Lecture 6
Beam Manipulations

(Ch. 10 of UP-ALP)

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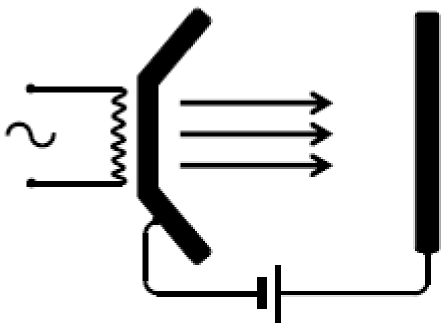
Bunch Compression

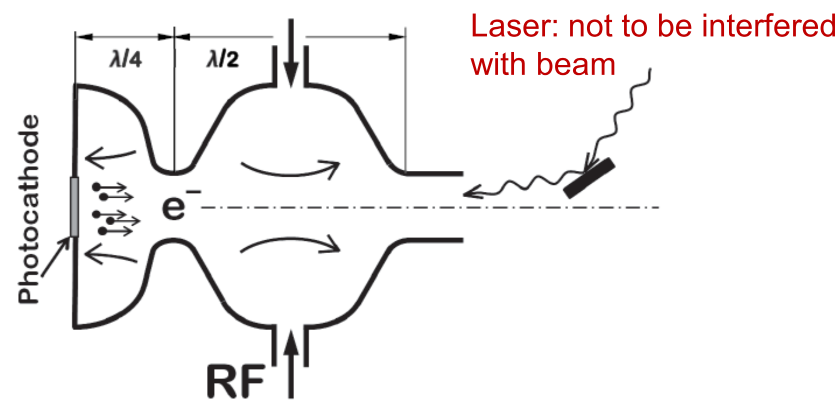
10.1.1 Bunch Compression

- Short electron bunches are typically produced by RF photo guns where the cathode is illuminated by a short laser pulse.

$$I = \text{Perveance} \times V^{3/2}$$



DC Thermal cathode e-gun



One-and-a-half cell RF photocathode e-gun

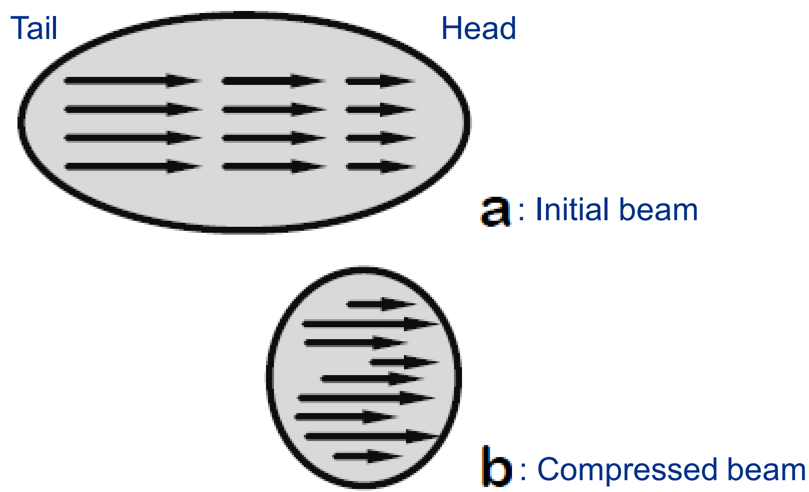
- While a pure metal photocathode is robust, its quantum efficiency (the number of electrons per number of incident photons) is usually quite low, of the order of 10^{-4}. Photocathodes with alkali metals such as cesium can have a quantum efficiency of 10% or higher.

- However, such photo injectors produce still relatively long pulses (a few ps), while for many applications, such bunches would be too long and fs bunches would be required instead.

Velocity Bunching

- By properly adjusting the timing when the laser illuminates the cathode with the RF field in the gun, one can create correlation between the energy of electrons and the longitudinal position within the bunch.

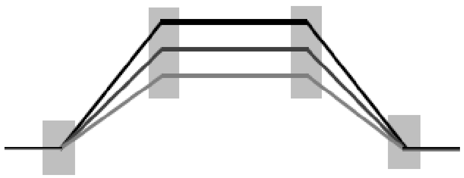
- The difference of energy for beams that are still weakly relativistic can create a difference in longitudinal velocities.



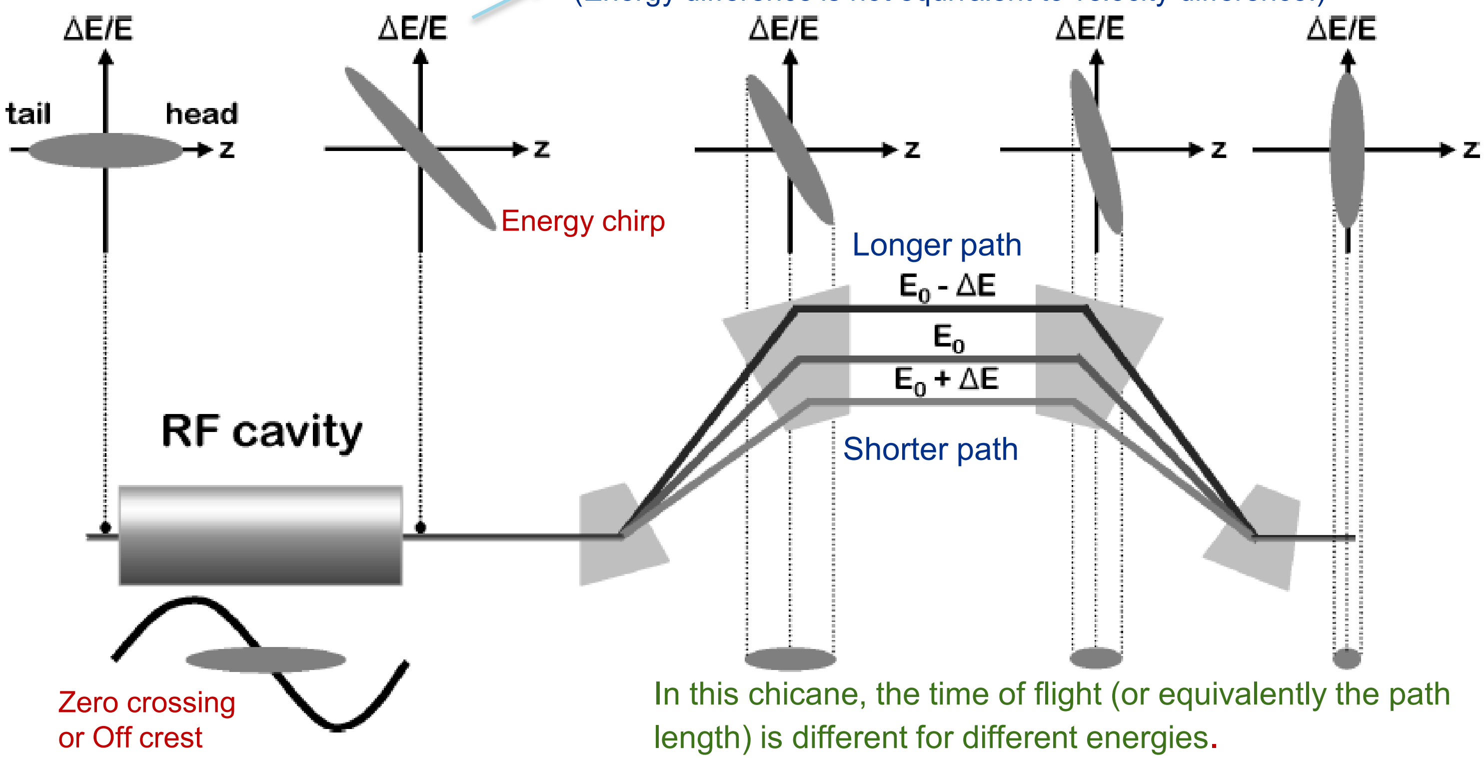
- Because velocity bunching is based on the velocity's dependence on energy, it can work only for weakly relativistic beams.
- Weakly relativistic electron beams can, in particular, suffer strongly from space charge effects, limiting the degree of compression

Chicane (자동차 속도를 줄이게 하기 위한 이중 급커브길)

- Achieving ultra-short electron bunches is usually done at higher energies, when the beam is relativistic and space charge effects are less severe.



Particle velocities are nearly constant at c
(Energy difference is not equivalent to velocity difference!)



In this chicane, the time of flight (or equivalently the path length) is different for different energies.

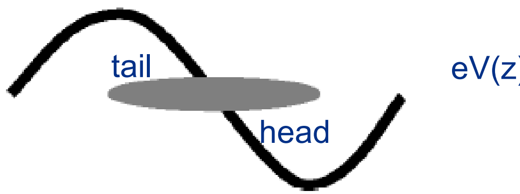
Analytical Description of Bunch Compressor

- Let's use the linear transfer matrices to evaluate the evolution of the longitudinal position and relative energy offset (z, δ).

- Passing through the RF cavity (assuming the cavity is thin), the longitudinal coordinate does not change, while the energy change depends on the initial position as follows:

$$
\begin{aligned}
z_1 &= z_0 \\
\delta_1 &= \delta_0 + \frac{eV_{RF}}{E_0} \sin(\pi + k_{RF} z_0) \approx \delta_0 - \frac{eV_{RF}}{E_0} k_{RF} z_0
\end{aligned}
$$

Reference beam energy



Assume zero crossing for simplicity

$$
\begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ R_{65} & 1 \end{pmatrix} \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}, \quad R_{65} = -\frac{eV_{RF}}{E_0} k_{RF}
$$

- The next step is to take into account the bunch compressor itself.

$\propto$ Momentum compaction

$$
\begin{aligned}
z_2 &= z_1 + R_{56}\delta_1 + \overbrace{T_{566}\delta_1^2 + U_{5666}\delta_1^3 + \cdots}^{\text{Nonlinear terms}} \\
\delta_2 &= \delta_1
\end{aligned}
$$

$$
\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \begin{pmatrix} 1 & R_{56} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix}
$$

Analytical Description of Bunch Compressor

- The full transformation is given by multiplying the matrices of each element:

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \mathbf{M} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} 1 + R_{65}R_{56} & R_{56} \\ R_{65} & 1 \end{pmatrix}$$

- Then the final bunch length is

$$\sigma_{z2} = \sqrt{\left(1 + R_{65}R_{56}\right)^2 \sigma_{z0}^2 + R_{56}^2 \sigma_{\delta 0}^2}$$

- In order to achieve maximal compression, we need should adjust R_{65} (Energy chirp induced by the RF cavity) or RF voltage in such a way that

$$R_{65}R_{56} \approx -1$$

[Note]

- Taking second and higher-order terms into account will give increased final bunch lengths.
- Linear transformation preserves the longitudinal beam emittance:

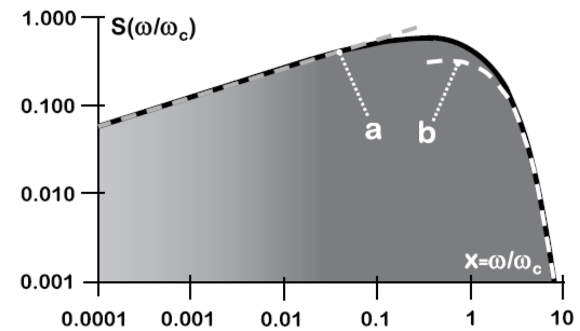
$$\varepsilon = \sqrt{\sigma_z^2 \sigma_\delta^2 - \sigma_{z\delta}^2}$$

- The achievable compression is limited due to the effects associated with beam's high peak current: longitudinal space-charge (LSC), wakefields and coherent synchrotron radiation (CSR).

10.1.2 Coherent Synchrotron Radiation (CSR)

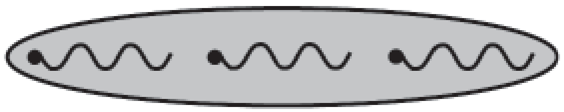
- Characteristic frequency of Synchrotron Radiation (SR):

$$\omega \lesssim \omega_c = \frac{3}{2}\frac{c\gamma^3}{R}$$

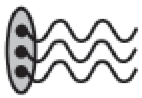


- Wavelength of the radiation $\sim c/\omega$ and bunch length σ:

$\frac{c}{\omega} < \sigma$



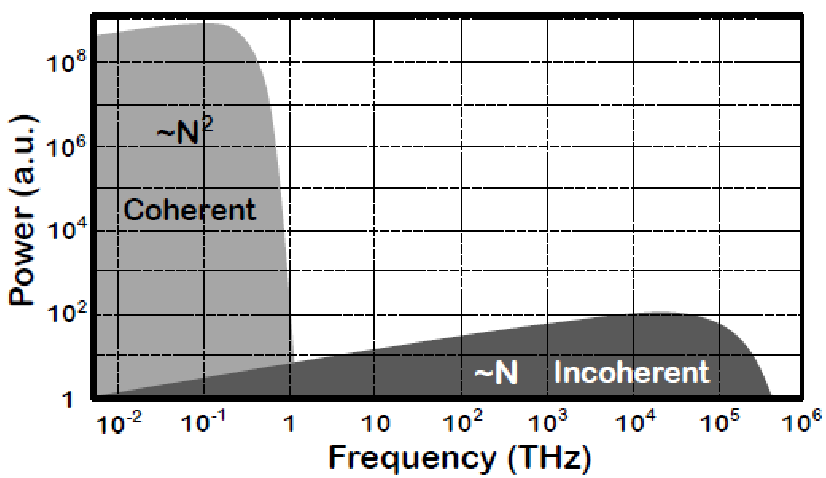
Incoherent radiation



$\sigma < \frac{c}{\omega}$

Coherent radiation

- Radiation power:



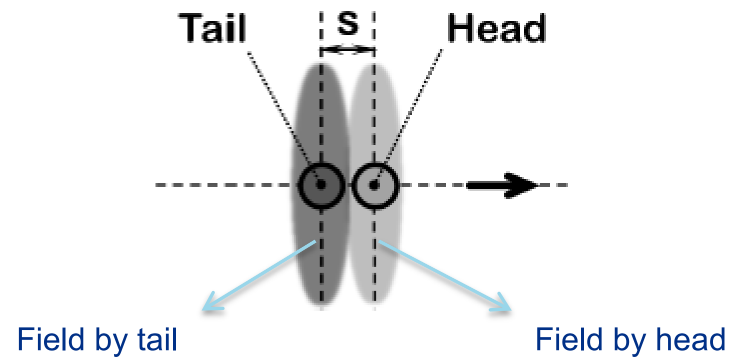
$$\text{Radiation power} \propto \left(\sum_{j=1}^{N} E_j e^{i\phi_j}\right)^2$$

$$= \sum_{j=1}^{N} E_j^2 + \sum_{j=1}^{N}\sum_{k\neq j}^{N} E_j E_k^* e^{i(\phi_j-\phi_k)}$$

If $\phi_j \approx \phi_k$, then the 2nd term >> 1st term as there are N^2 of them and results in coherent radiation

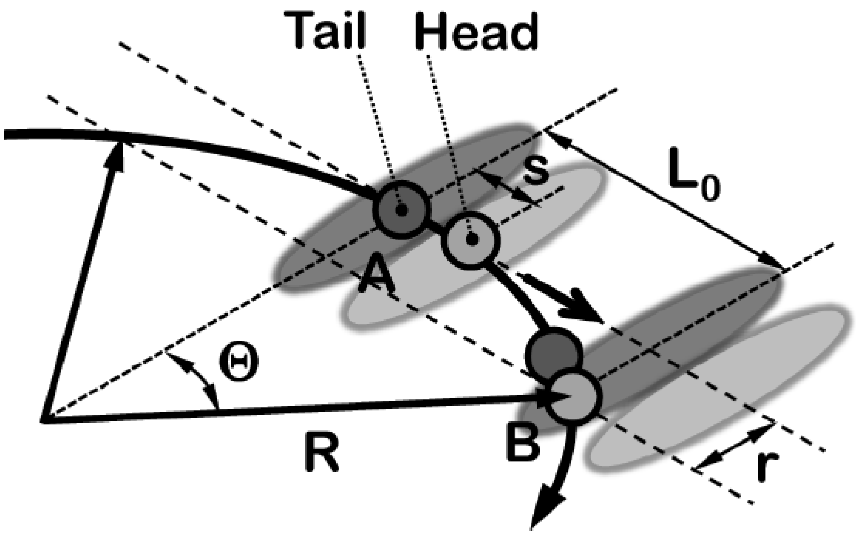
10.1.3 CSR Effects on the Longitudinal Motion

- Two particle model: the beam is represented just by two particles (at the head and in the tail of the bunch).



- The CSR effects are essentially caused by the possibility for the tail field to overtake the head particle while the beam is moving on a curved trajectory.

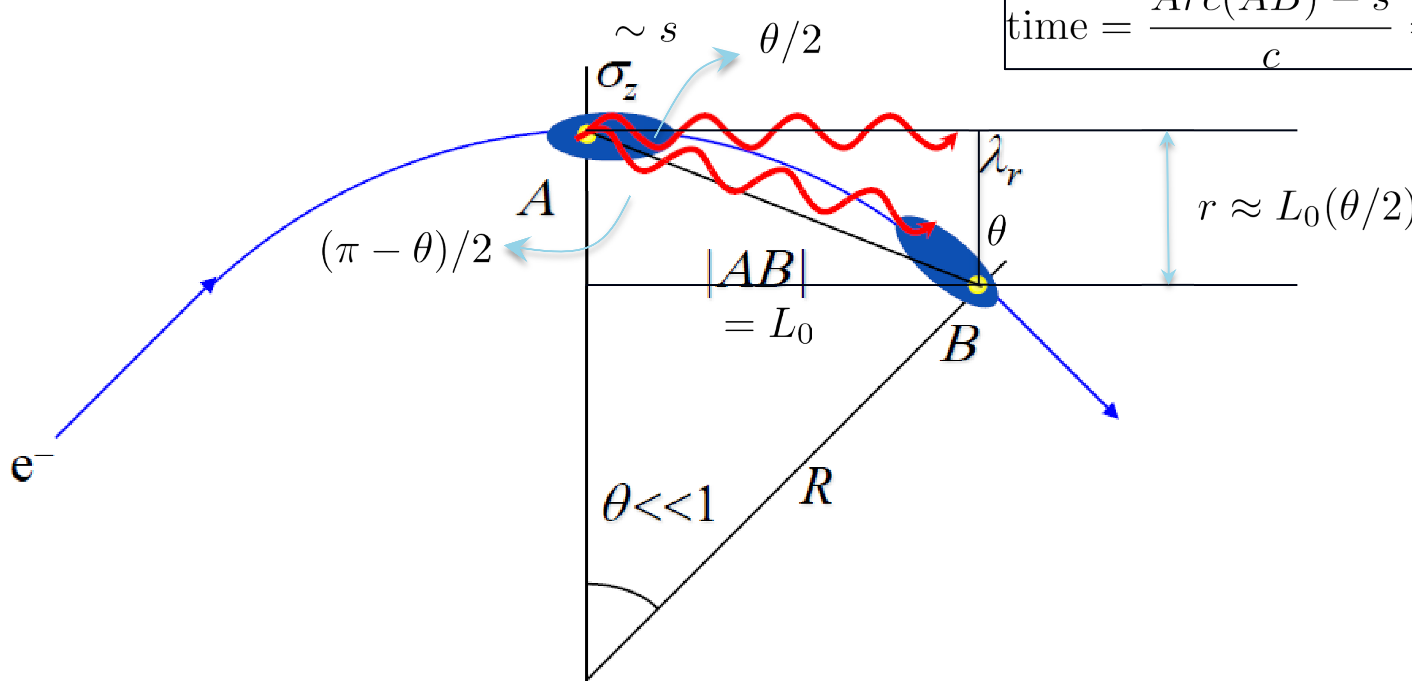
We assume both particles and fields are moving with c



10.1.3 CSR Effects on the Longitudinal Motion

- The condition for the catch-up (overtake) to happen in point B:

$$\boxed{\text{time} = \frac{Arc(AB) - s}{c} = \frac{|AB|}{c}}$$



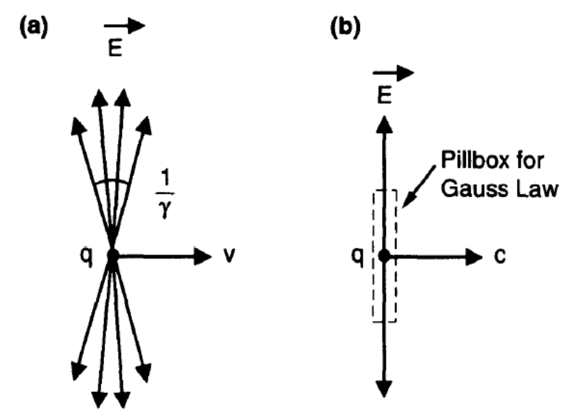
$$Arc(AB) - s = |\mathbf{AB}| \qquad R\,\theta - s = 2R\sin(\theta/2), \quad \text{which gives} \quad s \approx R\,\theta^3/24$$

- Overtaking distance L_0 can be estimated as

$$L_0 = |\mathbf{AB}| = R\,\theta \approx 2\left(3sR^2\right)^{1/3}$$

10.1.3 CSR Effects on the Longitudinal Motion

- Values of the transvers field (MKS unit):



$$E_\perp \approx \frac{\text{Line charge density}}{2\pi\epsilon_0 r} \approx \frac{eN\lambda}{2\pi\epsilon_0 r}$$

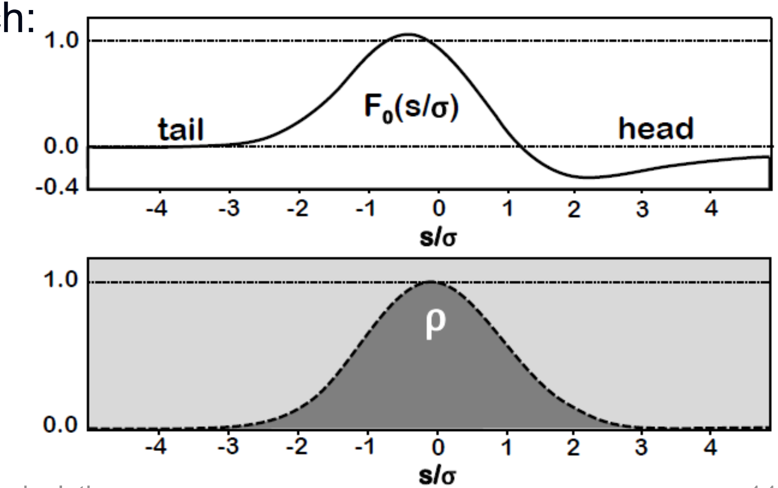
- Longitudinal force acting on the head:

$$F_{||} = eE_\perp \sin\theta \approx eE_\perp \theta = \frac{Ne^2\lambda\theta}{2\pi\epsilon_0 r} = \frac{Ne^2\lambda}{2\pi\epsilon_0 (3sR^2)^{1/3}}$$

- With more accurate derivation for a Gaussian bunch:

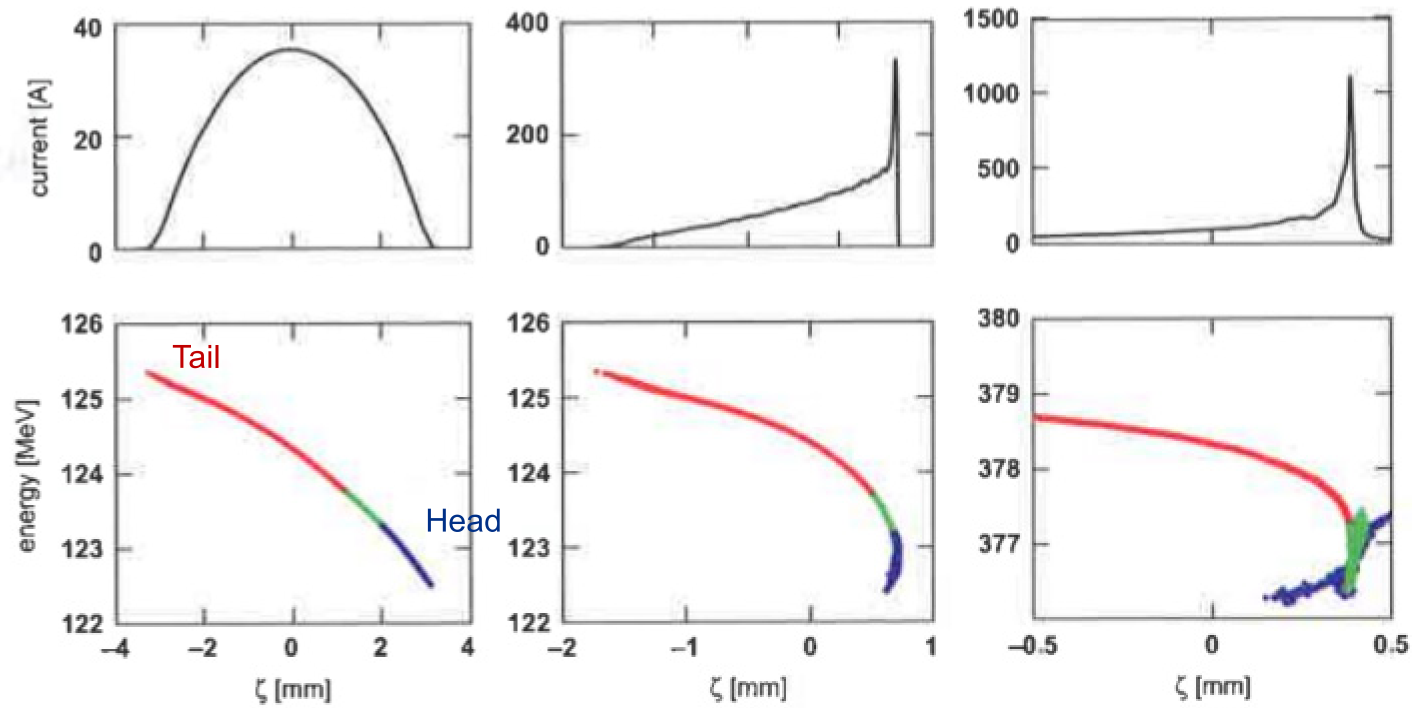
$$F_{||} = const. \times F_0(\text{Shape function})$$

  - The shape function changes its sign.
  - The head of the bunch slightly accelerates.
  - The major part of the bunch decelerates.



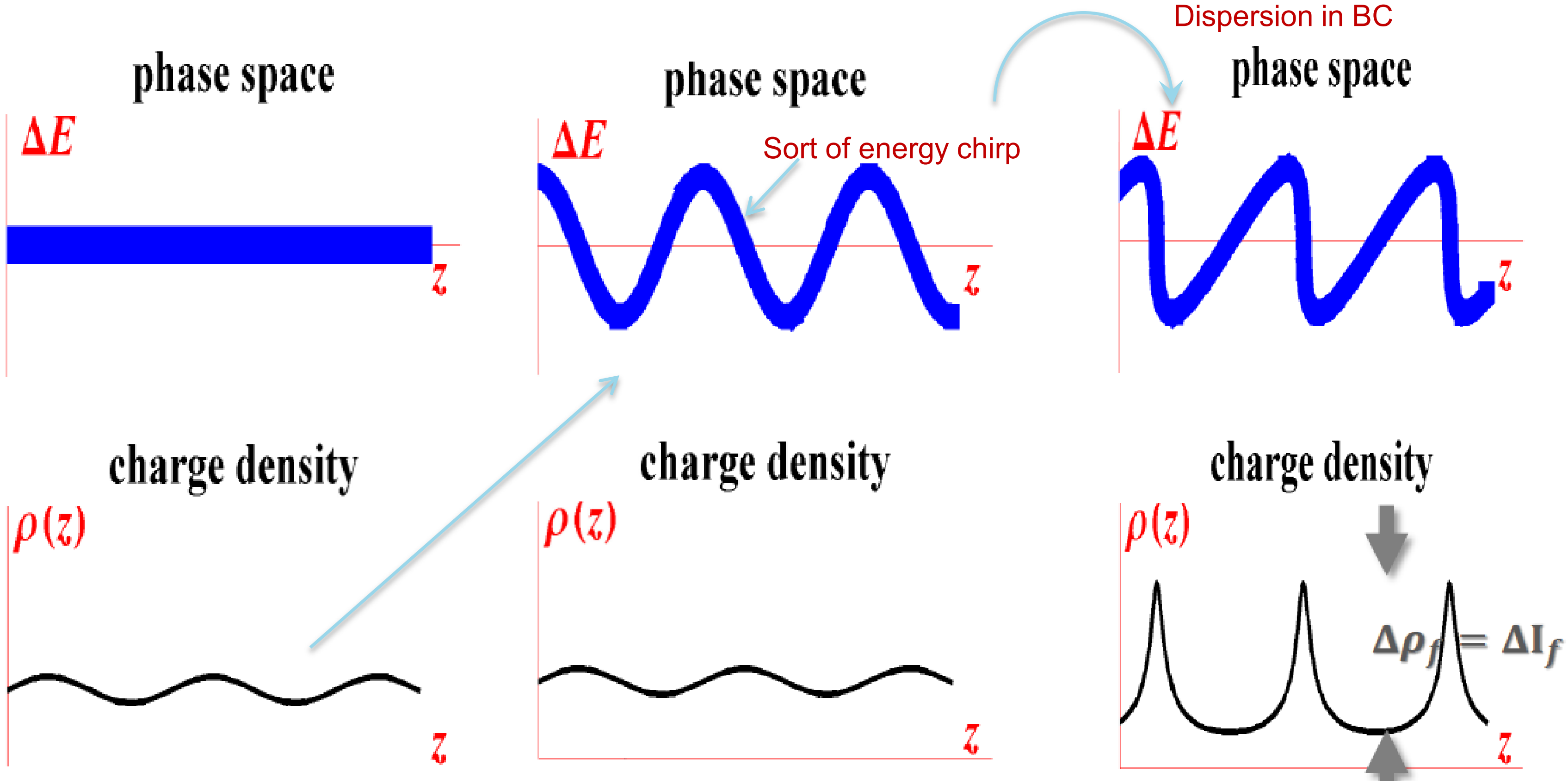
10.1.3 CSR Effects on the Longitudinal Motion

- The effects of CSR are particularly important in bunch compressors and can prevent achieving ultra-short bunches.
- CSR can cause bunch instability and microbunching, and can therefore deteriorate the longitudinal phase space of the beam.



Micro-bunching Instability

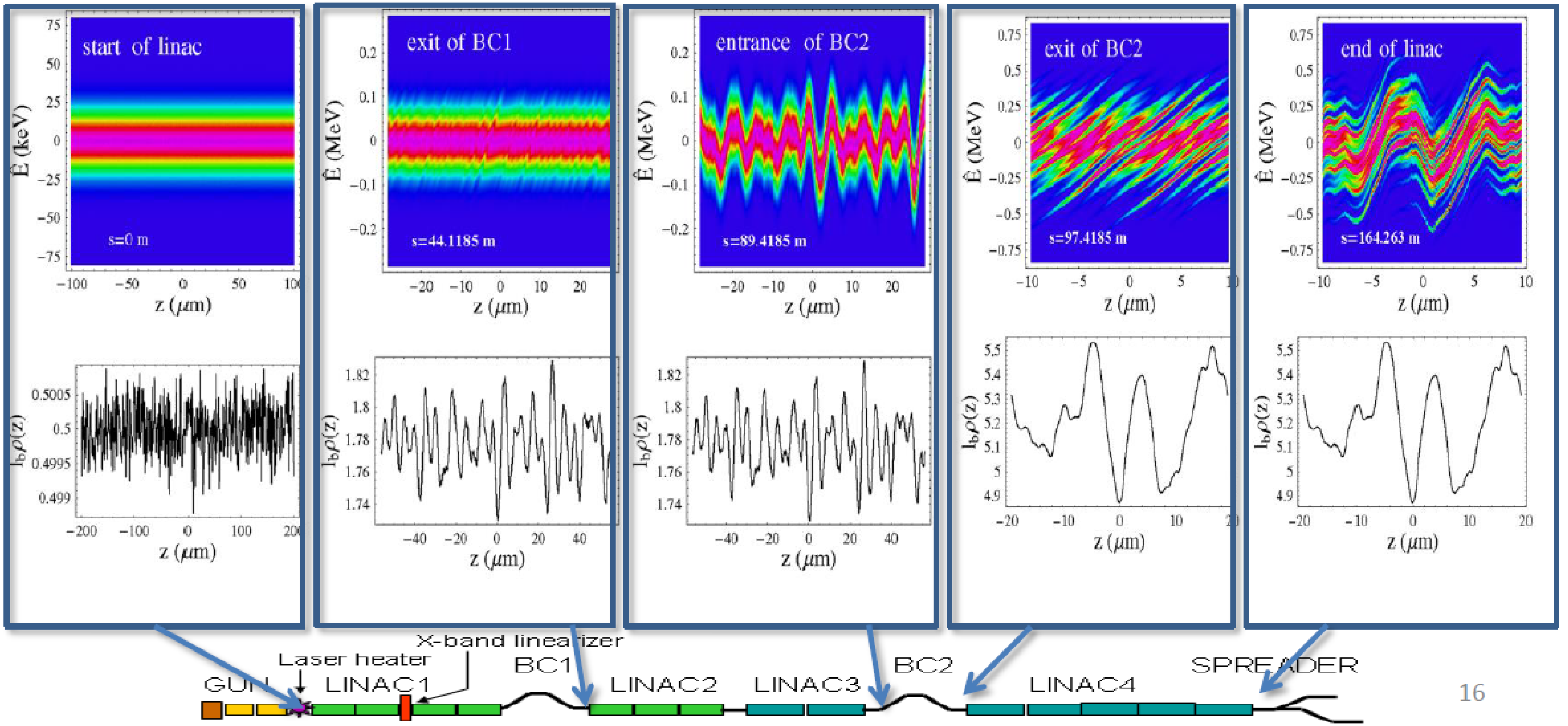
- Collective effects (Longitudinal Space Charge and CSR) turn ripples of charge-density into energy modulation.
- Dispersion turns energy modulation into larger charge-density ripples.
- Main adverse effect of micro-bunching instability is growth in energy spread.



Due to either LSC or CSR
(LSC is more relevant as dipoles are short compared to rest of machine)

Micro-bunching Instability

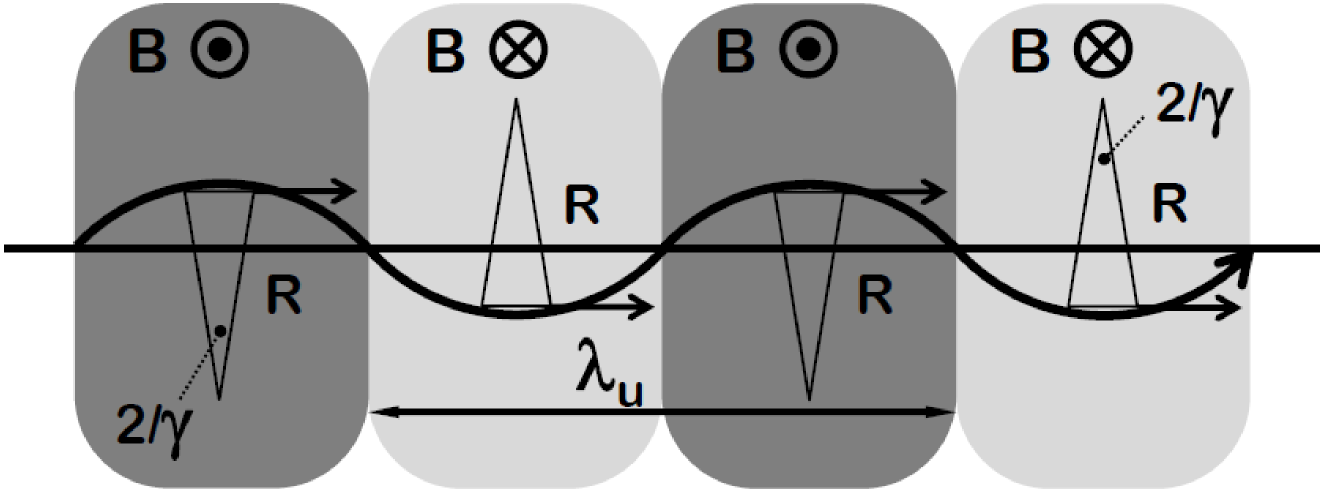
- Multiple-stage bunch compression enhances instability



Laser-Beam Interactions

Wiggler VS Undulator

- Bending magnets arranged in a sequence with NSNS polarity with period λ_u:



- Wiggler regime: If the length of the emitting region is much less than the length of an individual.

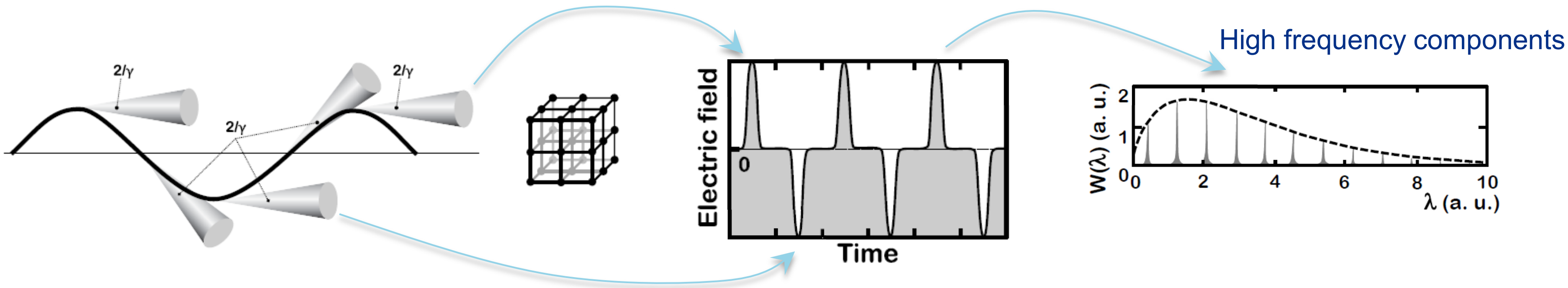
$$R\frac{2}{\gamma} \ll \frac{\lambda_u}{2} \rightarrow \text{the radiation emitted in each bend is independent}$$

- Undulator regime: The entire wiggling trajectory contributes to radiation, therefore interference leads coherence of radiation.

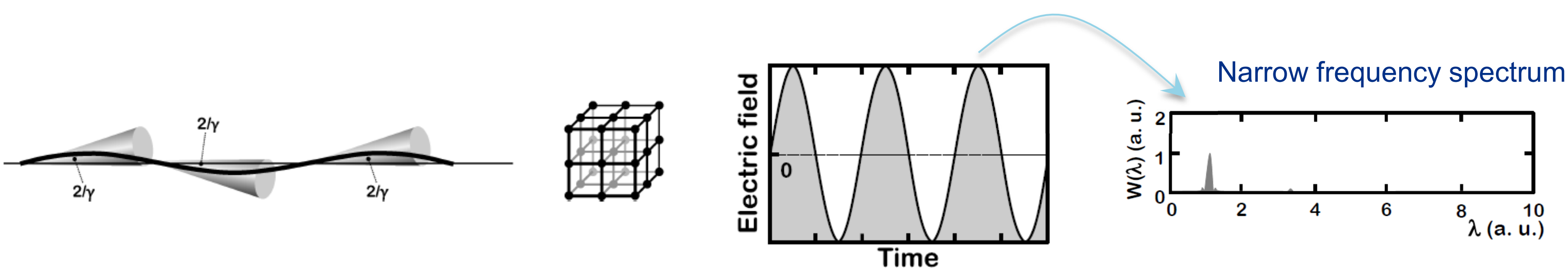
$$R\frac{2}{\gamma} \gg \frac{\lambda_u}{2} \rightarrow \text{the radiation is coherently build up}$$

Wiggler VS Undulator

- **Wiggler:** $R\dfrac{2}{\gamma} \ll \dfrac{\lambda_u}{2}$, or $K(\text{undulator parameter}) = \dfrac{\gamma\lambda_u}{2\pi R} \gg 1$



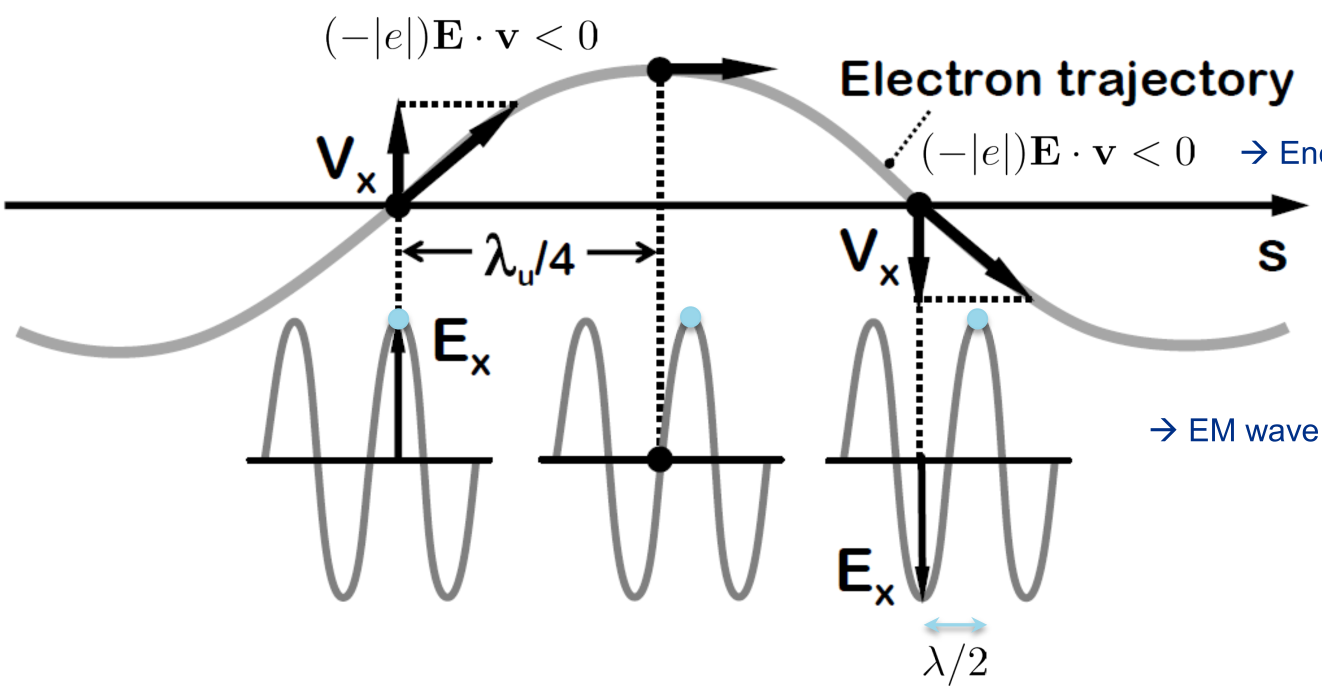
- **Undulator:** $R\dfrac{2}{\gamma} \gg \dfrac{\lambda_u}{2}$, or $K(\text{undulator parameter}) = \dfrac{\gamma\lambda_u}{2\pi R} \ll 1$



Particle and Field Energy Exchange

- There is longitudinal retardation of the particle in the undulator:

$$\langle v_z \rangle = v_z 0 + \text{sin-like trajectory contribution} = c\left(1 - \frac{1}{2\gamma^2}\right) - c\frac{K^2}{4\gamma^2} = c\left(1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right)$$



→ Energy transfer to wave

→ EM wave is moving faster

- The necessary condition for the resonant energy transfer is that the EM wave slips forward with respect to an electron by λ_u per half period of electron trajectory

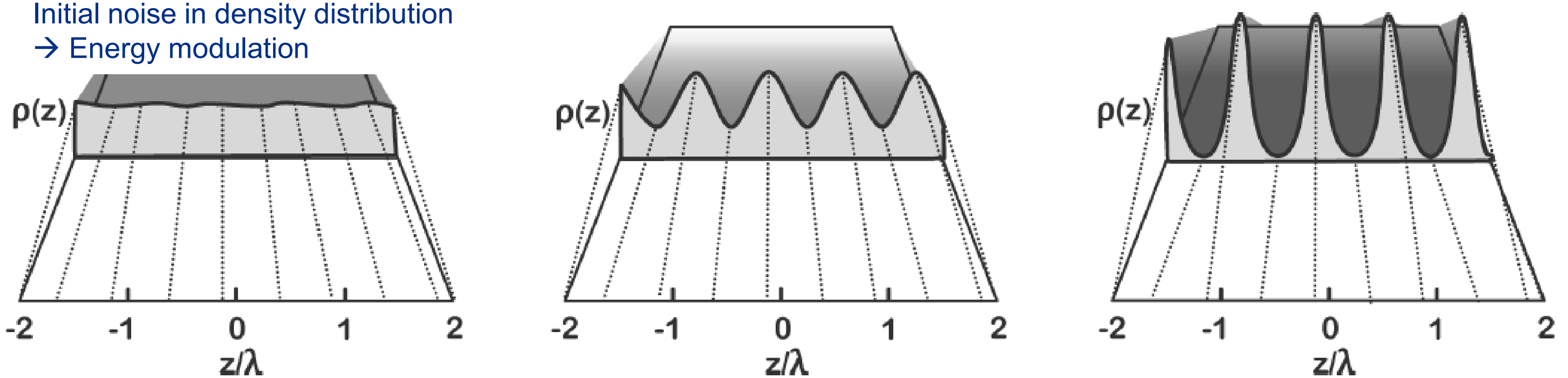
$$\frac{\lambda_u/2}{\langle v_z \rangle} = \frac{\lambda_u/2 + \lambda/2 + n\lambda}{c} \rightarrow \lambda = \frac{\lambda_u}{2n+1}\left(1 - \langle v_z \rangle / c\right) = \frac{\lambda_u}{m2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

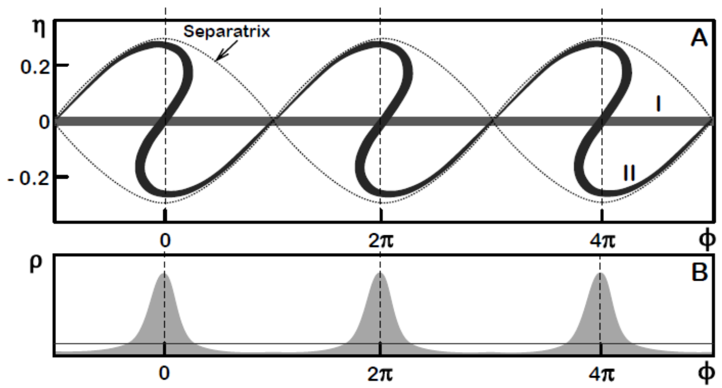
harmonics

Micro-bunching in the Undulator

- The interaction of particles with the resonant EM wave can create energy modulation in the particle beam.
- As particles move along the curved sine-like trajectory, the different path lengths can in turn create density modulations along the beam.

Initial noise in density distribution
→ Energy modulation

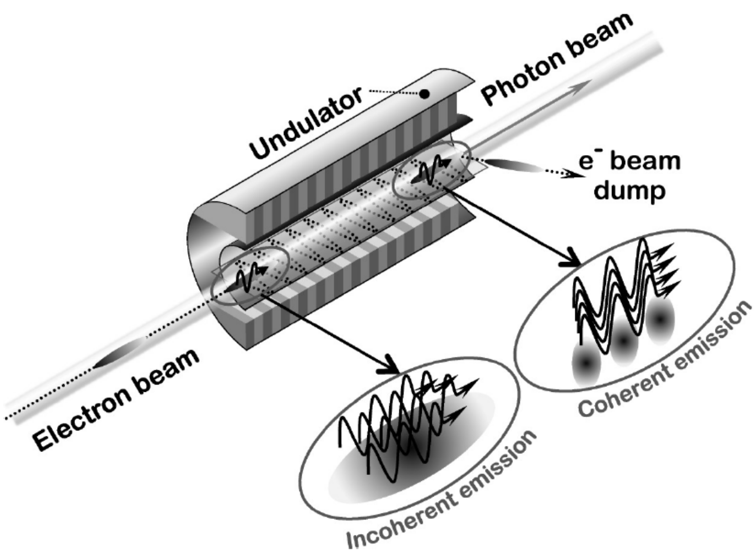


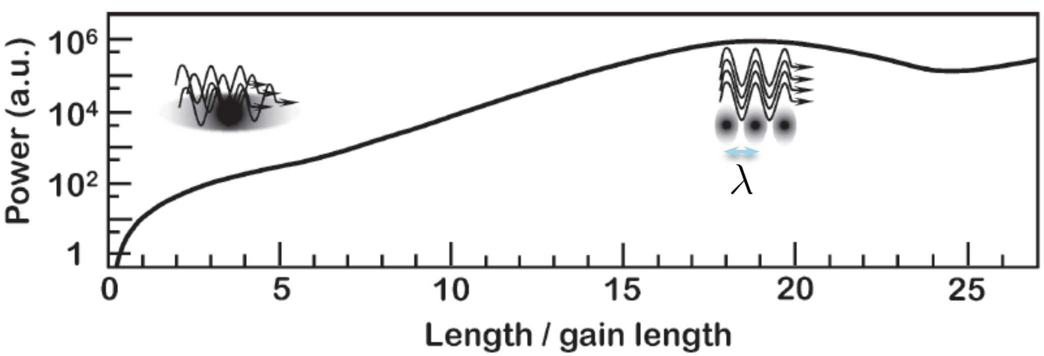


- An initial EM wave of resonant wavelength can be external (Seeding) or can emerge from the noise that is always present in the beam (SASE).

Single-pass FEL

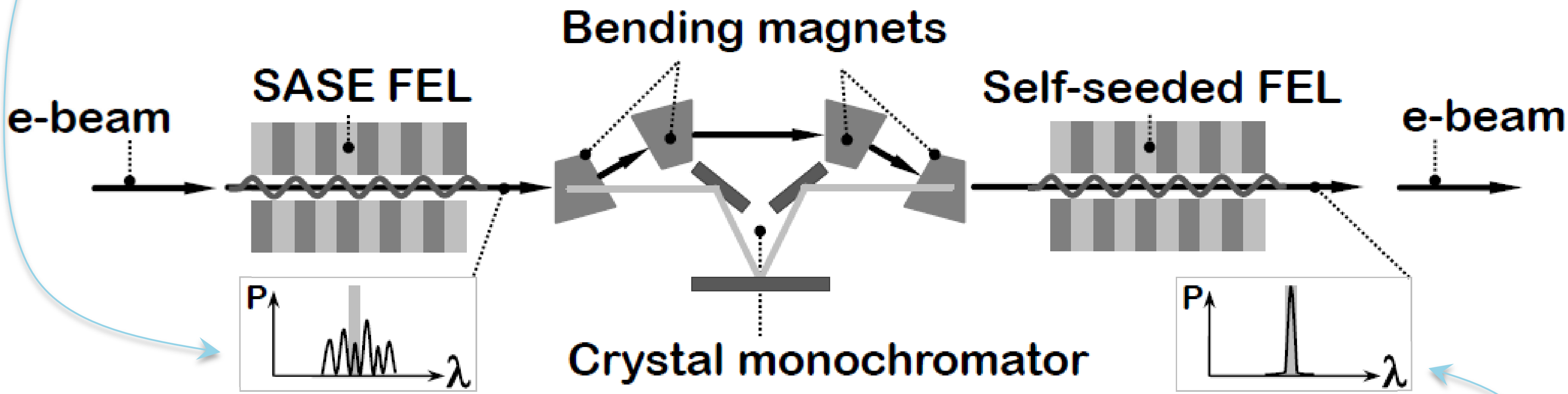
- In a single-pass FEL, the radiation has to grow within a single passage of the beam through the undulator (either seeded or SASE type).
- The need to use single-pass systems is actually a necessity dictated by an absence of good mirrors in the X-ray spectral region.
- The single-pass system has to be a high-gain system, which puts extreme constraints on the quality of the electron beam, as well as on the accuracy of the undulator.
- In the high-gain system, the gain is so large that the EM wave amplitude changes within a single pass in the undulator.





Self-seeded FEL

- In the SASE regime, the FEL lasing starts from noise → Each individual beam slice can generate radiation at slightly different wavelengths near the resonant wavelength, and, moreover, the amplitude of the radiation coming from each slice can be slightly different.



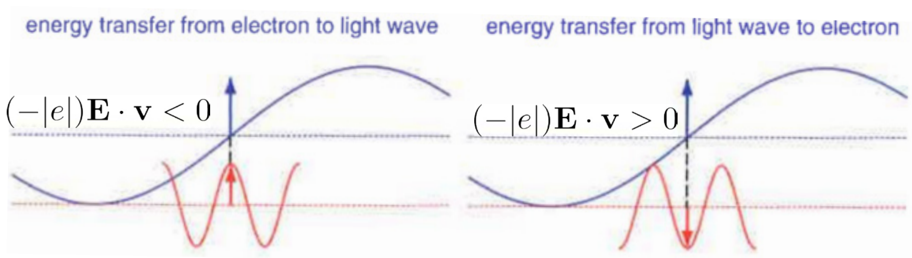
→ Filtering out other than the desired wavelength

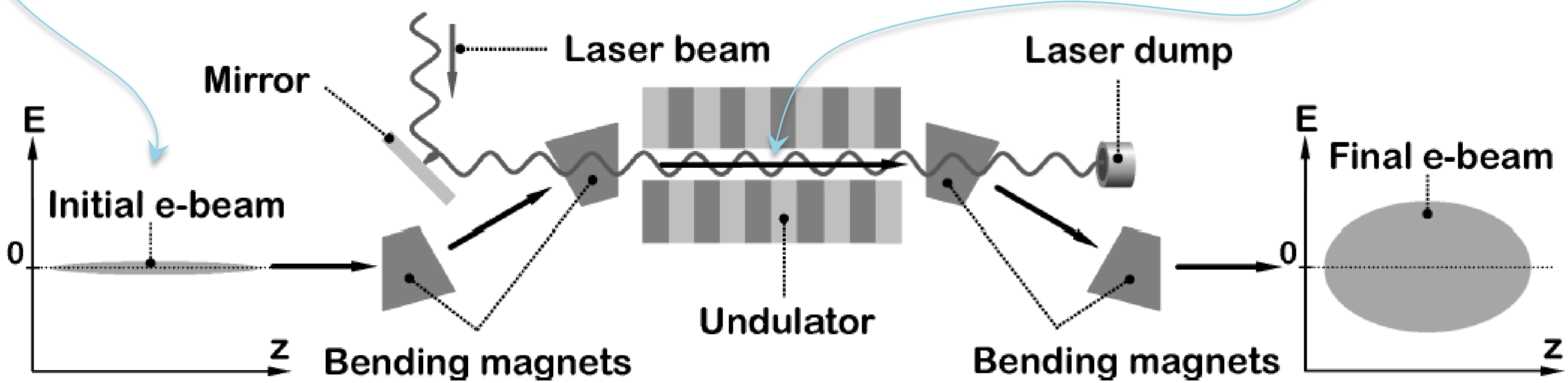
- In the self-seeded FEL, before entering the second part, radiation generated in the first part is passed through a crystal monochromator.

- The second part of the FEL is thus seeded with narrow spectrum radiation, which continues to be amplified along the way in the undulator; the resulting output spectrum of FEL then contains a narrow peak.

Beam laser heating

- The generation of femtosecond-short X-ray pulses requires fs-short electron bunches. However, the creation of short bunches in bunch compressors can be complicated due to CSR effects that can cause instability and microbunching.
- Instabilities can often be suppressed if the beam has sufficient spread of its relevant phase space coordinates (the energy spread in the case of CSR instability).
- The beam energy spread coming from a photocathode gun is extremely small, and insufficient for suppressing CSR instability.
- A method has been developed at SLAC to introduce additional uncorrelated energy spreads into the beam: Laser heater.





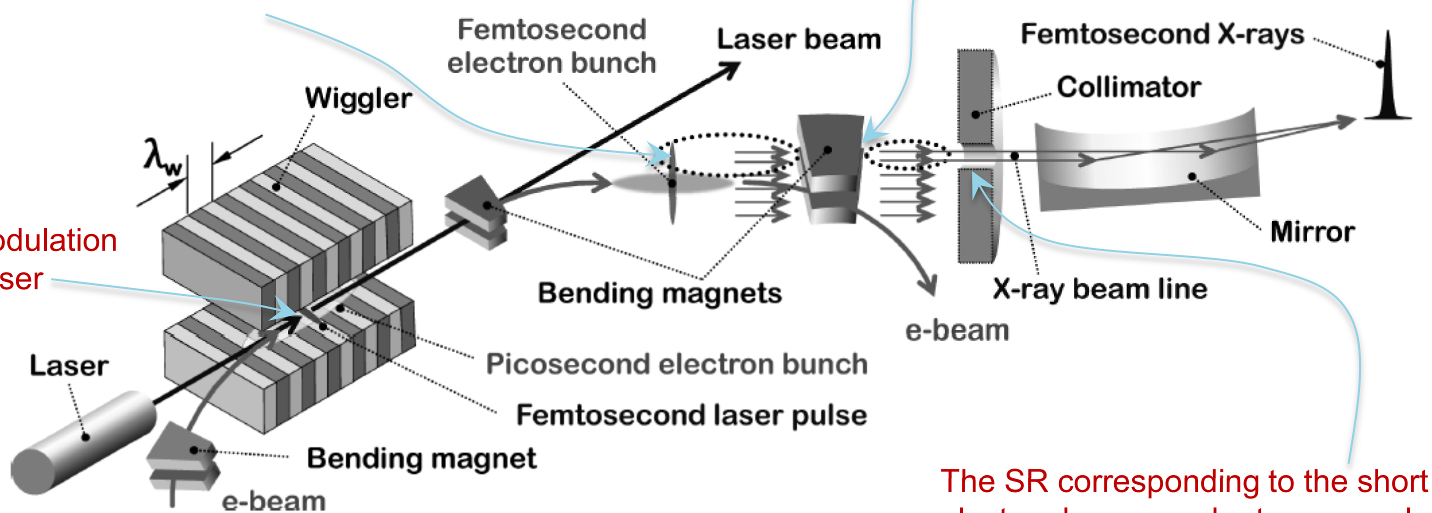
Beam laser slicing

- A technique that selects a femtosecond short portion of radiation from a much longer initial electron pulse. → Suitable for generating femtosecond synchrotron radiation pulses even for ring-based 3rd generation SR sources.
- A very short laser pulse overlaps with the center of a longer bunch in the undulator or wiggler. The laser wavelength matches the undulator resonance condition and produces modulation of energy in the short beam slice.

$$\lambda_L = \frac{\lambda_W}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Short energy modulation converted into spatial variation using a dispersion beamline

Synchrotron radiation in bending magnet

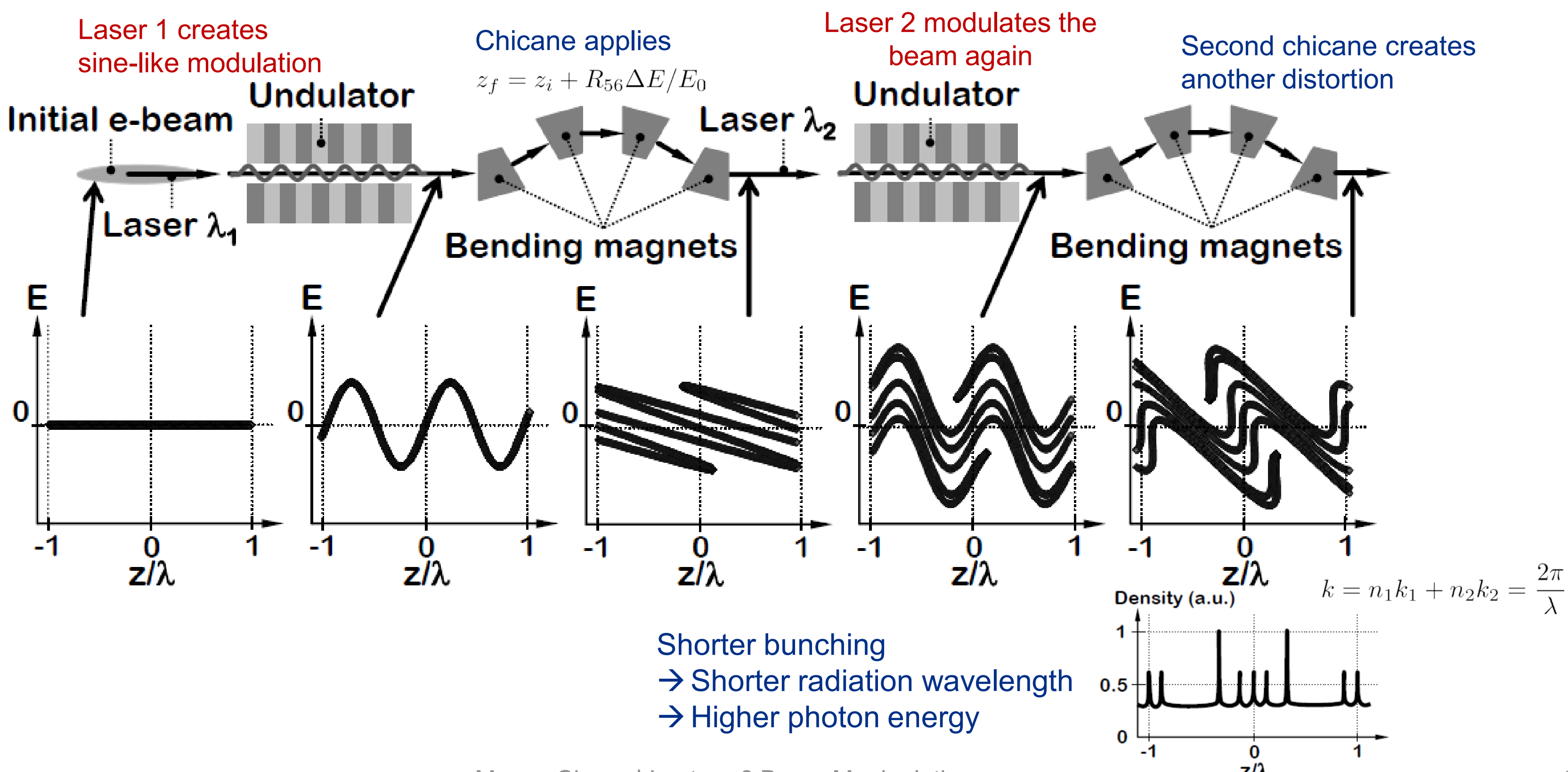


Energy modulation by short laser

The SR corresponding to the short portion of the electron beam can be transversely separated from the rest of the pulse using collimators.

Beam laser harmonic generation

- Harmonic generation is the technique that can help to produce X-ray photons of much higher energies.
- An FEL would normally generate primarily the first harmonic. In order to primarily generate a higher-order harmonic, external seeding is required. There are, however, no conventional lasers of appropriately short wavelengths that can be used for such seeding.
- A technique invented by G. Stupakov is currently solving the problem: echo-enabled harmonic generation (EEHG).

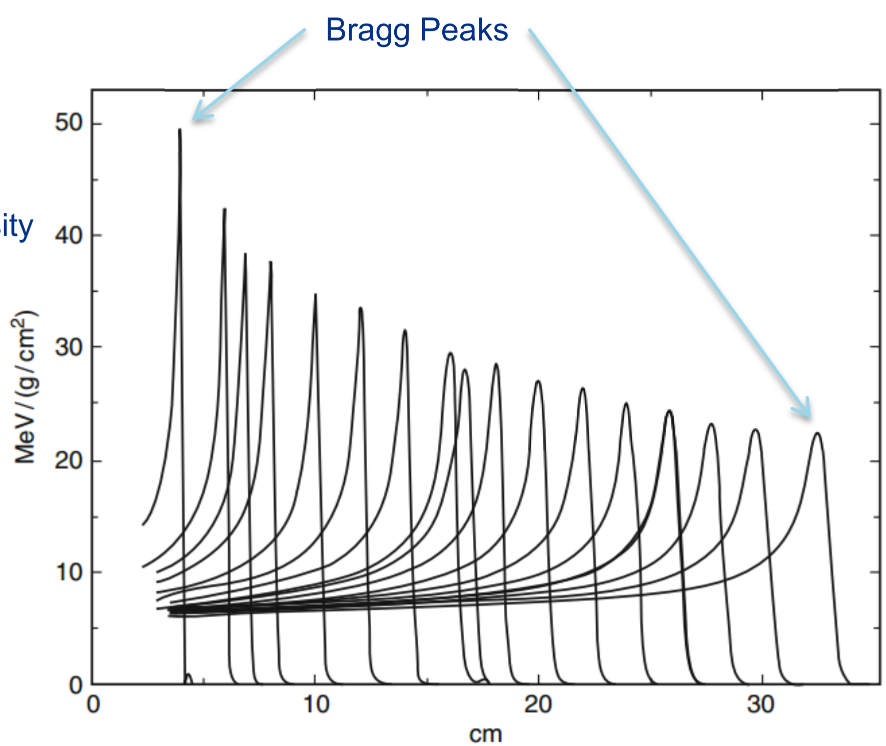


Beams for Cancer Therapy

Bragg Peak

- Charged particles lose their energy in matter primarily through a Coulomb interaction with the outer-shell electrons of the absorber's atoms.
- Excitation and ionization of atoms result in a gradual slowdown of the particle.
- A slower moving particle will interact with an atom for longer time, resulting in a larger energy transfer.
- Therefore, the charged particles will have an increased energy loss per unit length at the end of their passage through the medium.

Energy loss
per unit length
per unit mass density



Penetration ranges for protons in water:

250 MeV — 38 cm;
200 MeV — 26 cm;
150 MeV — 15.6 cm;
100 MeV — 7.6 cm;
50 MeV — 2.2 cm.

Stopping Power

- Quantitatively, the Bragg peak can be explained by the following formula for the mean energy loss (or stopping power) of moderately relativistic heavy particles (Bethe-Bloch equation):

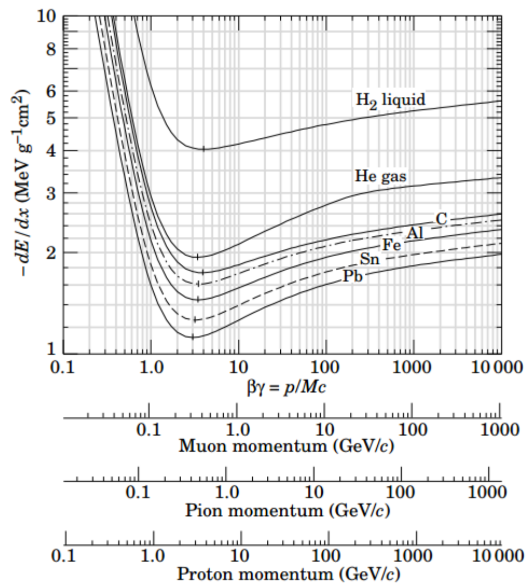
$$\left\langle -\frac{dE}{\rho ds}\right\rangle = \left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{2m_ec^2\beta^2\gamma^2W_{max}}{I^2}\right)-\beta^2-\frac{\delta}{2}\right]$$

Be careful, x is not length !
It is length times density

$$K = 4\pi N_A r_e^2 m_e c^2$$

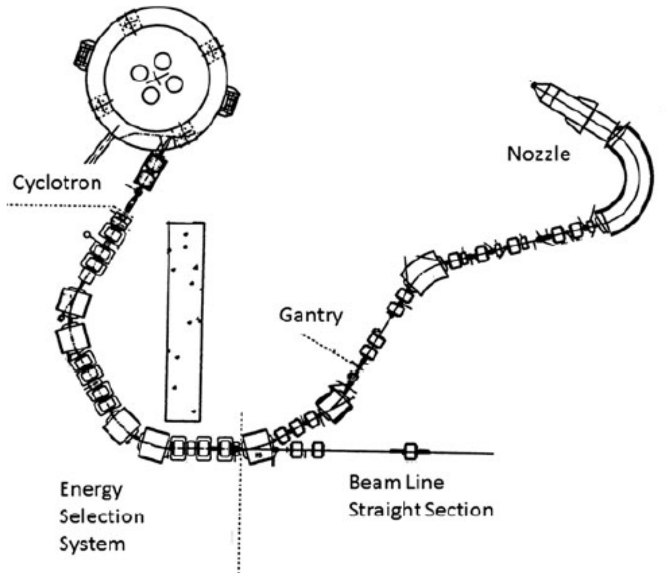
Density effect correction

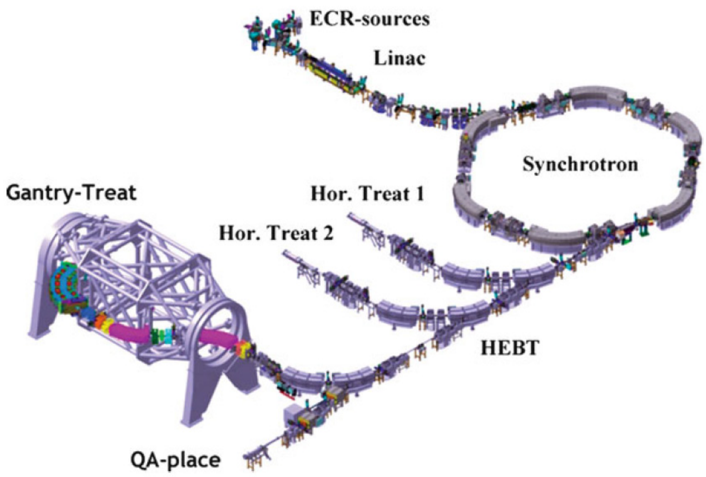
- Here, z is the charge number of an incident particle, Z and A are the charge number and atomic mass of the absorber, respectively. I — the mean excitation energy of the atom's electron, and W_{max} — the maximum energy transfer in a single collision.



Proton/Heavy Ion Therapy Facilities

- For proton machine: Cyclotron-based; For heavy ion (C) machine: Synchrotron-based.



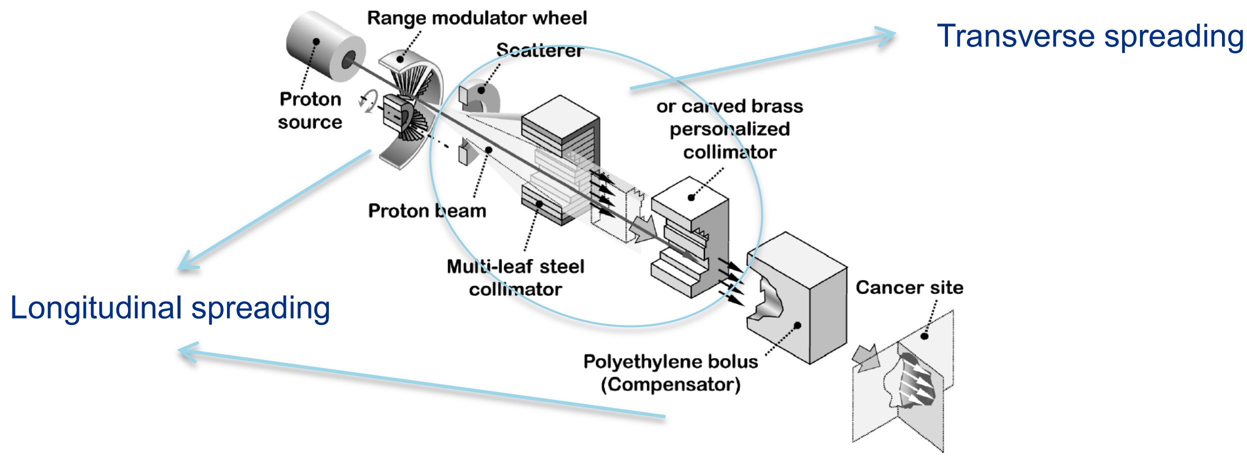


- To provide full geometrical flexibility for the tumor treatment, the beam should be able to enter the patient from different directions (done via rotating beam lines: gantry systems).
- The overall size of the proton/ion treatment facilities is defined by the size of the accelerator and beamlines, but, most notably, by the size and the height of the gantries.
- The energy of the beams and the achievable strength of the magnets would typically make the gantry on the order of ten meters in size and a hundred of tons in weight.
- The use of SC magnets in the gantries is possible and would typically reduce the weight of the gantry by around a factor of four.

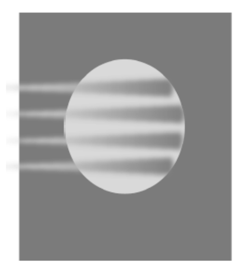
Beam Spreading

- The particle beam should cover the target volume with a prescribed dose distribution. The particle beam extracted from an accelerator normally has beam dimensions smaller than that of a typical target.
- The beam transported to the patient must be spread out in both transverse and longitudinal directions.

- For cyclotrons: the beam energy cannot be changed easily.



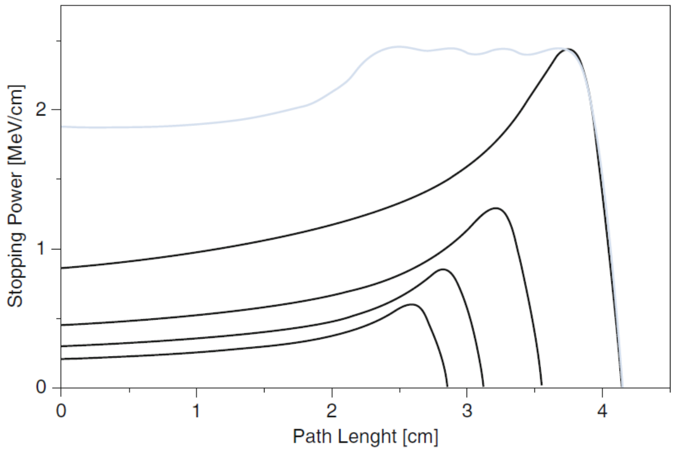
- For synchrotrons: the penetration depth can be adjusted by the beam energy.



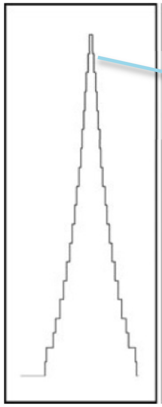
Pencil beam scanning

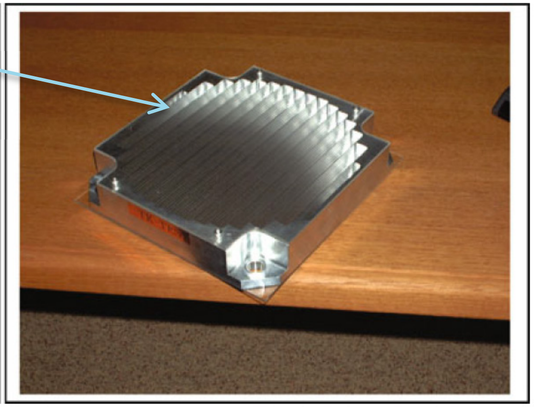
Longitudinal Beam Spreading

- Multiple Bragg peaks of different ranges can be added to approximate a flat spread-out depth dose distribution.



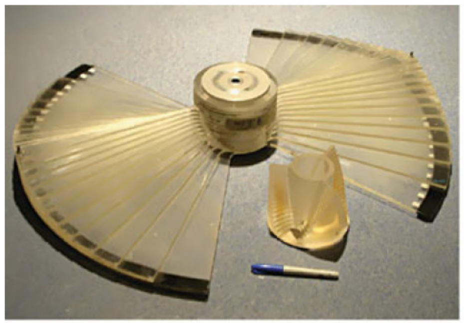
- Ridge (산등성이) Filter:

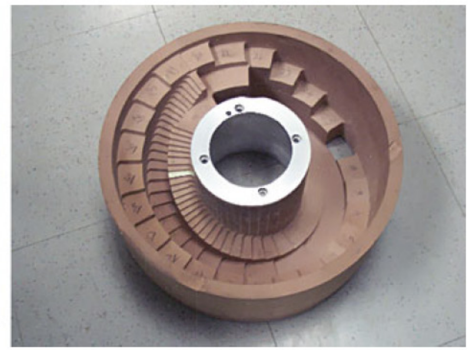




Longitudinal Beam Spreading

- Range Modulator Wheel: A dynamic version of longitudinal beam spreading.
  - Assuming that this wheel rotates at a constant rate that the incident beam flux is constant, then the number of ions that are degraded to a given energy will depend upon the amount of time a given thickness of the degrader wheel is in the path of the beam.

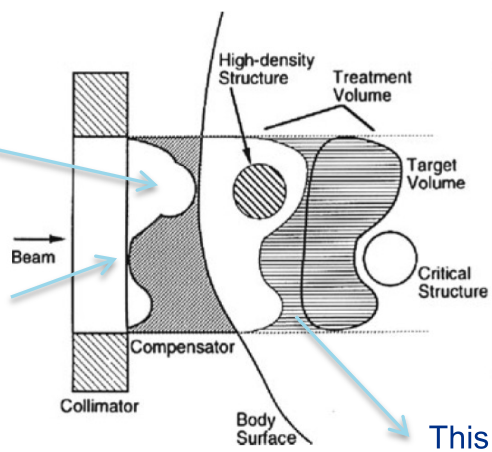


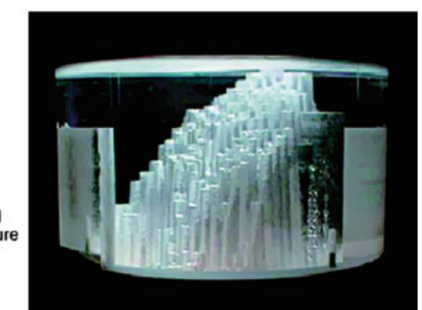


- Range Compensator: Compensate for the depth difference of different regions of the target volume.

Less energy loss

More energy loss

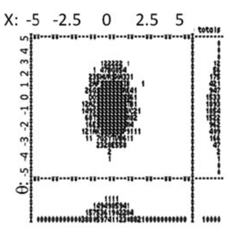




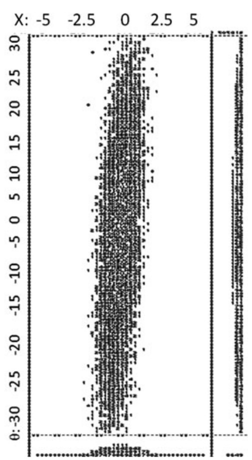
This range spread is achieved by range modulator

Transverse Beam Spreading

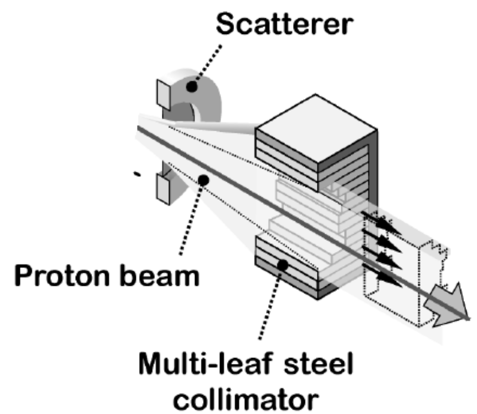
- First, the beam coming from the source passes through a scatterer, which increases the divergence of the beam.
- The best material to affect the scattering without losing too much energy is lead.



Before scattering

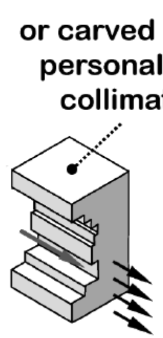


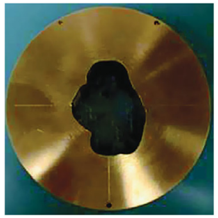
After scattering + drift



- The transverse beam shape is then adjusted by a multi-leaf steel collimator to match the shape of the target volume. (mechanically controlled on a sufficiently short time scale)

- Instead, a fixed-shape brass(황동) collimator can also be used.

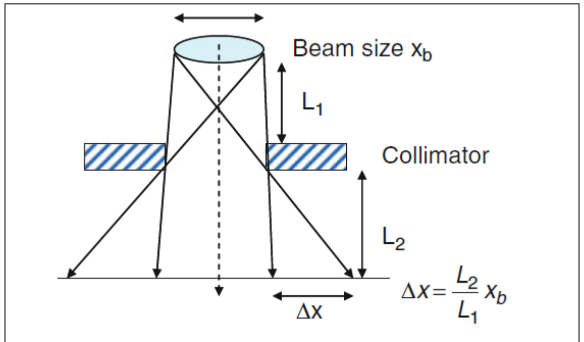




Transverse Beam Spreading

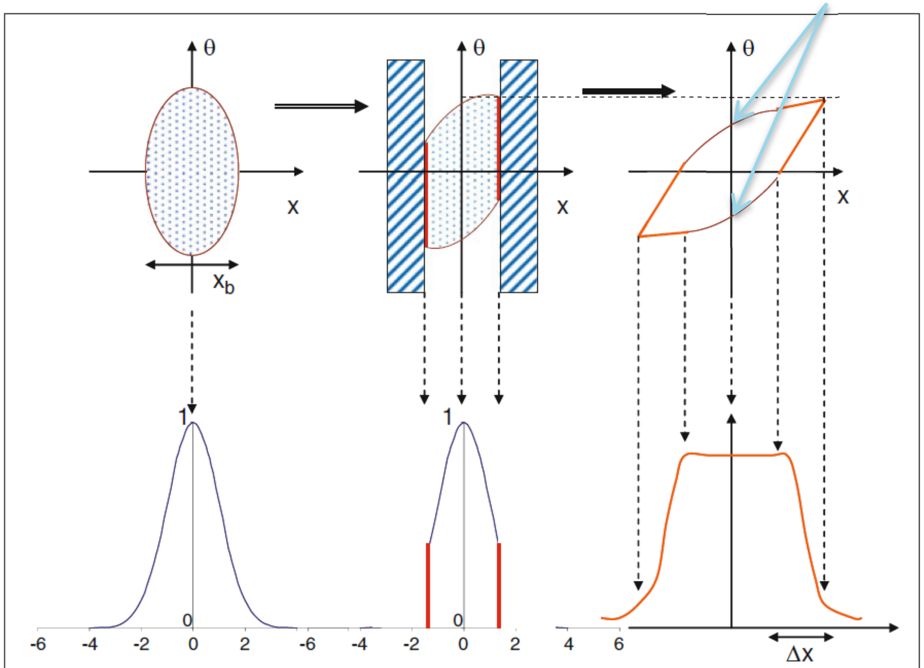
- When there is a finite distance between collimator and target, the final distribution of the beam contains an intensity fall-off at the edges.

- In the end one would achieve a flat central distribution (since the integral of all angles along a given x position results in the same number in the curved portion of the phase space area).



Geometric Representation of Collimation

Nearly parallel



Optical Representation of Collimation

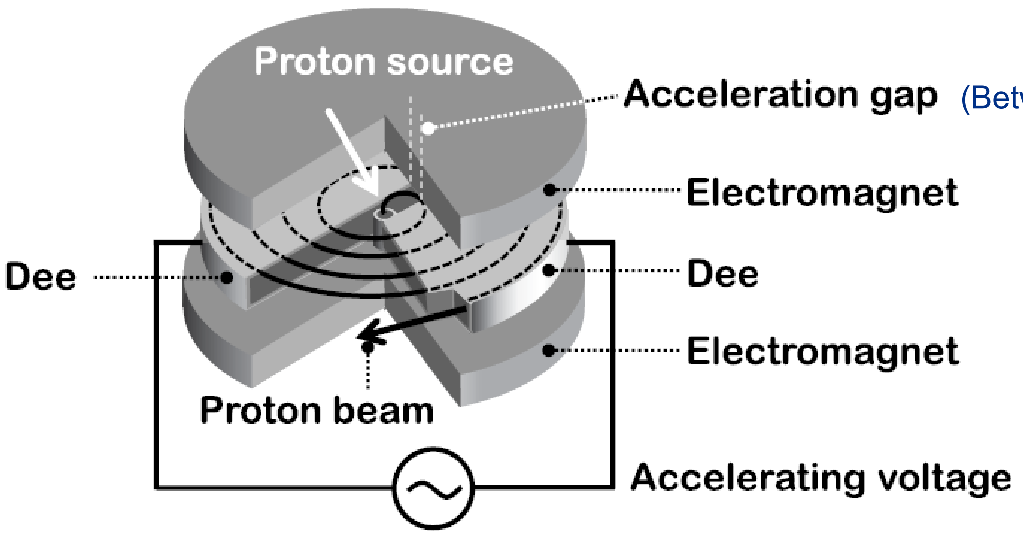
Standard Cyclotrons

- The standard cyclotron is intended for CW operation, and this is in fact one of its main advantages.

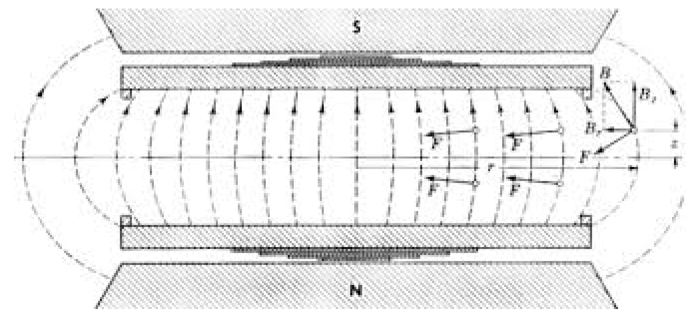
Constant frequency of accelerating RF voltage

Constant magnetic field

$$\omega_0 = const. = \frac{v}{R} = \frac{qB_0}{m\gamma}$$



(Between two Dee's)



Basically weak focusing

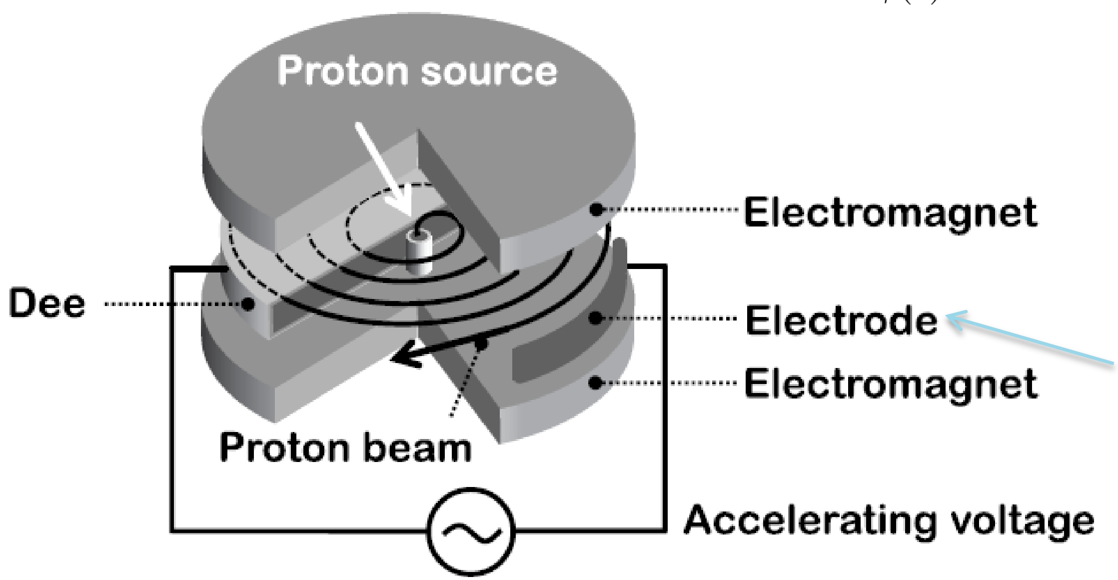
- Problems:
  - The final energy of the cyclotron is fixed by its geometry and cannot be changed.
  - Standard cyclotrons also suffer from the relativistic effects.
  - Particle orbits are getting closer for higher energies → Space-charge effects.
  - Extraction requires high electric fields → H^- is used to improve extraction.

Synchrocyclotron

- In contrast to a standard cyclotron, a synchrocyclotron can have variable energies of the accelerated beam and it can also achieve higher energies — several hundreds of MeV.

- The relativistic effects are compensated by continuously decreasing the frequency of the accelerating voltage during acceleration.

$$\omega(t) = \frac{\omega_0}{\gamma(t)}$$



Only one dee remains, while the other electrode has a modified open shape

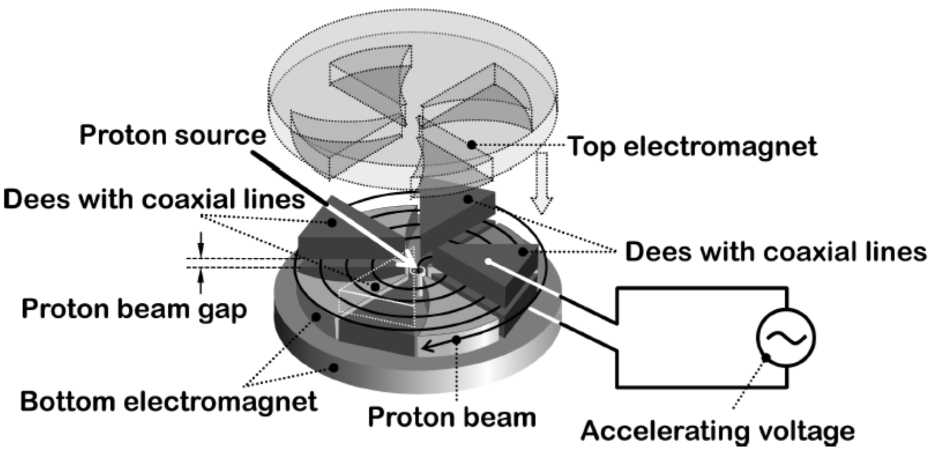
- Problems:
  - The time-varying accelerating frequency also means that only a bunch of a certain length can be in sync with the field—the synchrocyclotron therefore cannot accelerate CW current but can only produce pulsed beams (e.g., only few bunches for an injection-extraction cycle).

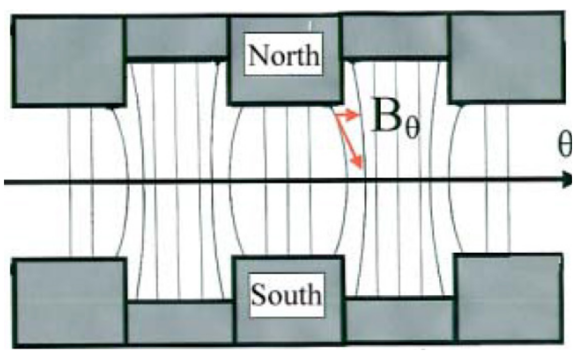
Isochronous cyclotron

- Isochronousim: Particle takes the same amount of time to travel one turn
- The relativistic effects are compensated by allowing the magnetic field to increase with the radius of the orbit.

$$B = \gamma(R) B_0$$

- The field increasing with the radius contradicts the requirements for weak focusing, and therefore strong focusing is arranged in isochronous cyclotrons by employing a spiral shape for the pole shims (often called *flutter* configuration).





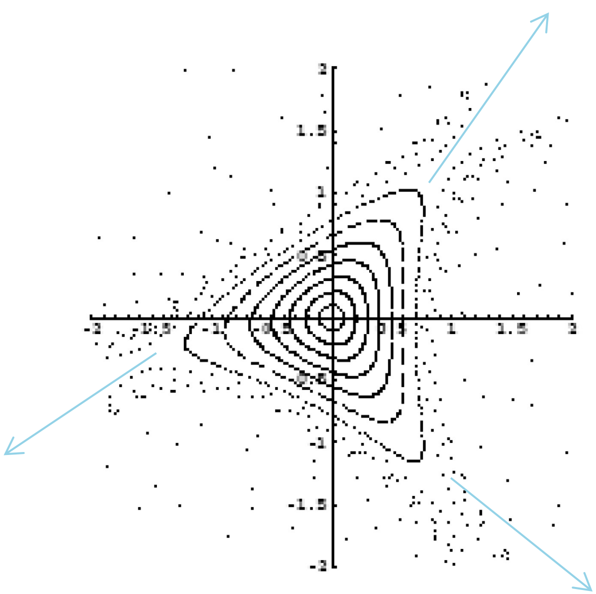
Sort of edge focusing

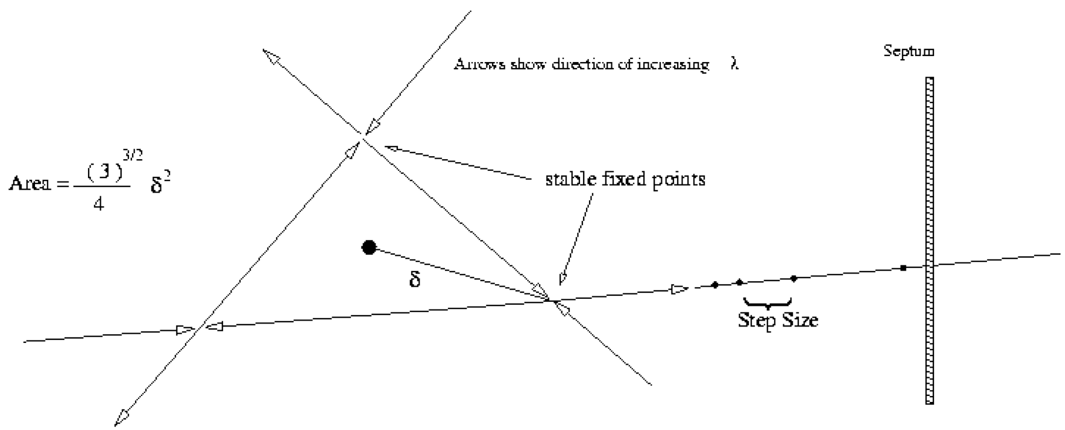
- The isochronous cyclotrons, as their acceleration frequency is constant, are suitable for CW operation.
- Proton energies of around a GeV can be achieved, but the final energy is fixed, as its adjustments require modification of the poles.

Slow Extraction from Synchrotron

- Synchrotron can provide variable energies for cancer therapy. However,
  - Synchrotrons are larger than cyclotrons
  - Synchrotrons are pulsed machines, while beam therapy benefits from a slow dose delivery

- Therefore, slow extraction methods, such as those based on excitation of nonlinear resonances, may need to be used.
  - Usually sextupoles are used to create a 3rd order resonant instability

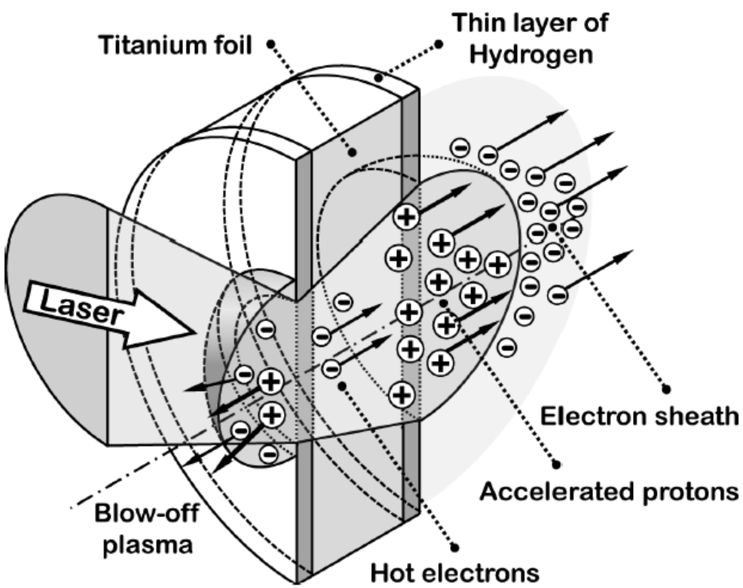


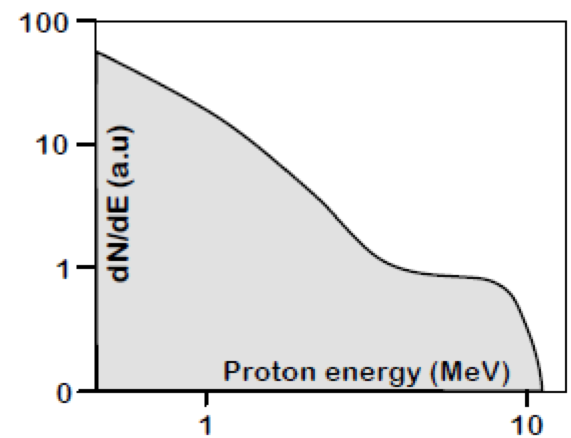


Particles will flow out of the stable region along lines in phase space into an electrostatic extraction field.

Laser-Driven Proton Generation

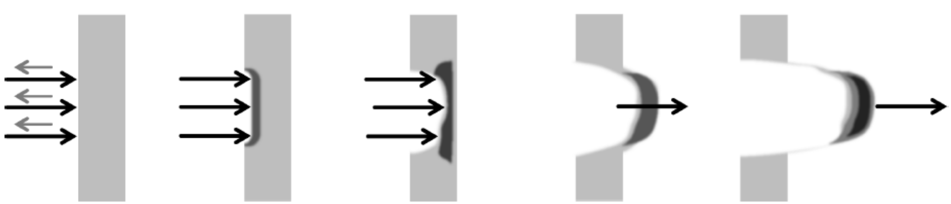
- Target normal sheath acceleration (TNSA): The plasma electrons quickly become relativistically hot and leave the foil, creating a sheath of charge, which then pulls out the ions and protons from the plasma.



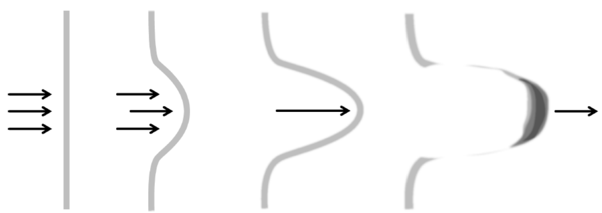


The spectrum of protons was, however, very broad and there were not many protons at the large energy side of the spectrum

- Radiation pressure:



Hole-boring



Light-sail