Lecture 2

Basics of Magnets

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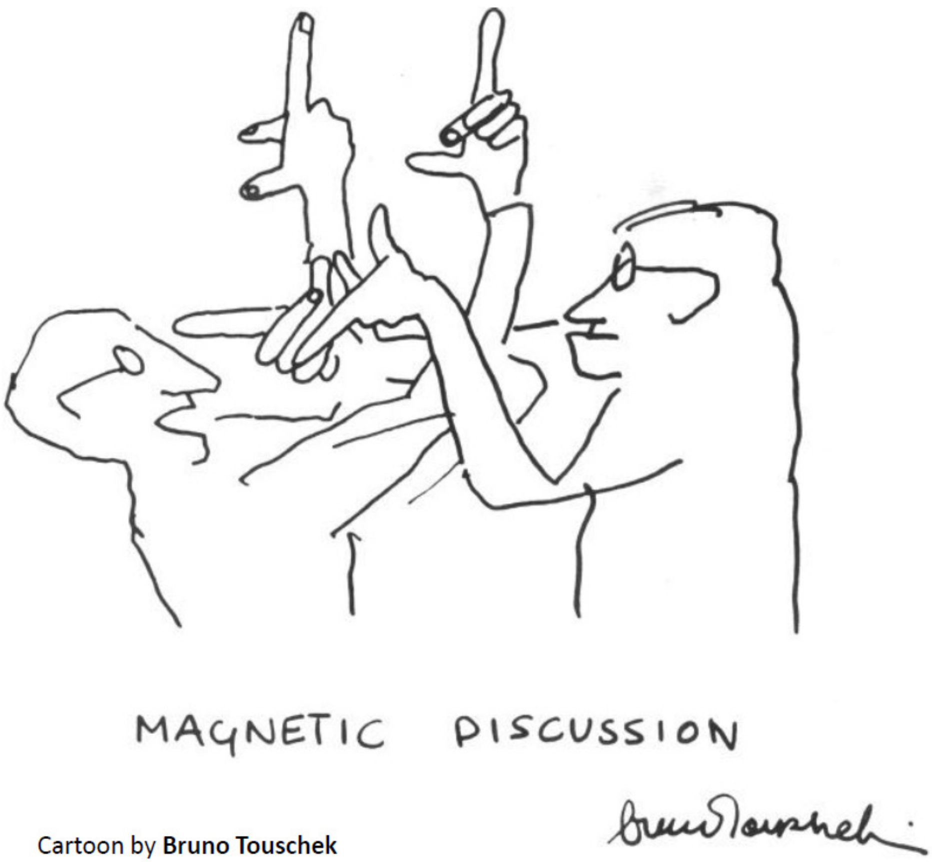
Lecture 2 Introduction

- In general, charged particles are focused and bent by use of magnets, and accelerated by use of electromagnetic waves in cavities.

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

- A transverse magnetic field will provide a much larger force than a field of the same magnitude which is almost in the direction of the particle's trajectory.
- Static magnetic fields for accelerators are produced either by electric currents or permanent magnet material (e.g., alloys of samarium-cobalt or neodymium-boron-iron).
- Iron ($\mu \approx 5000\mu_0$) is frequently placed around the currents or permanent magnet material to decrease the magnetic reluctance, guide the field distribution, and reduce the leakage flux.
- Electrically powered magnets can be made with warm conductors (water-cooled copper or aluminum), or superconductors (an alloy of niobium-titanium embedded in a copper stabilizer).

Please Use Your Right Hand !



Cartoon by **Bruno Touschek**
(3 February 1921–25 May 1978)

Austrian physicist who initiated e-p collider

Yoda is among the oldest and most powerful known Jedi Masters in the *Star Wars* universe

Magnitude of Magnetic Fields

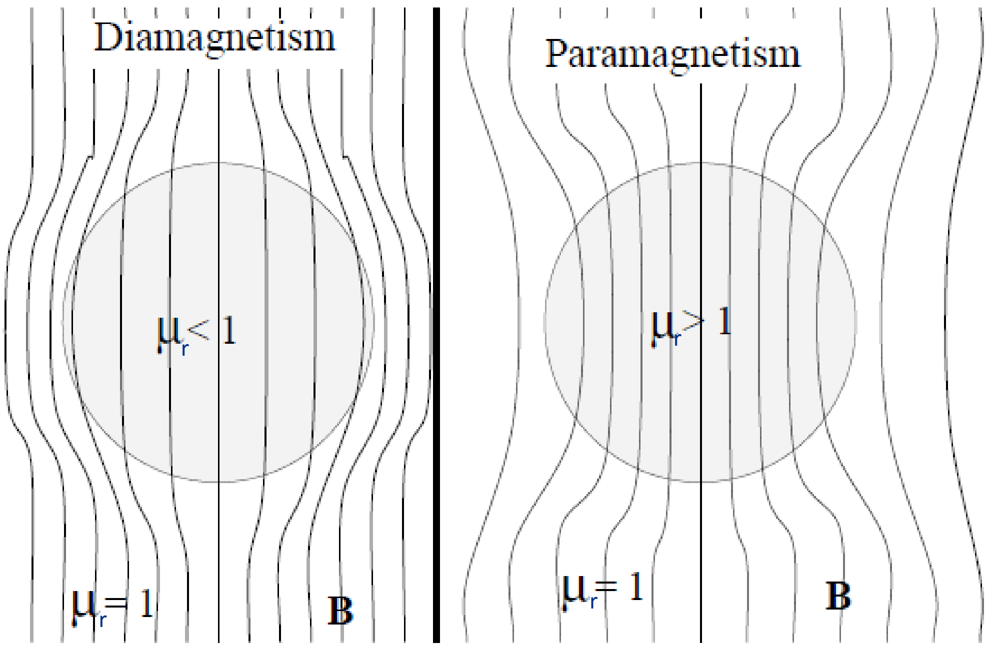
1 T = 10000 Gauss

Value	Item
0.1 - 1.0 pT	human brain magnetic field
24 μT	strength of magnetic tape near tape head
31-58 μT	strength of Earth's magnetic field at 0° latitude (on the equator)
0.5 mT	the suggested exposure limit for cardiac pacemakers by American Conference of Governmental Industrial Hygienists (ACGIH)
5 mT	the strength of a typical refrigerator magnet
0.15 T	the magnetic field strength of a sunspot
1 T to 2.4 T	coil gap of a typical loudspeaker magnet
1.25 T	strength of a modern neodymium-iron-boron ($Nd_2Fe_{14}B$) rare earth magnet.
1.5 T to 3 T	strength of medical magnetic resonance imaging systems in practice, experimentally up to 8 T
9.4 T	modern high resolution research magnetic resonance imaging system
11.7 T	field strength of a 500 MHz NMR spectrometer
16 T	strength used to levitate a frog
36.2 T	strongest continuous magnetic field produced by non-superconductive resistive magnet
45 T	strongest continuous magnetic field yet produced in a laboratory (Florida State University's National High Magnetic Field Laboratory in Tallahassee, USA)
100.75 T	strongest (pulsed) magnetic field yet obtained non-destructively in a laboratory (National High Magnetic Field Laboratory, Los Alamos National Laboratory, USA)
730 T	strongest pulsed magnetic field yet obtained in a laboratory, destroying the used equipment, but not the laboratory itself (Institute for Solid State Physics, Tokyo)
2.8 kT	strongest (pulsed) magnetic field ever obtained (with explosives) in a laboratory (VNIIEF in Sarov, Russia, 1998)
1 to 100 MT	strength of a neutron star
0.1 to 100 GT	strength of a magnetar

Classes of Magnetic Materials

- Diamagnetic ($\mu_r < 1$) and Paramagnetic ($\mu_r > 1$) Materials:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(\mathbf{H} + \chi\mathbf{H}) = \mu_r\mu_0\mathbf{H}$$

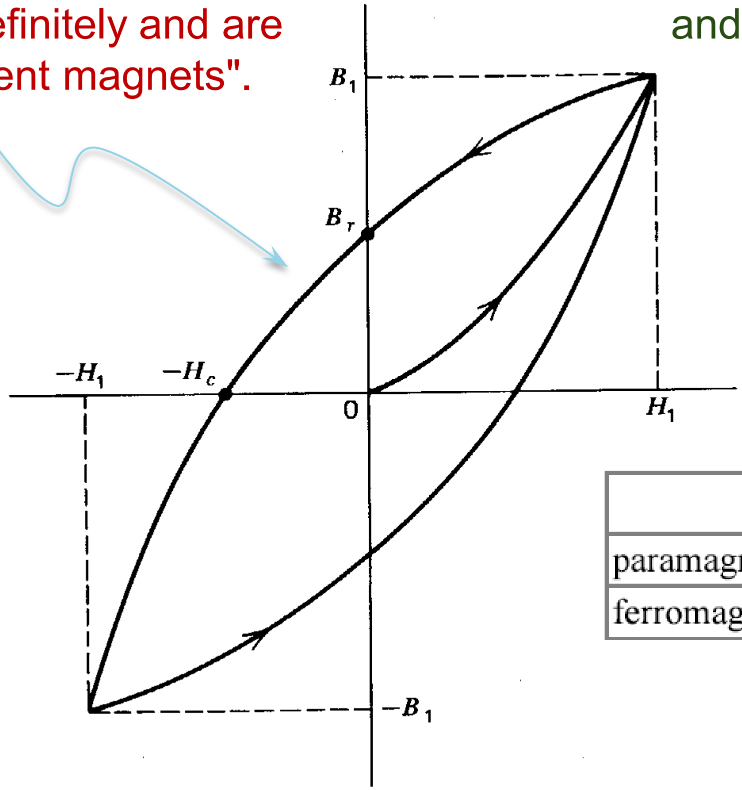


- Diamagnetism is a fundamental property of all matter, although it is very weak.
- Paramagnetism follows Curie's law: $\chi \propto \frac{1}{T} > 0$.

Classes of Magnetic Materials

- Ferromagnetic Materials: Iron, Cobalt, Nickel
- Ferrimagnetic Materials: very similar behavior to ferromagnetic, but have very different magnetic ordering (e.g., Ferrites)

Some compositions of ferromagnetic materials will retain an imposed magnetization indefinitely and are useful as "permanent magnets".



Magnetization in ferromagnetic materials is saturated in moderate ***H*** and room temperature.

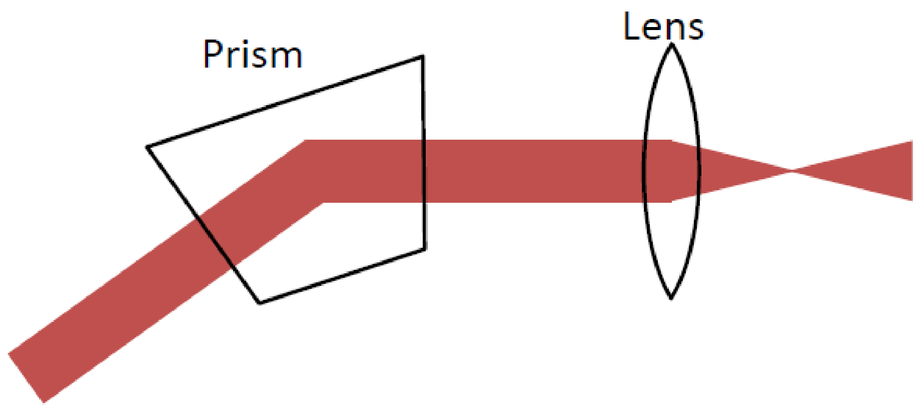
$$\mathbf{M} \rightarrow \text{const.} = \text{saturated}$$

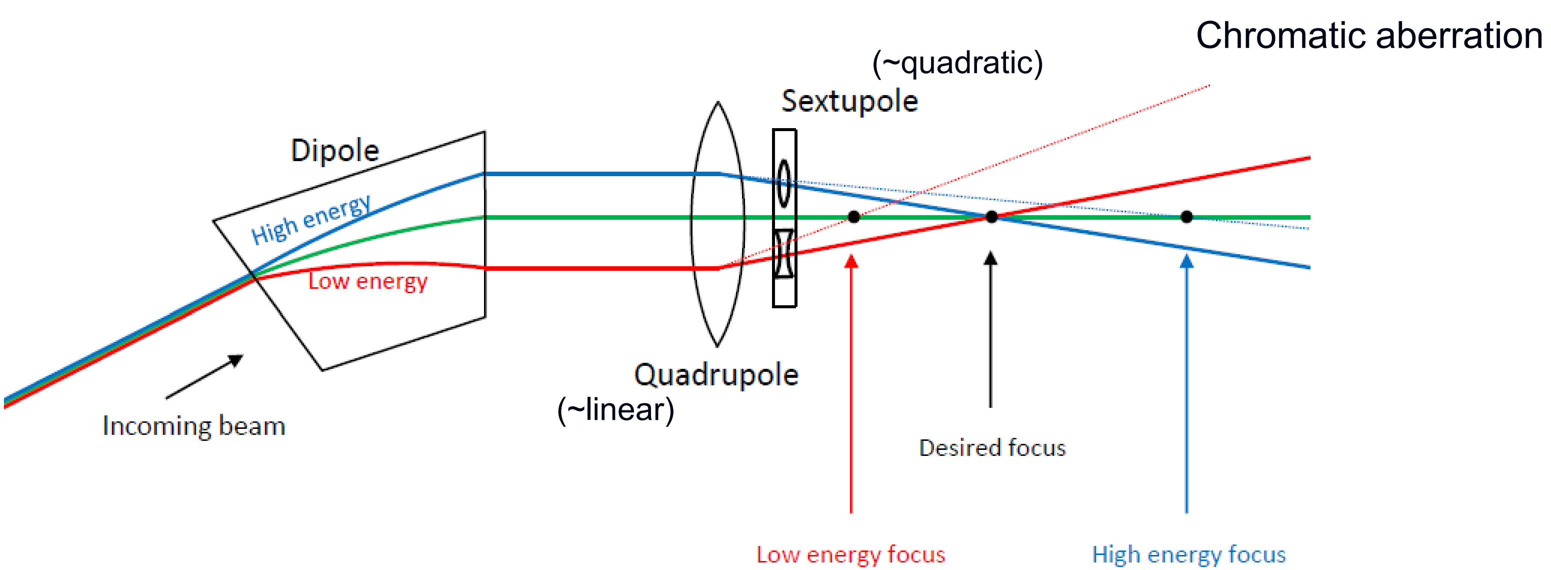
$$\mathbf{B} = \mu_0 \mathbf{H} + const.$$

Applied field ($\propto I$)

	H_{sat} Tesla	T range (K)	χ 10^{-8}m^3/kg
paramagnets	>10	<<100	~50
ferromagnets	~1	~300	1000-10000

Optics Analogy





Equations of Magnetostatics

- Field free ($\mathbf{J} = 0$) vacuum region ($\mu = \mu_0$):

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0$$

  - Method 1: Directly by magnetic induction for 2D

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0, \; \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} \quad \longrightarrow \quad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) B_x = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) B_y = 0$$

  - Method 2: In terms of scalar potential (sometimes, "-" sign is dropped for simplicity)

$$\mathbf{H} = -\nabla \Phi_m \quad \longrightarrow \quad \nabla^2 \Phi_m = 0$$

  - Method 3: In terms of vector potential

$$\mathbf{B} = \nabla \times \mathbf{A}, \nabla \cdot \mathbf{A} = 0 \quad \longrightarrow \quad \nabla^2 \mathbf{A} = 0$$

Solutions in 2D

- Using separation of variables in cylindrical coordinates ($r < a, \theta, z$):

$$B_x = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta)\,, \quad B_y = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (b_n \cos n\theta - a_n \sin n\theta)$$

  - The central field is given by B_0 (only $a_0 = 1$; skew dipole, only $b_0 = 1$; dipole), and $a \approx$ (order of the beam pipe radius) is a reference radius of the expansion.

- Fields in the cylindrical coordinates are

$$\begin{aligned} B_r &= B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n [a_n \cos(n+1)\theta + b_n \sin(n+1)\theta] \\ B_\theta &= B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n [b_n \cos(n+1)\theta - a_n \sin(n+1)\theta] \end{aligned}$$

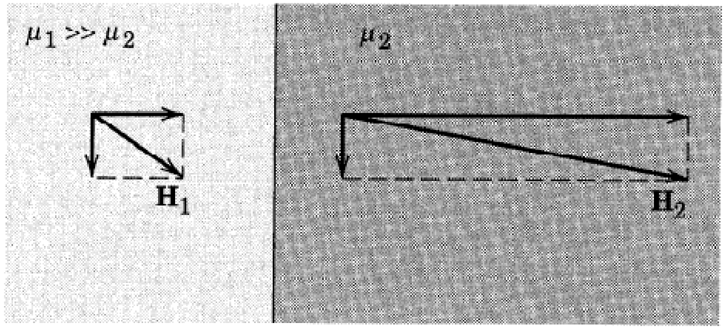
- Or, in terms of scalar potential:

$$\Phi_m = -\sum_{n=0}^{\infty} \frac{a}{n+1} \left(\frac{r}{a}\right)^{n+1} [F_n \cos(n+1)\theta + G_n \sin(n+1)\theta]$$

  - We have let $F_n = B_0 a_n/\mu_0$, $G_n = B_0 b_n/\mu_0$ to be the respective skew and normal strengths $2(n+1)$th for multipoles.

Equipotential Surfaces

- The magnetic field must be perpendicular to the equipotential lines of Φ_m.
- If we assume that our magnet were made of iron ($\mu \gg \mu_0$), then for the pole face of the magnet would be nearly the equipotential surface.



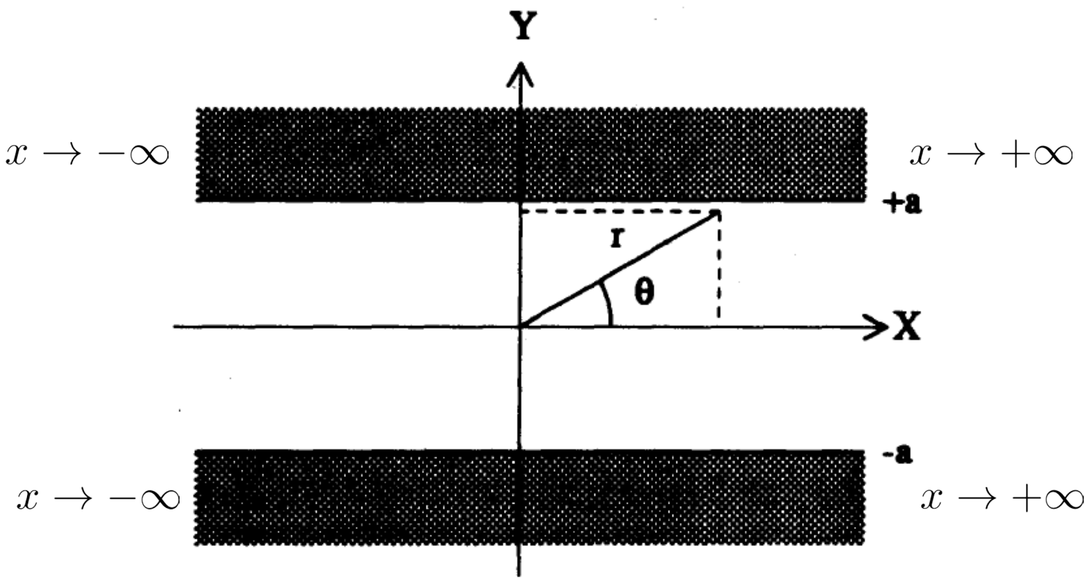
- For a pure normal multipoles, the optimum shape of the pole face would be the same as the equipotential of the term proportional to G_n.

$$\left(\frac{r}{a}\right)^{n+1} \sin(n+1)\theta = \pm 1$$

Examples: Dipole

- For normal dipole magnet ($n = 0$):

$$\left(\frac{r}{a}\right)^1 \sin\theta = \pm 1 \longrightarrow r\sin\theta = y = const. = \pm a$$

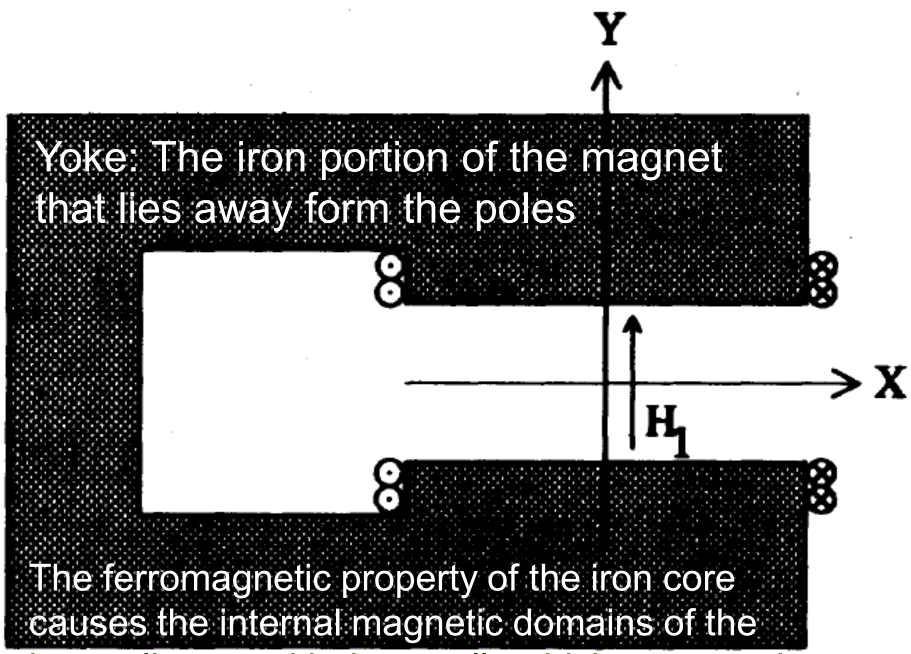


More Realistic “C” Magnet

- Magnetic circuit:

$$I_{\text{enc}} = \underbrace{2NI}_{Ampere-turns} = \oint \mathbf{H} \cdot d\mathbf{l} = \underbrace{\Phi}_{Flux} \times \underbrace{\mathcal{R}}_{Reluctance}$$

$$\begin{aligned} I_{\text{enc}} &= \frac{B_g}{\mu} l_g + \frac{1}{\mu} \int_{\text{Fe}} \mathbf{B} \cdot d\mathbf{l} \approx \Phi \left[\frac{l_g}{\mu_0 S} + \frac{(l - l_g)}{\mu S} \right] \\ &\approx \frac{B_g}{\mu} l_g \end{aligned}$$



Yoke: The iron portion of the magnet that lies away form the poles

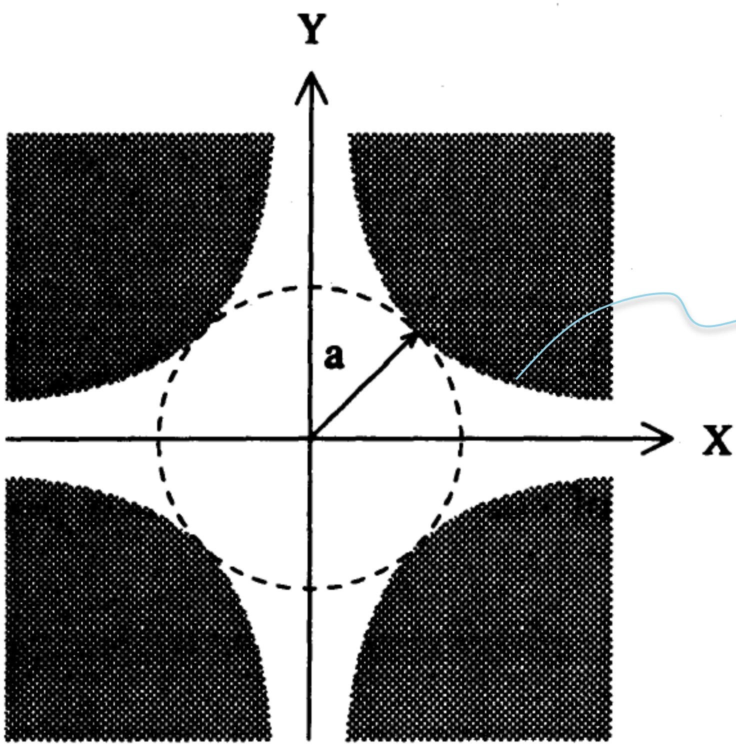
The ferromagnetic property of the iron core causes the internal magnetic domains of the iron to line up with the smaller driving magnetic field produced by the current

- Most of the reluctance is due to the vacuum (or air) gap, that is, the iron is more permeable.
- Essentially, all of the magnetomotive force producing the flux is applied across the gap, even though the gap is physically very small.

Examples: Quadrupole

- For normal quadrupole magnet ($n = 1$):

$$\left(\frac{r}{a}\right)^2 \sin 2\theta = \pm 1 \longrightarrow r^2 \sin 2\theta = 2(r\sin\theta)(r\cos\theta) = 2xy = \pm a^2$$



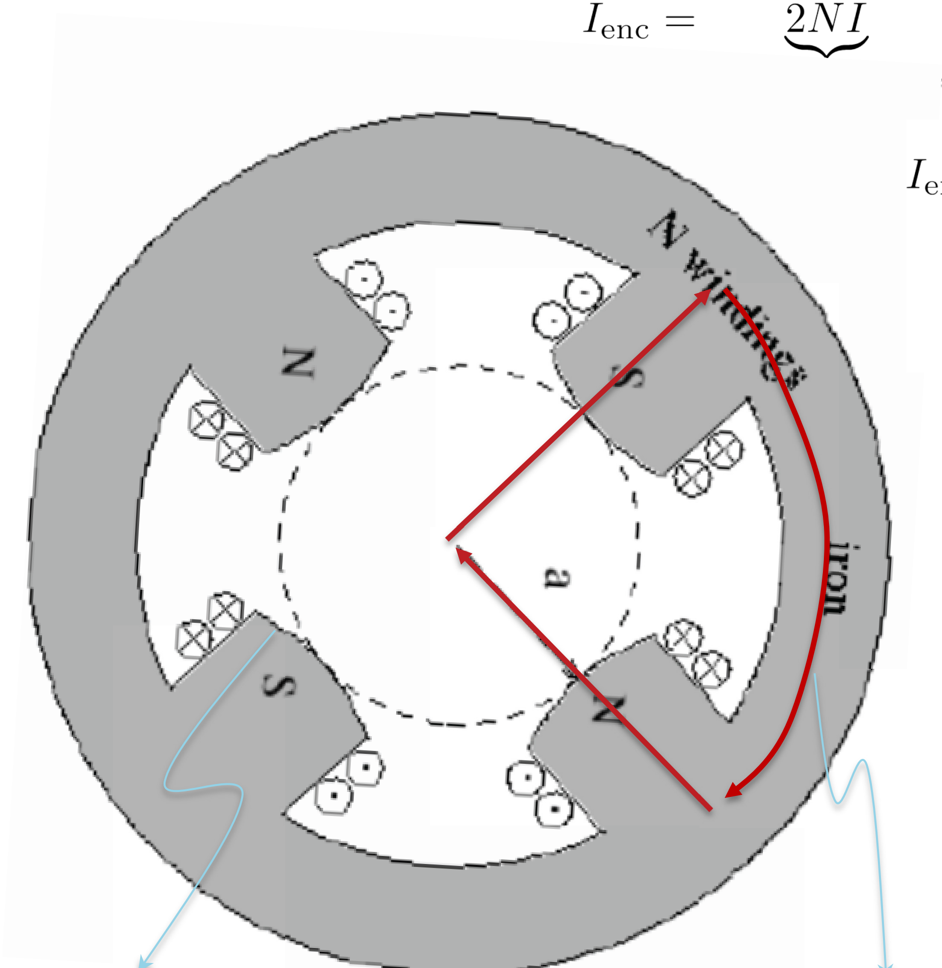
Hyperbolic pole surface

More Realistic Quadrupole Coils

- Applying Ampere's law with a proper Amperian loop:

$$I_{\text{enc}} = \underbrace{2NI} = \oint \mathbf{H} \cdot d\mathbf{l} = \frac{1}{\mu}\int_{\text{Fe}} \mathbf{B} \cdot d\mathbf{l} - \int \nabla\Phi_m \cdot d\mathbf{l}$$

$$\begin{aligned} I_{\text{enc}} &\approx -\int_N^S d\Phi_m = \Phi_m(N) - \Phi_m(S) \\ &= -\frac{a}{2} G_1 \left[\sin(-2\times\pi/4) - \sin(2\times\pi/4)\right] \\ &= G_1 a \end{aligned}$$



Hyperbolic pole surface

Field return path (Yoke)

$$\Phi_m = -\frac{a}{2}\left(\frac{r}{a}\right)^2 G_1 \sin(2\theta) = -\frac{I_{\text{enc}}}{a^2} xy$$

$$(\text{Check } \Phi_m > 0 \quad \text{for} \quad xy < 0)$$

$$\begin{aligned} \mathbf{H} &= \frac{I_{\text{enc}}}{a^2}(y\hat{x} + x\hat{y}) \\ \mathbf{B} &= \frac{\mu_0 I_{\text{enc}}}{a^2}(y\hat{x} + x\hat{y}) \end{aligned}$$

$$(\text{Check } B_y > 0 \quad \text{for} \quad x > 0)$$

Summary of Multipoles

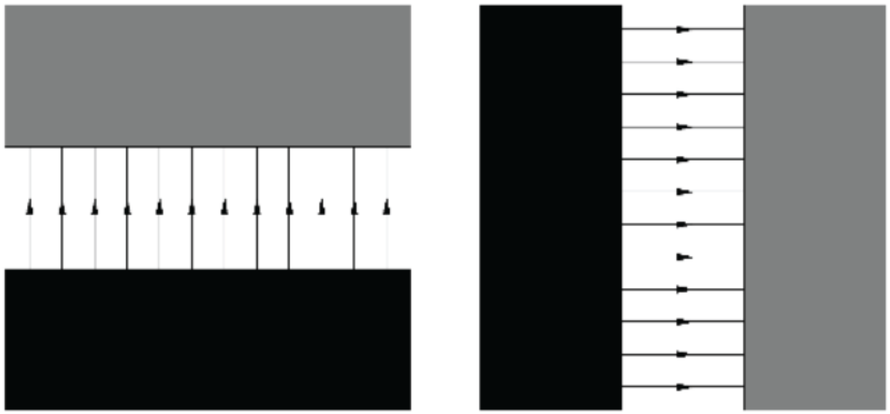
- US Convention:

$$(a_n, b_n) \text{ in US} = (a_{n+1}, b_{n+1}) \text{ in EU, for } n = 0, 1, 2, \cdots$$

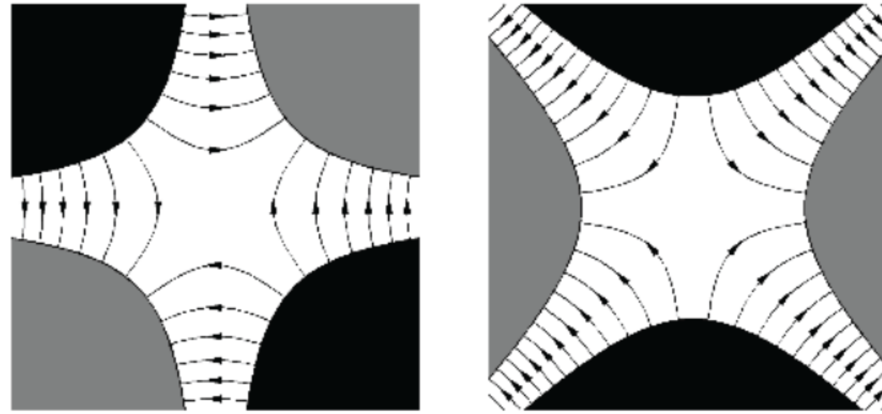
coefficient	multipole	field	notes
b_0	normal dipole	$B_y = B_0 b_0$	horz. bending
a_0	skew dipole	$B_x = B_0 a_0$	vert. bending
b_1	normal quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) b_1 \sin\theta = B_0 \left(\frac{y}{a}\right) b_1$ $B_y = B_0 \left(\frac{r}{a}\right) b_1 \cos\theta = B_0 \left(\frac{x}{a}\right) b_1$	focusing defocusing
a_1	skew quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) a_1 \cos\theta = B_0 \left(\frac{x}{a}\right) a_1$ $B_y = -B_0 \left(\frac{r}{a}\right) a_1 \sin\theta = -B_0 \left(\frac{y}{a}\right) a_1$	coupling
b_2	normal sextupole	$B_x = B_0 \left(\frac{r}{a}\right)^2 b_2 \sin(2\theta)$ $B_y = B_0 \left(\frac{r}{a}\right)^2 b_1 \cos(2\theta)$	nonlinear! coupling

Summary of Multipoles

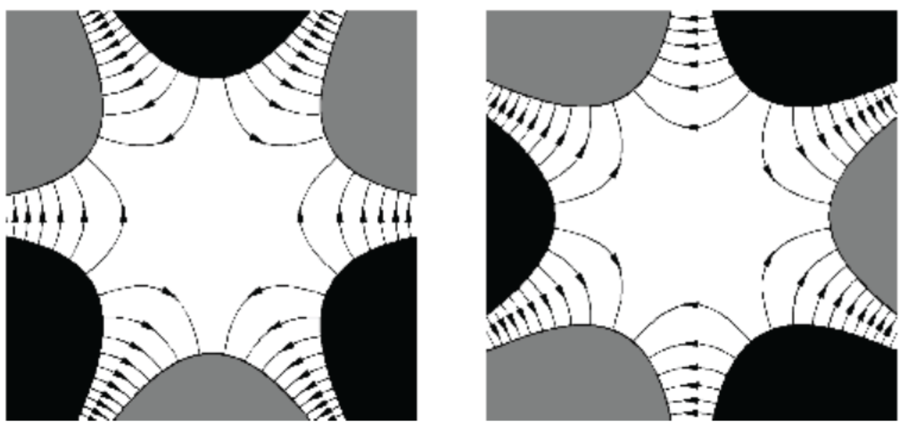
Dipole and Skew dipole



Quadrupole and Skew Quadrupole



Sextupole and Skew sextupole



Vector Potential Solutions

- In terms of vector potentials for $r < a$:

$$A_z = B_0 \sum_{n=0}^{\infty} \frac{a}{n+1}\left(\frac{r}{a}\right)^{n+1}\left[a_n \sin(n+1)\theta - b_n \cos(n+1)\theta\right]$$

- A more general form in arbitrary current-free region is given by

$$A_z = \mathrm{Re}\sum_{n=1}^{\infty}\left[\mathrm{A}_n r^n + \mathrm{B}_n r^{-n}\right]\exp(in\theta)$$

  - We assume a localized current sheet distribution at $r = a$.
  - Note that $\mathrm{A}_n, \mathrm{B}_n$ are complex in general.

- As an example, we examine the case of a dipole with strength B_0:

$$A_z = \begin{cases} \mathrm{A}_1 r\cos\theta, & r < a \\ \mathrm{B}_1 r^{-1}\cos\theta, & r > a \end{cases} \xrightarrow{\mathrm{A}_1 = -B_0,\ \mathrm{B}_1 = \mathrm{A}_1 a^3,\ \mathbf{B} = \nabla\times(A_z\hat{z})} \mathbf{B} = B_0 \begin{cases} \sin\theta\hat{r} + \cos\theta\hat{\theta} = \hat{y}, & r < a \\ (a/r)^2(\sin\theta\hat{r} - \cos\theta\hat{\theta}), & r > a \end{cases}$$

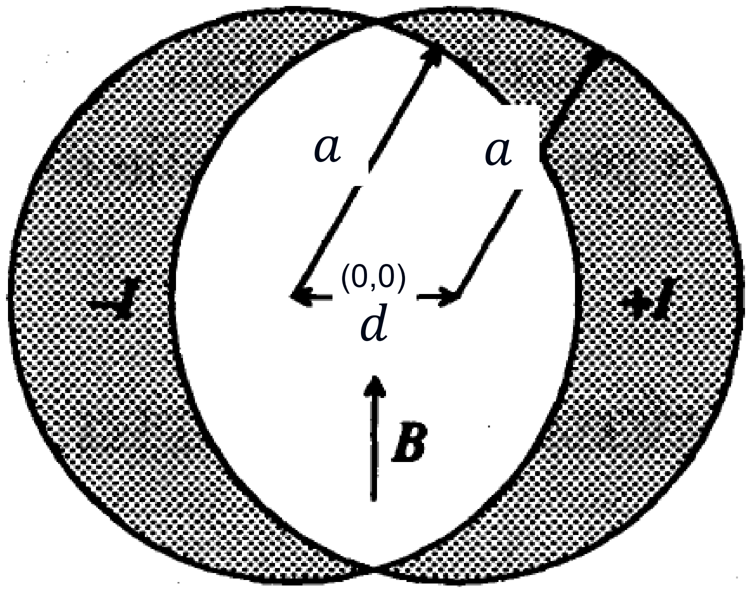
$$B_\theta(r = a^+) - B_\theta(r = a^-) = \hat{r}\times\mu_0 K_z\hat{z} \xrightarrow{\text{Discontinuity}} 2B_0\cos\theta = \mu_0 K_z(\theta)$$

$$J_z(r,\theta) = K_z(\theta)\delta(r-a) \ = (2B_0/\mu_0)\cos\theta\,\delta(r-a)$$

More Realistic "$\cos\theta$" Magnet

- Using Ampere's law for a uniform current distribution in a long cylinder:

$$\mathbf{B} = \frac{\pm\mu_0 J_z}{2}\left(r\hat{r} \mp \frac{d}{2}\hat{x}\right) \times \hat{z}$$

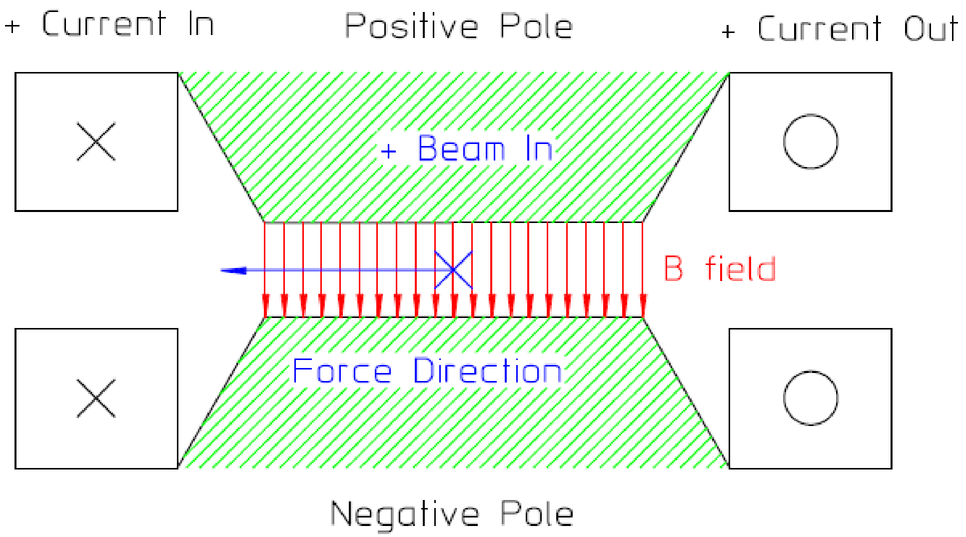


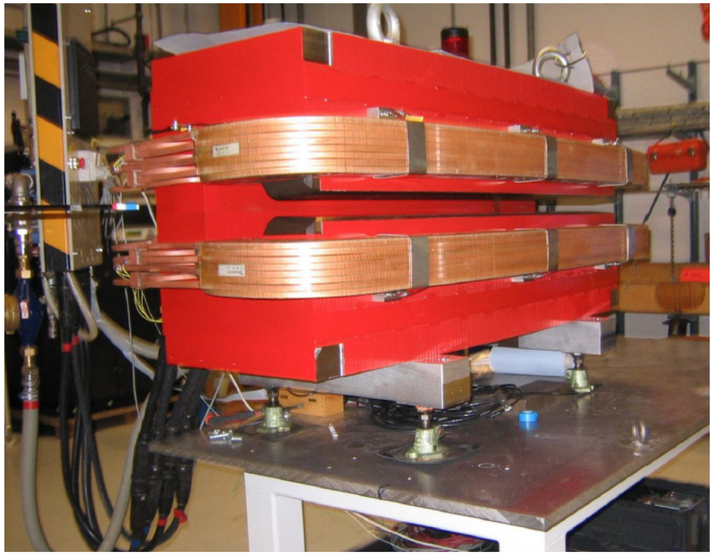
- The superposition of the fields from two opposing current densities:

$$\begin{aligned}\mathbf{B} &= \frac{+\mu_0 J_z}{2}\left(r\hat{r} - \frac{d}{2}\hat{x}\right) \times \hat{z} + \frac{-\mu_0 J_z}{2}\left(r\hat{r} + \frac{d}{2}\hat{x}\right) \times \hat{z} \\ &= \frac{-\mu_0 d J_z}{2}\hat{x} \times \hat{z} = \frac{\mu_0 d J_z}{2}\hat{y}\end{aligned}$$

Dipole

- Field is constant.



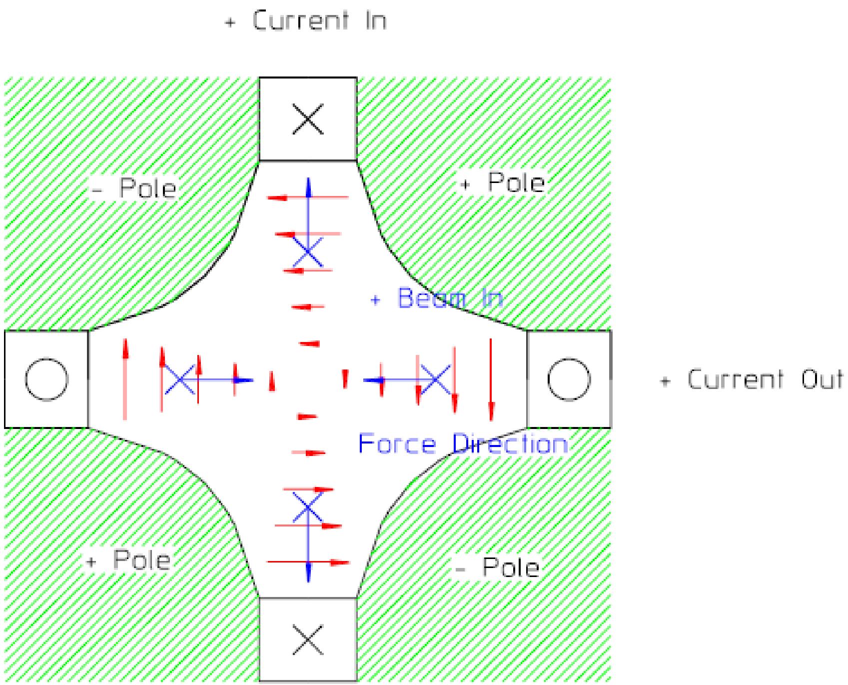


$$B_x = 0, \quad B_y = B_0 b_0 \quad (< 0 \ \text{ for this Fig.})$$

$$A_z = -B_0 b_0 x$$

Quadrupole

- Field varies linearly.



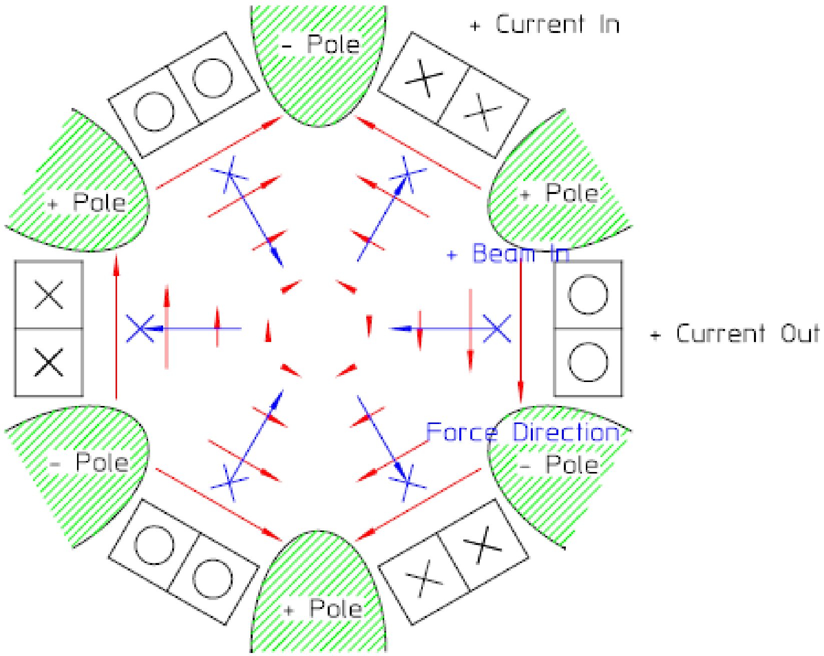
$$B_x = B_0 b_1 \frac{y}{a}, \quad B_y = B_0 b_1 \frac{x}{a} \quad (B_0 b_1 < 0 \ \text{ for this Fig.})$$

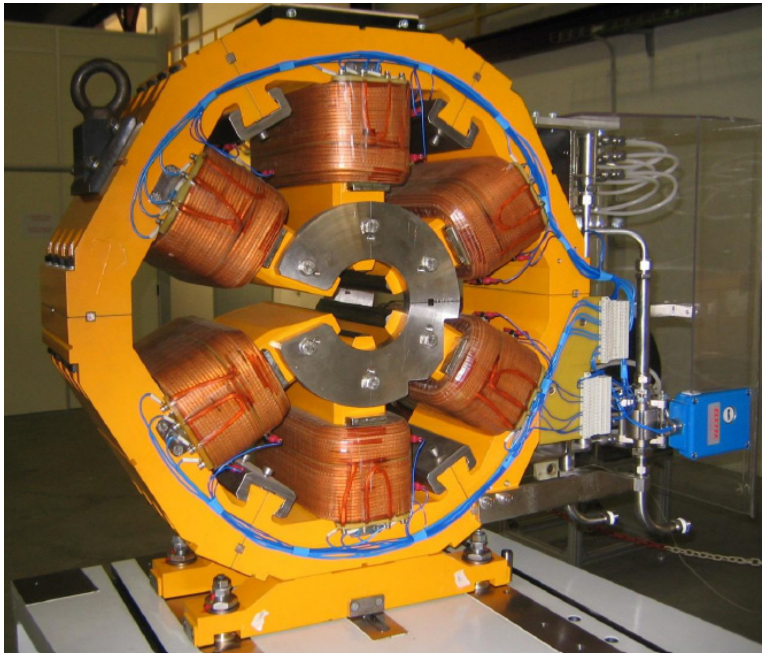
$$\mathbf{B} = B_q' \left(y\hat{x} + x\hat{y} \right)$$

$$\mathbf{A} = -\frac{1}{2} B_q' \left(x^2 - y^2 \right) \hat{z}$$

Sextupole

- Field varies quadratically.

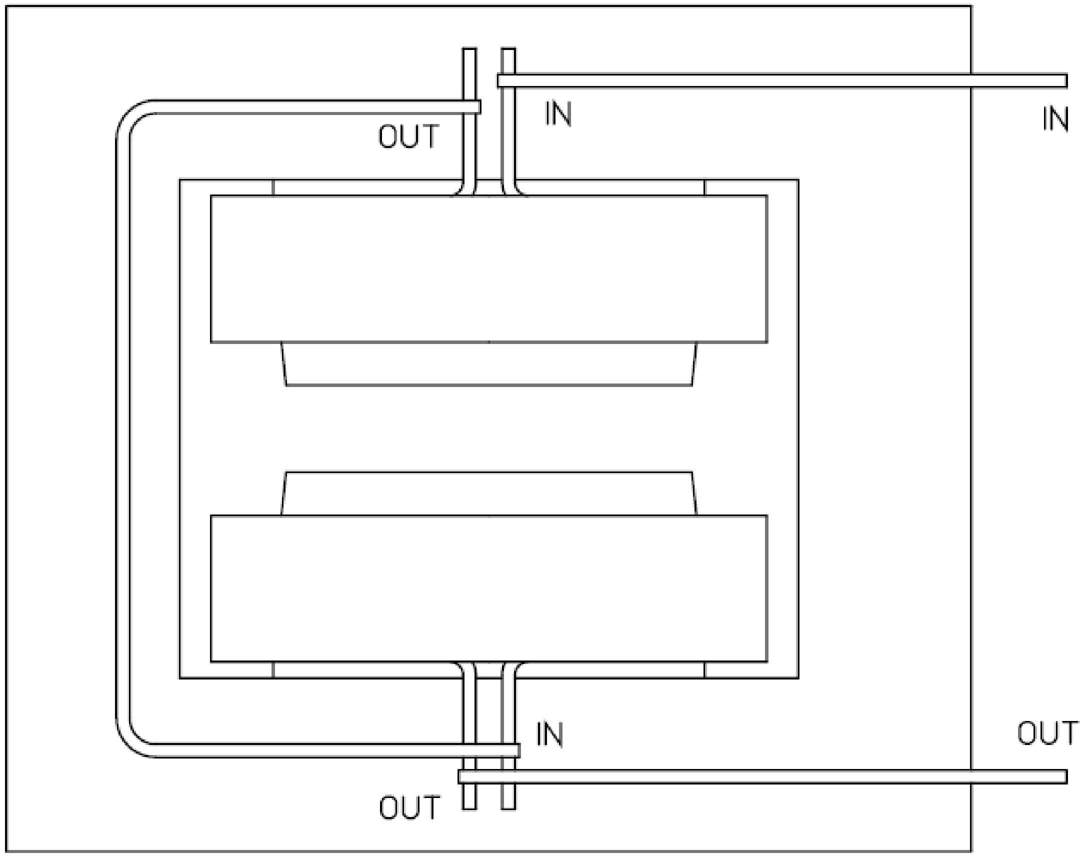




A standard technique for insulating magnet coils is to use epoxy resin, reinforced with fiberglass.

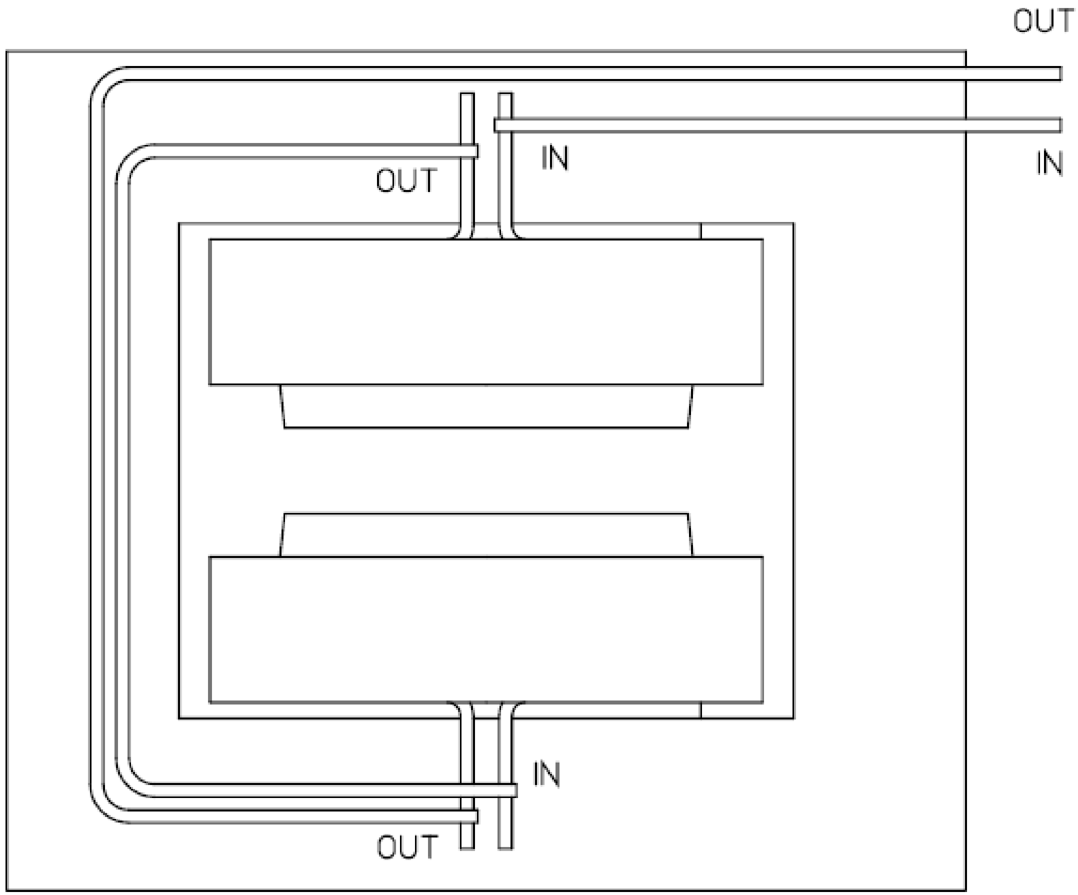
Magnetic Bussing

- What's wrong with this picture ?



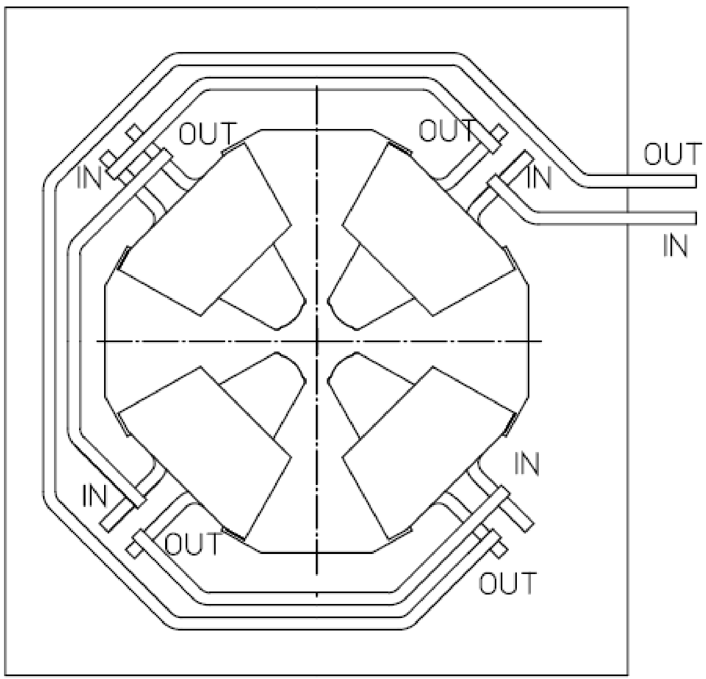
Magnetic Bussing

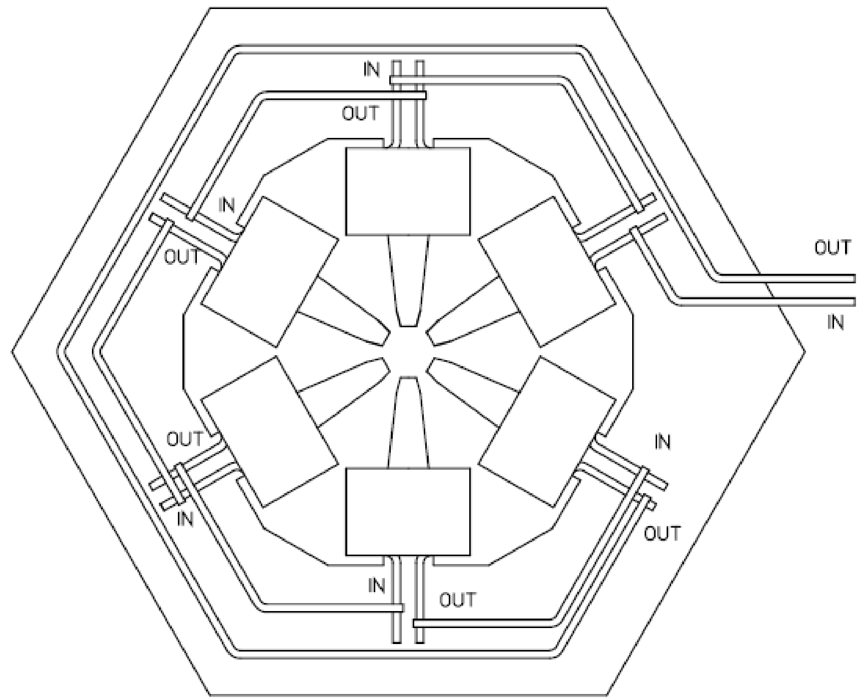
- The electrical bussing connection creates a loop around the beam line, resulting in a small solenoidal field. This longitudinal field can rotate the beam.



Quad & Sextupole Bussing

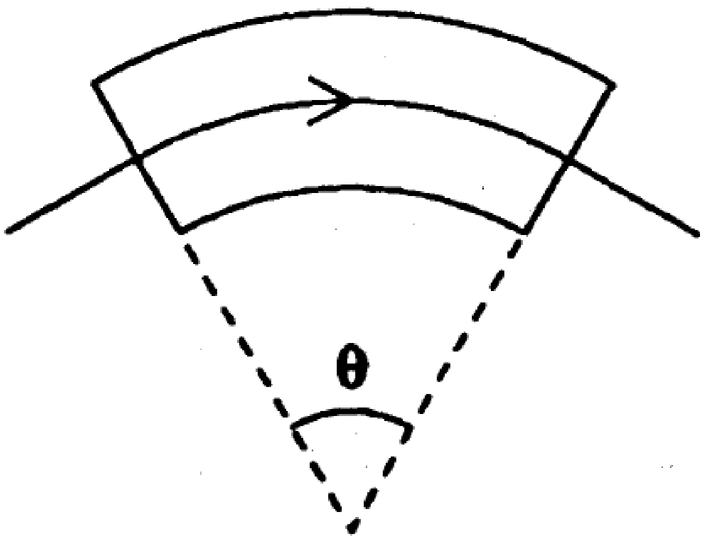
- The in and out conductors should be placed close to each other so that longitudinal fields are minimized



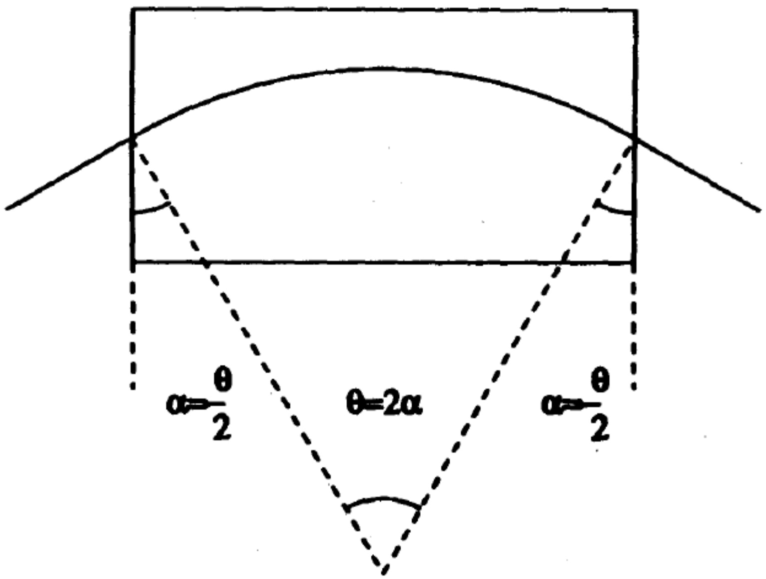


Dipoles are Not Infinitely Long !

- Sector bend (sbend):
  - Simpler to conceptualize, but harder to build
  - Beam design entry/exit angles are $\perp$ to end faces
- Rectangular bend (rbend):
  - Easier to build, but must include effects of edge focusing
  - Beam design entry/exit angles are half of bend angle



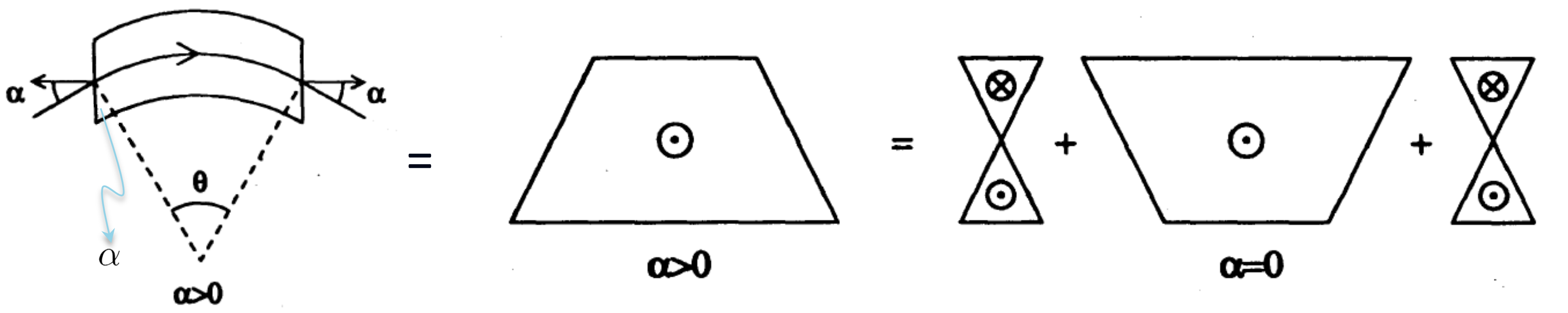
$\alpha = 0$



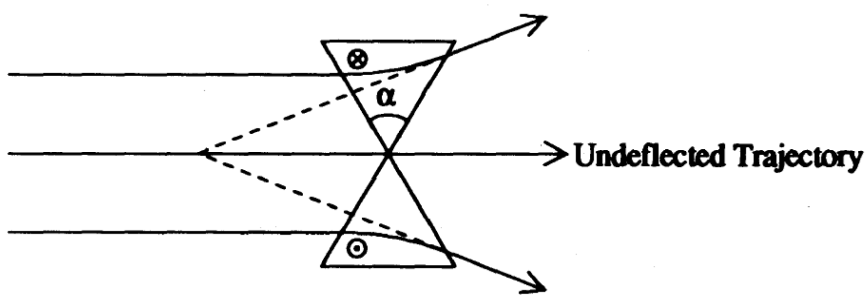
$\alpha = \frac{\pi}{2} > 0$

Dipole Fringe Fields: Horizontal Motion

- For $\alpha \neq 0$, we need to include edge focusing effects.



- Defocusing effect of a thin wedge in horizontal direction with $\alpha > 0$.



Dipole Fringe Fields: Vertical Motion

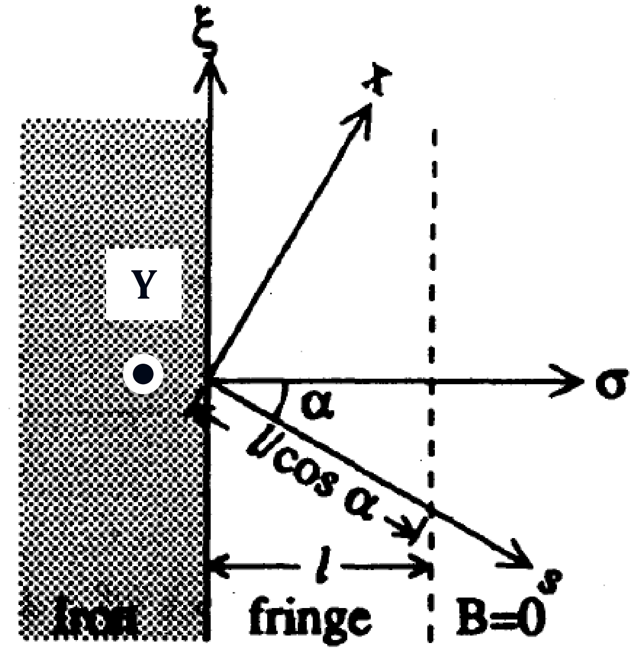
- There is a finite transverse field which induces vertical kicks:

$$B_y \approx B_0 \left(1 - \frac{\sigma}{l}\right) \quad \text{for} \quad 0 < \sigma < l$$

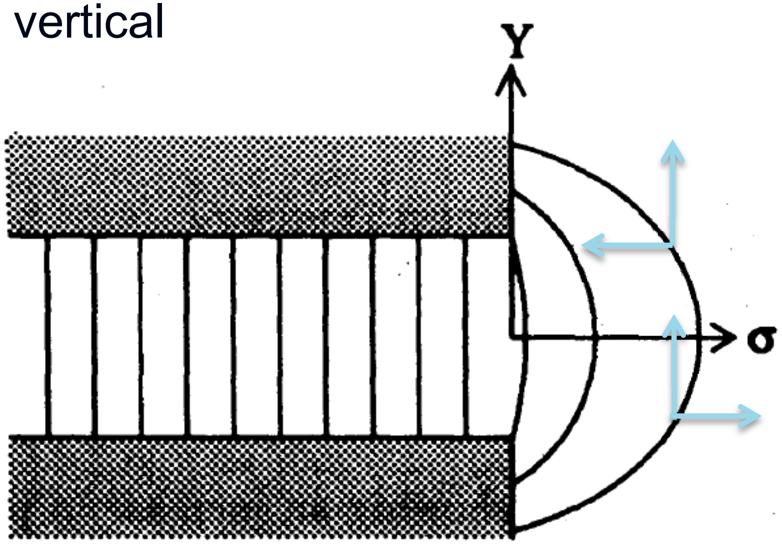
$$B_\xi \approx 0 \quad \text{(i.e., wide poles)}$$

$$B_\sigma \simeq \left(\frac{\partial B_\sigma}{\partial y}\right) y = \left(\frac{\partial B_y}{\partial \sigma}\right) y = -\frac{B_0}{l} y$$

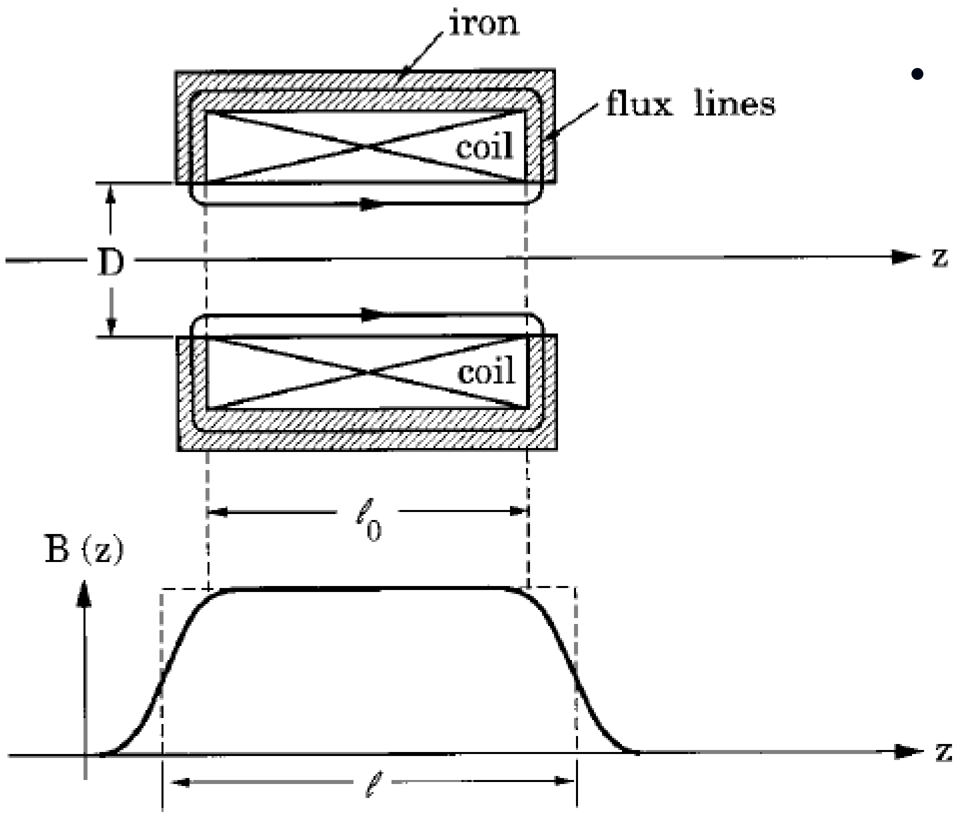
$$B_x = B_\xi \cos\alpha + B_\sigma \sin\alpha = -\frac{B_0 \sin\alpha}{l} y$$



- Focusing effect of a fringe field in the vertical direction with $\alpha > 0$.



Solenoidal Fields



- There is a finite radial field in the fringe field region:

$$
\begin{aligned}
\nabla \cdot \mathbf{B} &= \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\
&= 2\frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} \\
&= 0
\end{aligned}
$$

$$
\longrightarrow \frac{\partial B_x}{\partial x} = \frac{1}{2}\frac{\partial B_z}{\partial z}
$$

$$
\mathbf{B} = -\frac{1}{2}B_z'(z)\,(x\hat{x} + y\hat{y}) + B_z(z)\hat{z} = -\frac{1}{2}B_z'(z)r\hat{r} + B_z(z)\hat{z}
$$

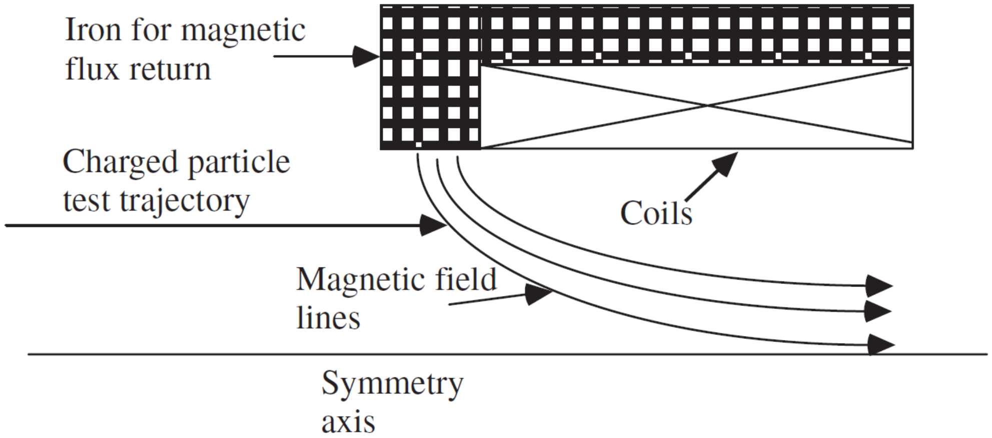
$$
B_z'(z) \equiv \left(\frac{\partial B_z}{\partial z}\right)_{(0,0)}
$$

$$
\mathbf{A} = \frac{1}{2}B_z(z)\,(x\hat{y} - y\hat{x}) = \frac{1}{2}B_z(z)r\hat{\theta}
$$

Solenoidal Fields

- At first glance, one would expect that a particle entering a uniform magnetic field on a straight path parallel to the field lines should not be deflected radially.
- However, there is a net radial focusing due to radial magnetic fields in the fringe regions.
- Rotation in the axisymmetric system due to conservation of canonical angular momentum or Busch's theorem .

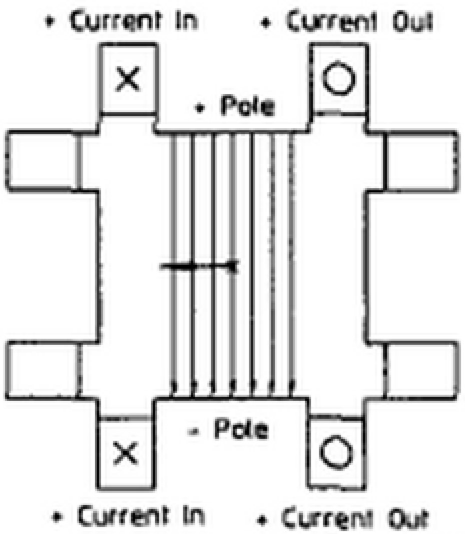
$$p_\theta = \gamma m r^2 \dot{\theta} + q r A_\theta = const. \qquad \gamma m r^2 \dot{\theta} + \frac{q}{2\pi}\Phi = const.$$

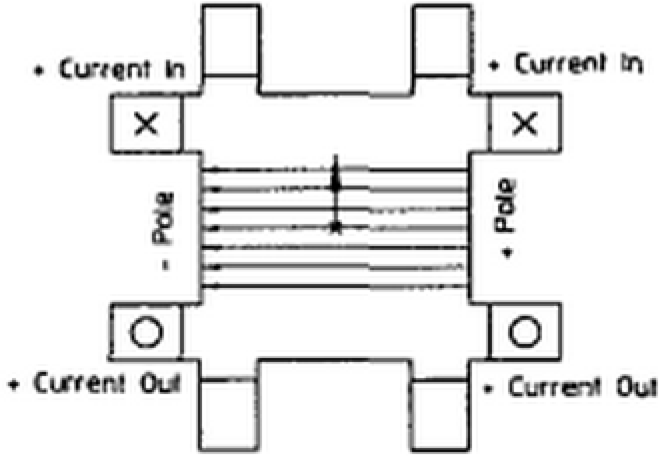


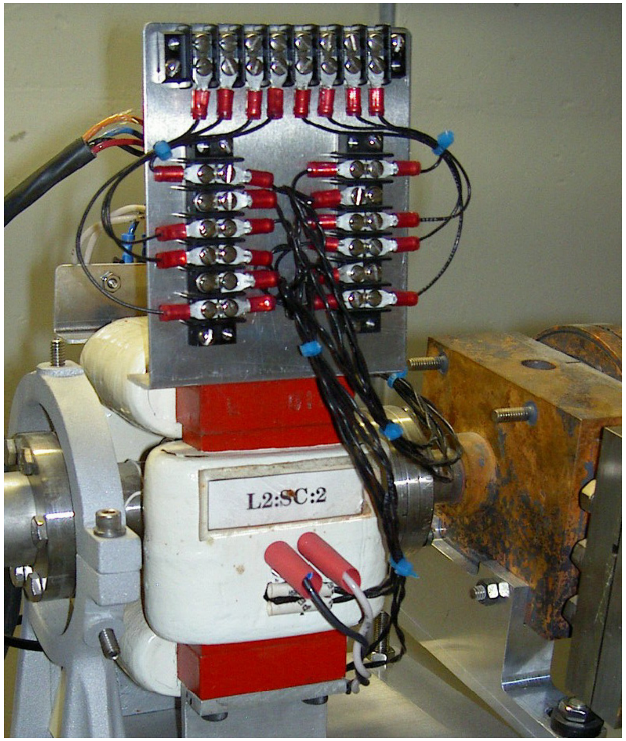
$$\Phi = \int \mathbf{B} \cdot d\mathbf{a} = \int \nabla \times \mathbf{A} \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$

Special Magnets: Corrector

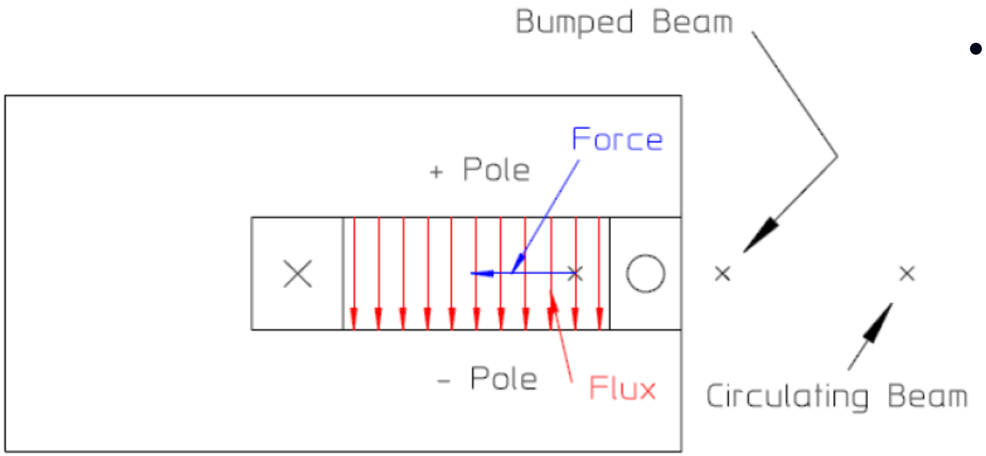
- Corrector magnets are used to provide minor horizontal and vertical steering (typically less than 1.5 mrad).
- They are normally located upstream from the main quadrupoles and are used to steer the beam to the center of the quadrupole.
- In order to conserve lattice space and to simplify operation, horizontal and vertical steering are often combined in a single magnet.



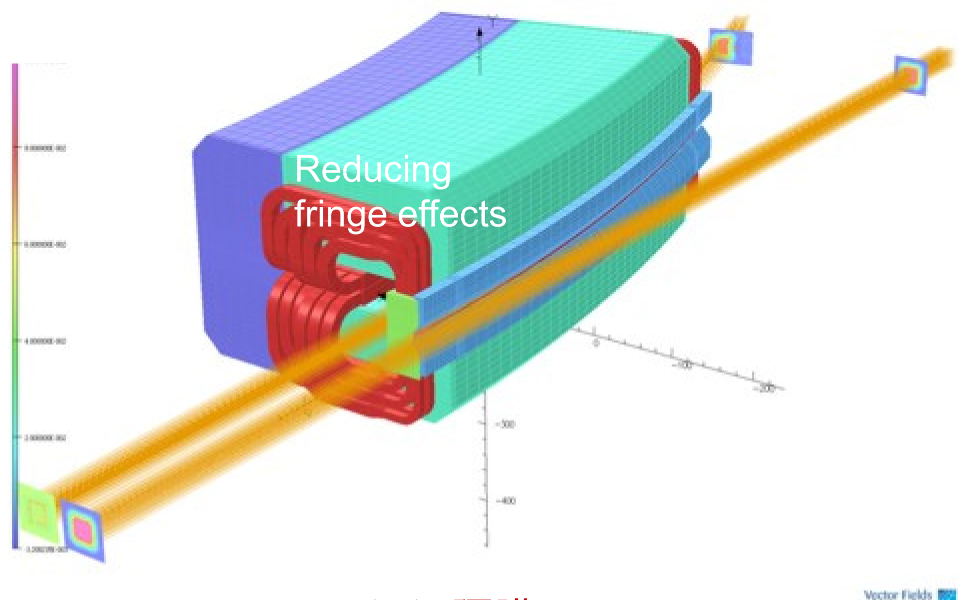




Special Magnets: Septum



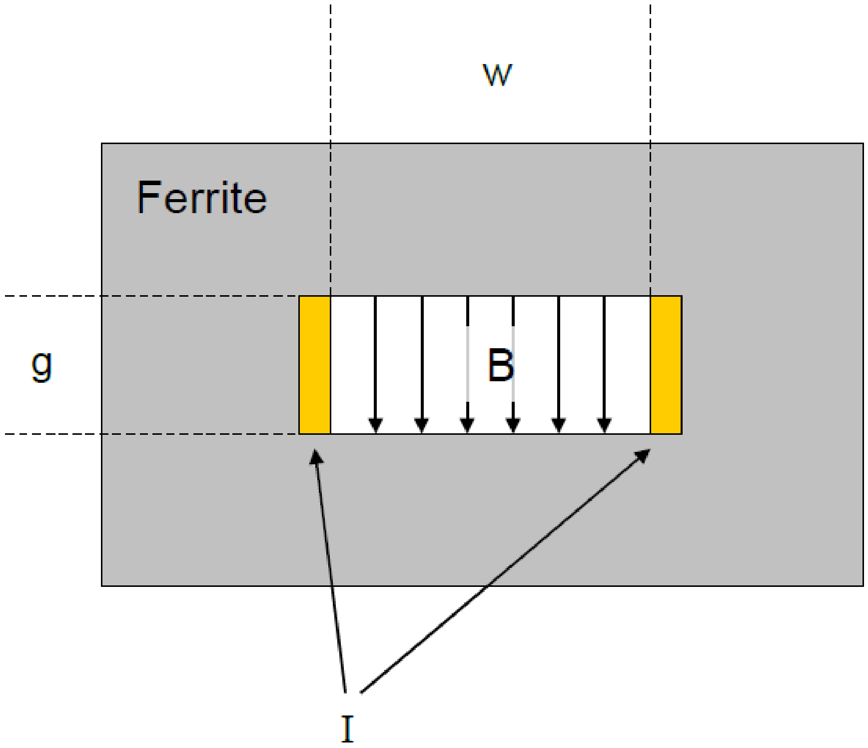
- Pulsed or DC magnet with thin (2-20mm) septum* between zero field and high field region



*septum: a wall or partition dividing a body space or cavity (격막, 隔膜)

Special Magnets: Kicker

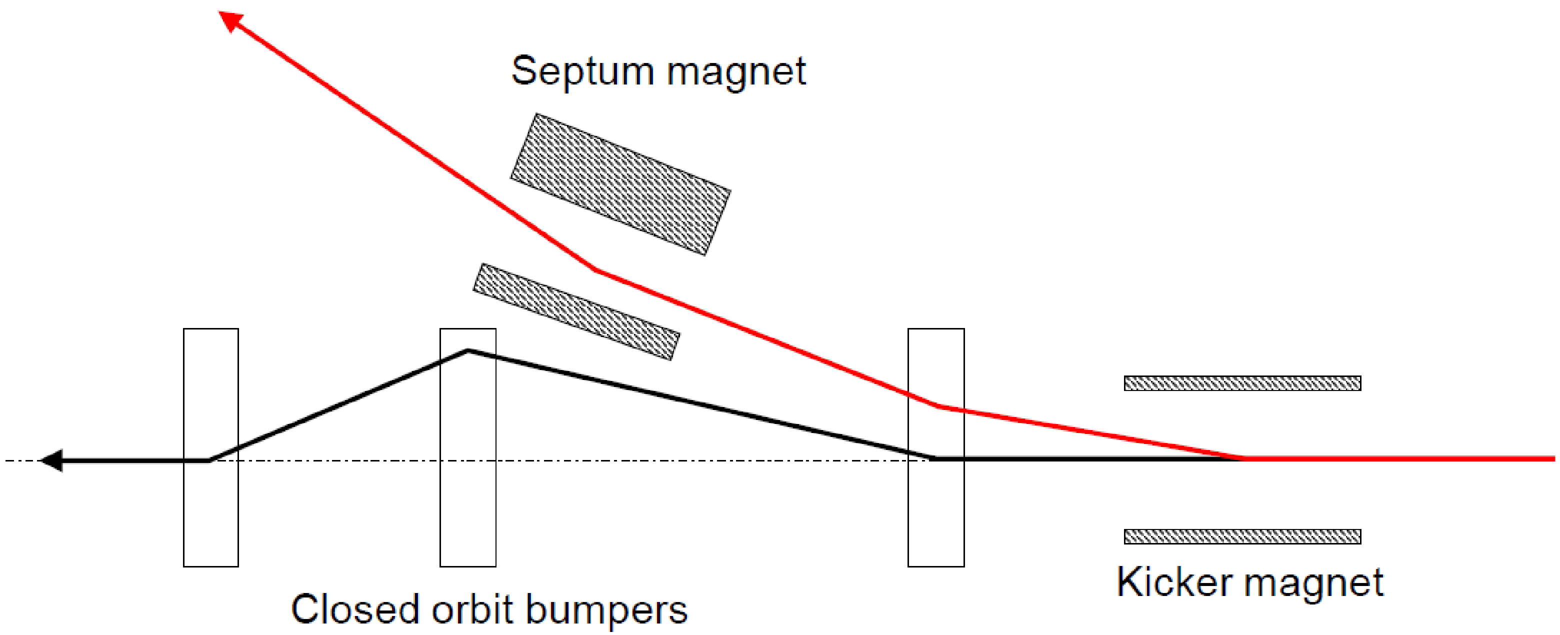
- Pulsed magnet with very fast rise time (100 ns –few μs)



- Ferrite core (ferrimagnetic) has comparatively low losses at high frequencies, and used for fast, low field magnet cores.
- In many cases, insulated laminated core are used for time varying field magnets in order to reduce or eliminate eddy current effects.

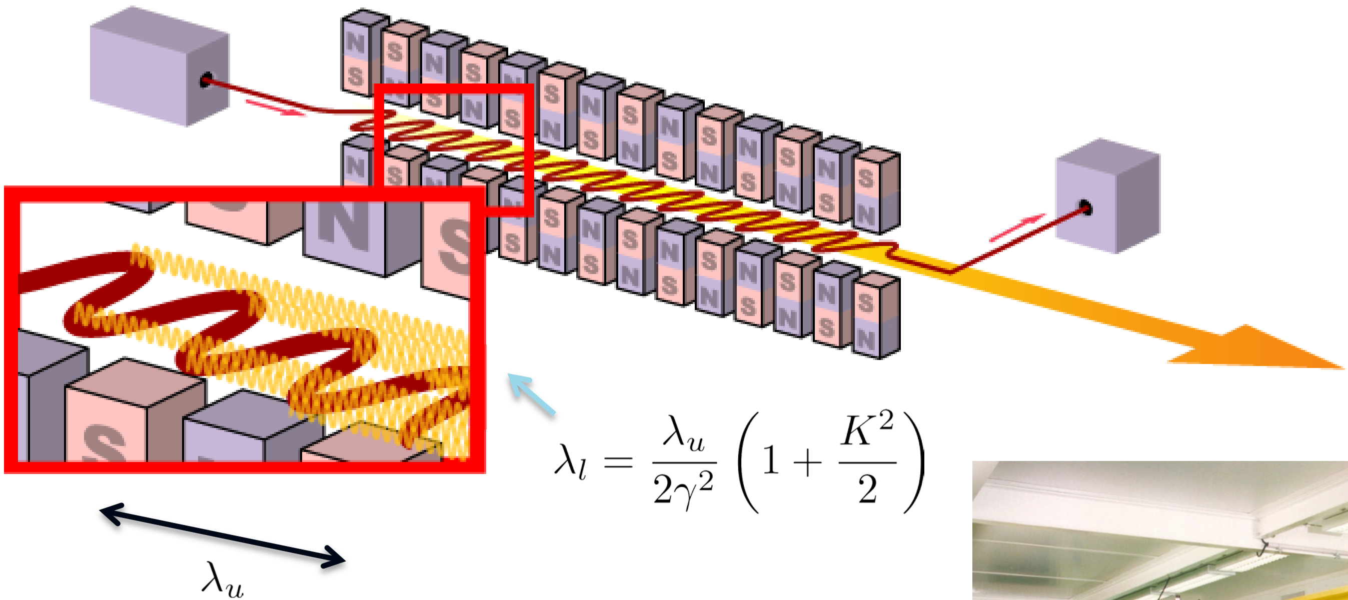
Use of Kicker and Septum

- Kicker deflects the entire beam into the septum in a single turn.
- Septum deflects the beam entire into the transfer line.



Special Magnets: Undulator

- An array of dipoles with alternating polarity of the magnetic field:



$$\lambda_l = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

$$K = \frac{eB_0\lambda_u}{2\pi m_e c} = \text{Undulator parameter}$$

$$\theta_{max} \approx \frac{K}{\gamma} = \text{Opening angle of radiation}$$

$$= \begin{cases} \leq \frac{1}{\gamma}, & K \leq 1 \text{ (Undulator)} \\ > \frac{1}{\gamma}, & K \gg 1 \text{ (Wiggler)} \end{cases}$$

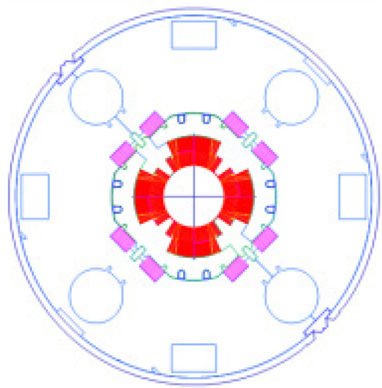
FLASH Undulator Magnet

Magnet Technology for Tighter Focus

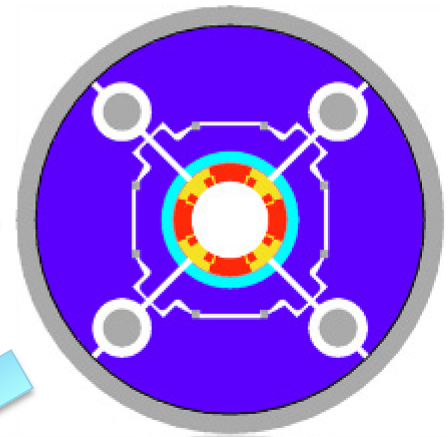
$$\mathcal{R} = \mathcal{L}\sigma(E), \quad \mathcal{L} = \text{Luminosity} = \frac{f_0 N_1 N_2}{4\pi\sigma_x\sigma_y} N_b$$

Fermilab KEK

LHC (USA & JP, 5-6 m)
$\varnothing$70 mm, B_{peak}~ 8 T
1992-2005



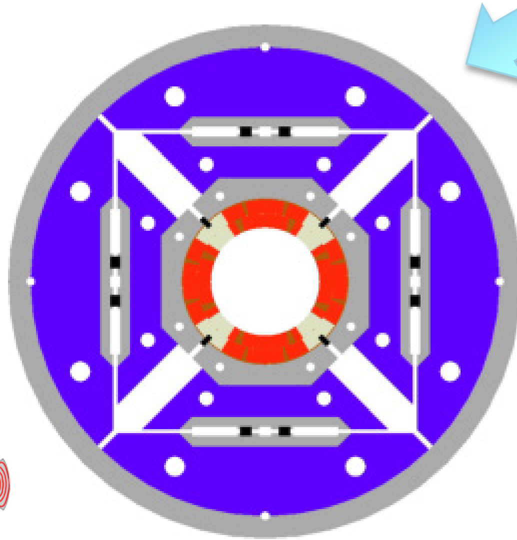
LARP

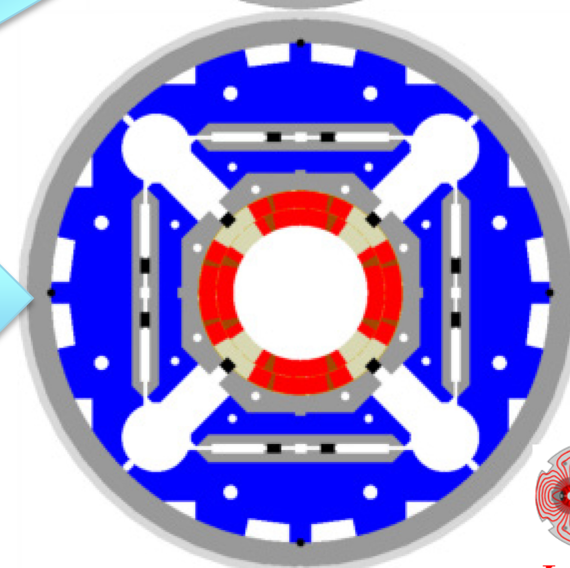


LARP TQS & LQ (4m)
$\varnothing$90 mm, B_{peak}~ 11 T
2004-2010

LARP HQ
$\varnothing$120 mm,
B_{peak}~ 12 T
2008-2014

LARP



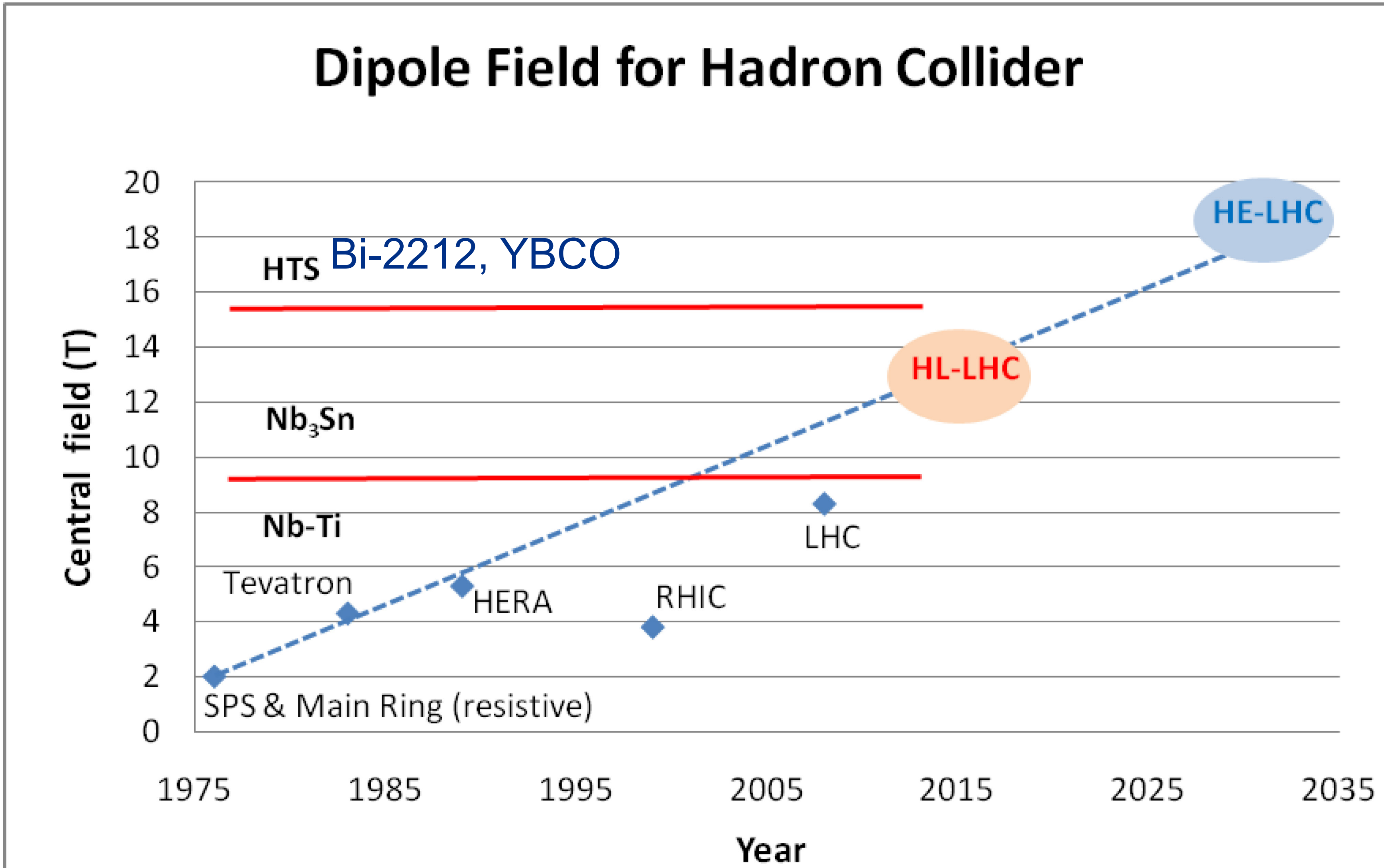


LARP & CERN
MQXF
$\varnothing$150 mm,
B_{peak}~ 12.1 T
2013-2020

LARP

Magnet Technology for Higher Energy

$$B\rho = \frac{p}{q} = \text{Magnetic Rigidity}$$



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