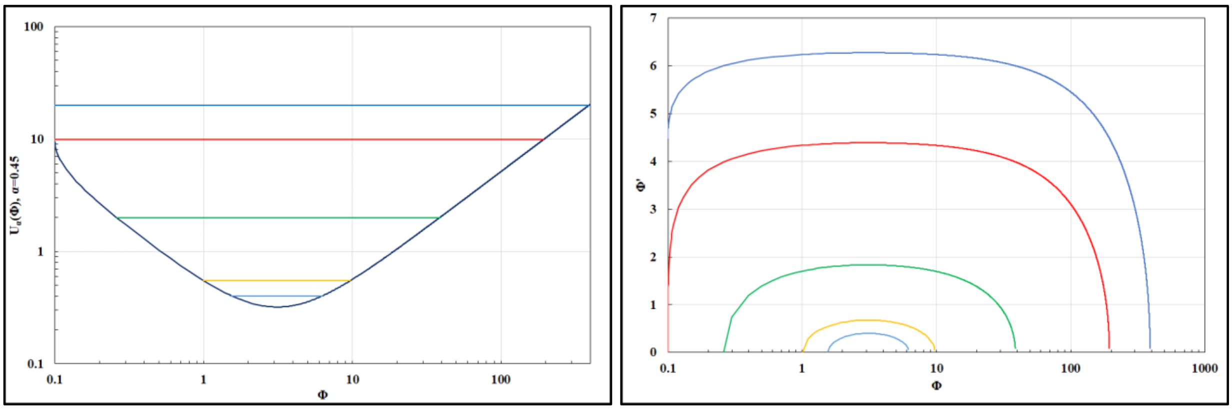

Fig. 2. Phase point motion in case of the parameter $\alpha = 0.45$, left: potential well, right: phase trajectories at different energies. Energy from top to bottom: 20, 9.955, 2, 0.55, and 0.4. Energy 9.955 corresponds to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$ (marked by red). The energy 0.55 corresponds to $U_\alpha(1) = 1 - \alpha$ (marked by yellow).

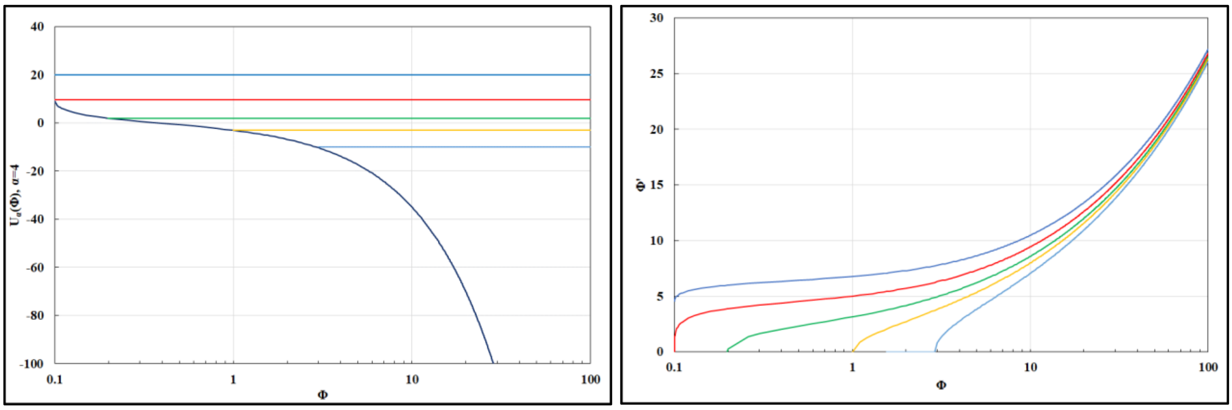

Fig. 3. Phase point motion in case of the parameter $\alpha = 4$, left: potential well, right: phase trajectories at different energies. Energy from top to bottom: 20, 9.6, 0, −3, and −10. Energy 9.6 corresponds to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$ (marked by red). Energy − 3 corresponds to $U_\alpha(1) = 1 - \alpha$ (marked by yellow)

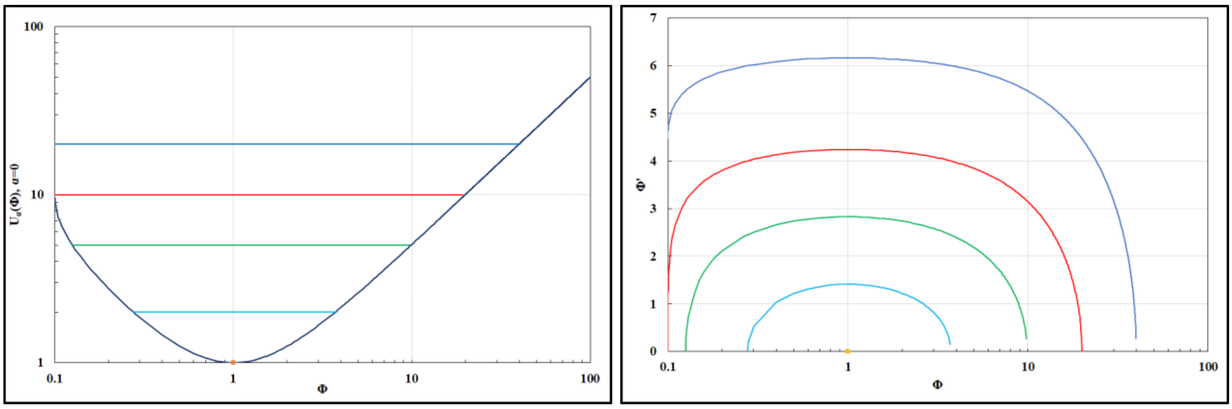

Fig. 4. Phase point motion in case of the parameter $\alpha = 0$, left: potential well, right: phase trajectories at different energies. Energy from top to bottom: 20, 10, 5, 2, and 1. Energy 10 corresponds to $U_{\alpha=0}(\gamma_0^{-1}) = \gamma_0$ (marked by red). Energy 1 corresponds to $U_\alpha(1) = 1$ (marked by yellow)

3. Wakefield of structured bunches

For the bunch of piecewise constant functions of density, one can construct the motion of this particle using motion in potential well formalism beginning with the front bunch and initial conditions Φ=1 and Φ'=0.

Let us consider that a structured bunch consists of N sub-bunches, each of a charge density α_i and a length of d_i (i varies from 1 to N). The wakefield of this structured bunch, and later, can be constructed as a successive movement of the phase point in the corresponding set of potential wells. The transition from one well to another is formed using the initial values of (Φ, Φ') formed at the exit of the previous well. The constructed solution should be physical if, during the motion in each well condition $\Phi > \gamma_0^{-1}$ is not violated and, at the exit of the bunch energy of the phase point, it does not exceed value γ_0 idea was first proposed in [22, 23]).

Notably, this caused the presence of a term $-\alpha\Phi$ in the formula (15) transition of the phase point from one well into another, accompanied by the particle energy jump $-(\alpha_{i+1} - \alpha_i)\Phi$. Therefore, here we mean the energy of the phase point in the free plasma.

Figure 5 shows an example of such calculations for a bunch consisting of three sub-bunches with parameter α: 10, −10, 5, and corresponding lengths of sub-bunches: 15, 20, and 15. At the entrance of the first bunch, the phase point had initial energy of ε=1 at the entrance to the second bunch ε=10712, at the entrance to the third bunch, ε=3183, and finally, at the entrance to the free plasma after the bunch, ε=662. The last value goes beyond the limits of the range of values of the existence of stable wake waves and leads to the breakdown of the wake wave at a distance of 72.83 behind the bunch.

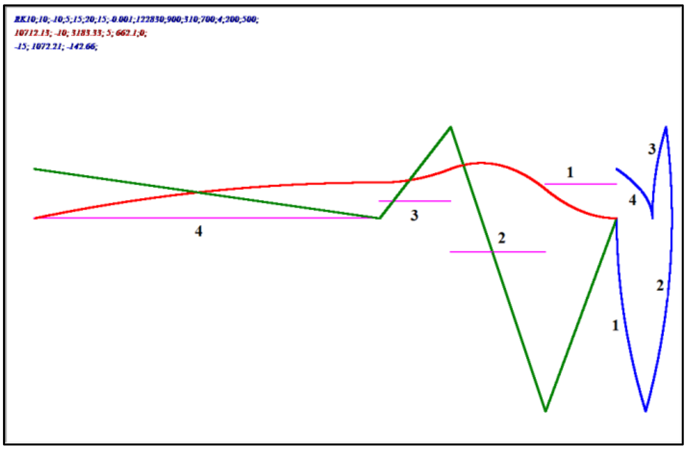

Fig. 5. Wakefield of the structured bunch consists of three sub-bunches with the following parameters: α_1=10, d_1=15, α_2=−10, d_2=20, α_3=5, and d_3=15

Wakefield of the structured bunch (fig. 5) consists of three sub-bunches with the following parameters: $\alpha_1 = 10$, $d_1 = 15$, $\alpha_2 = -10$, $d_2 = 20$, $\alpha_3 = 5$, and $d_3 = 15$. Sub-bunches are marked in magenta and denoted as 1, 2, and 3. 4 is denoted by free plasma behind the bunch. The bunch movement direction is from left to right. In the marked red function, Φ, depending on ξ is presented. In the marked green function, Φ' depending on ξ is presented. The space right from the front point of the bunch is used to present the phase point trajectory marked in blue (axis Φ is a horizontal direction to the right, and the axis Φ' is vertically directed up). Numbers correspond to the presented bunch structure. All values are presented in arbitrary units.

The distance behind the bunch, after which the breaking of the wake wave occurs, depends on the initial values (Φ, Φ') and can be quite significant, as shown in Figure 6. Here the phase point enters the free plasma with an energy of 3765. Then, the phase point moves to the right of the potential well boundary depicted in Figure 4*a*, and then is reflected from it and continues to move in the direction of the breakdown limit. The whole process of the wake wave breakdown occurs at a distance of 322 dimensionless units.

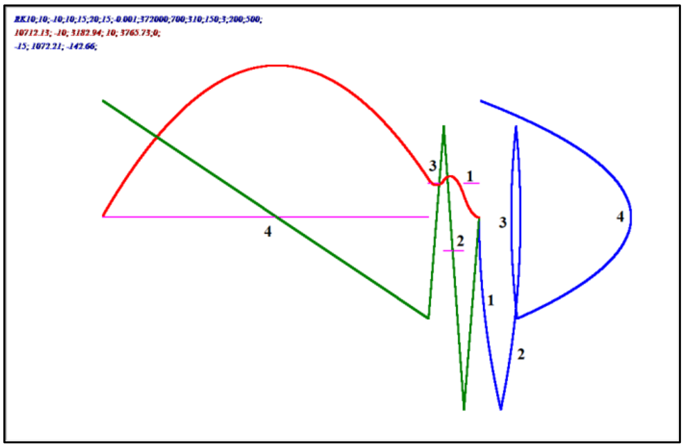

Fig. 6. Wakefield of structured bunch consists of three sub-bunches with the following parameters: α_1=10, d_1=15, α_2=−10, d_2=20, α_3=10, and other notifications are the same as in Figure 5

By the selection of parameters $\alpha_1, d_1, \alpha_2, d_2, \alpha_3$, and d_3, it is possible to obtain a situation where the wake behind the structured bunch forms stable wake waves. In Figure 7, the practically perfect entrance of the phase point after the bunch into the free plasma is presented (entrance energy is 1.04, and the density of the second bunch is −0.997). For estimation, the rigidity of the conditions in accordance with the parameters of ideal conditions, we present Figure 8, where the entrance energy is 8.49, and the density of the second bunch is −0.979.

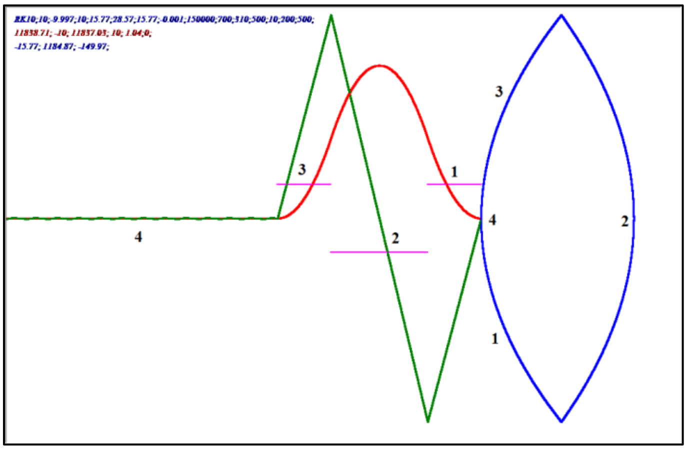

Fig. 7. The ideal entrance of the phase point into free plasma behind the structure of the bunches.

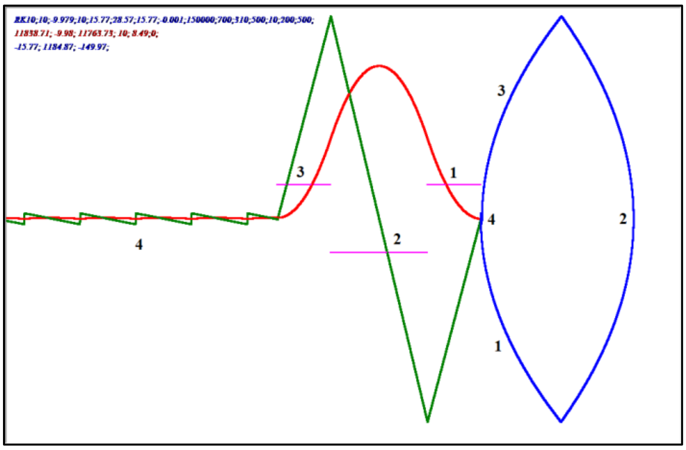

Fig. 8. Entrance into free plasma on the level near the plasma breakdown.

Notably, the role of the second bunch (tilting a wave after the first bunch) can be played by free plasma (see Figure 9).

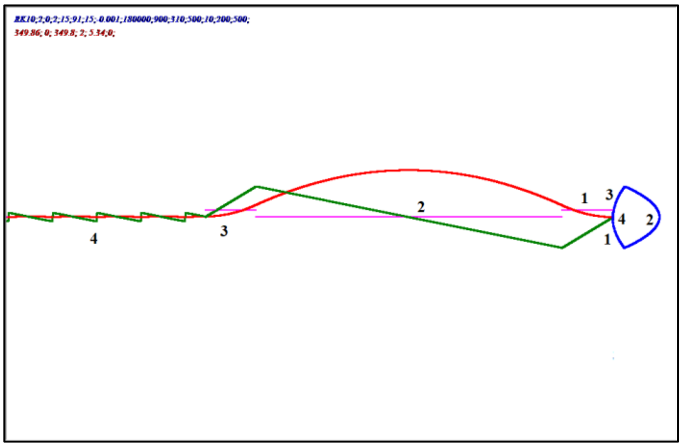

Fig. 9. The second bunch can be used as free plasma.

4. Perturbation of the bunch density

The above model is limited, as it concerns the assumption that the leading beam is not perturbed when moving in plasma. Moreover, this assumption is applicable to proton (inert) beams and is not suitable for electron bunches.

Amatuni [24] introduces two-time scales: the rise time of the waves in the plasma $\tau_e \approx \omega_p^{-1}$ and the characteristic time of the change in the momentum $\tau_b \approx p_b / qE$ of the beam particles, where E is the value of an electric field of a wakefield, p_b is beam particles momentum, and q is the beam particles charge. According to (9):

$$E = -\frac{mc^2}{e}\frac{\omega_p}{c}\Phi'. \qquad (20)$$

Some estimates of these times for various plasma densities are given in Table 1 for electron and proton beams. The Lorentz factor of the beam charges, γ_0, is set equal to 10. The values of the electric field are taken from the 1D model calculations.

Table 1. Values of characteristic time parameters (plasma parameters are chosen near to AWAKE values).

n_e, 1/cm^3	ω_p, s^{-1}	τ_e, s	λ_p, cm	Φ'_{MAX}	E_{MAX}, eV/cm	τ_b (electrons), s	τ_b (protons), s
2.1E + 14	8.2E + 11	1.2E − 12	2.3E − 01	100	1.4E + 09	1.2E − 13	2.2E − 10
7.7E + 14	1.6E + 12	6.4E − 13	1.2E − 01	100	2.7E + 09	6.4E − 14	1.2E − 10
1.0E + 15	1.8E + 12	5.6E − 13	1.1E − 01	100	3.0E + 09	5.6E − 14	1.0E − 10
2.1E + 14	8.2E + 11	1.2E − 12	2.3E − 01	100	1.4E + 09	1.2E − 13	2.2E − 10

Where $\lambda_p = 2\pi c / \omega_p$ is the length of the plasma waves. One can see that, for all case conditions, $\tau_e \ll \tau_b$ is valid only for the case of a proton beam. This signifies that the assumption concerning rigidity of the beam can be applied more or less for protons beam. H^- can be considered as a candidate for a heavy particle beam with negative charge hydrogen ions.

For the following calculations, we present a system of equations for the model in which the plasma remains cold. However, the assumption of beam rigidity is absent.

The equation of the motion of plasma electrons is as follows:

$$\frac{\partial(\beta_e\gamma_e)}{c\partial t}+\frac{\partial(\beta_e\gamma_e)}{\partial z}=-\frac{e}{mc^2}\frac{\partial\varphi}{\partial z}, \quad (21)$$

and bunch charges:

$$\frac{\partial(\beta_b\gamma_b)}{c\partial t}+\frac{\partial(\beta_b\gamma_b)}{\partial z}=-\frac{q}{mc^2}\frac{\partial\varphi}{\partial z}. \quad (22)$$

The continuity equation for plasma electrons and bunch charges is as follows:

$$\frac{\partial n_e}{c\partial t}+\frac{\partial(\beta_e n_e)}{\partial z}=0, \quad (23)$$

$$\frac{\partial n_b}{c\partial t}+\frac{\partial(\beta_b n_b)}{\partial z}=0 \quad (24)$$

and finally, the Poisson's equation:

$$\frac{\partial^2\varphi}{\partial z^2}=-4\pi e(n_e+\frac{q}{e}n_b-n_0). \quad (25)$$

Depriving the dimension of all equations, as above, using the following notation:

$$z=\frac{c}{\omega_p}\xi\,,\ t=\frac{1}{\omega_p}\tau\,,\ \varphi=\frac{mc^2}{e}(1-\Phi)\,,$$

$$n_e=\alpha_e n_0\,,\ n_b=(q/e)\alpha_b n_0. \quad (26)$$

We arrive at the complete system of equations:

$$\frac{\partial(\beta_e\gamma_e)}{\partial\tau}+\beta_e\frac{\partial(\beta_e\gamma_e)}{\partial\xi}=\frac{\partial\Phi}{\partial\xi}, \quad (27)$$

$$\frac{\partial(\beta_b\gamma_b)}{\partial\tau}+\beta_b\frac{\partial(\beta_b\gamma_b)}{\partial\xi}=\frac{q}{e}\frac{\partial\Phi}{\partial\xi}, \quad (28)$$

$$\frac{\partial\alpha_e}{\partial\tau}+\frac{\partial(\beta_e\alpha_e)}{\partial\xi}=0, \quad (29)$$

$$\frac{\partial\alpha_b}{\partial\tau}+\frac{\partial(\beta_b\alpha_b)}{\partial\xi}=0, \quad (30)$$

$$\frac{\partial^2\Phi}{\partial\xi^2}=\alpha_e+\alpha_b-1. \quad (31)$$

The system of equations is proposed to be solved as follows: a beam of charged particles limited in length enters into a semi-infinite homogeneous cold plasma. Such a statement allows us to solve the problem as an initial one with an alternate calculation of parameters of the plasma and beam electrons.

5. Conclusion

In a cold, uniform, and collision-less 1D plasma model with fixed ions and rigid external beams, a solution has been found when in space behind the last bunch of a train, consisting of three bunches, wakefields are not formed at all. In particular, the train can consist of two bunches separated by the corresponding length of free space. At the same time, the density of bunches can significantly exceed the limit of the characteristic density at which the wake wave overturns behind one bunch model. The electric field inside such a train can be used to accelerate charged particles.

Acknowledgments

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Appendix 1. One-dimensional plasma equation in [20] notation

In [20], the following dimensionless coordinate was introduced:

$$\tau=\omega_p(t-\frac{z}{\beta_0 c}) \quad (a1)$$

(we replaced β_{ph} of [20] by our notation β_0).

Basic equation of [20] (Eq. 3):

$$\frac{\partial^2}{\partial\tau^2}\left(\frac{1-\beta_0\beta_e}{\sqrt{1-\beta_e^2}}\right)=\beta_0^2\left(\frac{\beta_e}{\beta_0-\beta_e}+\alpha\right) \quad (a2)$$

(we replaced β of [20] by our notation β_e).

By usage of (a1), equation (a2) can be rewritten as:

$$\frac{\partial^2}{\partial\xi^2}\left(\frac{1-\beta_0\beta_e}{\sqrt{1-\beta_e^2}}\right)=\frac{\beta_e}{\beta_0-\beta_e}+\alpha. \quad (a3)$$

(here we use notation ξ of our equation (14))

It is easy to see that:

$$\Phi^2-\gamma_0^{-2}=\frac{(1-\beta_0\beta_e)^2}{1-\beta_e^2}-(1-\beta_0^2)=\frac{(\beta_0-\beta_e)^2}{1-\beta_e^2}. \quad (a4)$$

Introducing this into (a1), we find:

$$\frac{\partial^2\Phi}{\partial\xi^2}=\alpha+\frac{\beta_e}{\beta_0-\beta_e}. \quad (a5)$$

This equation is equal to the basic equation of [20].

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