소염 근처 비예혼합 튜브형 화염의 열-확산 불안정성에 대한 수치 해석적 연구

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A Numerical Study of the Diffusive-Thermal Instability of Nonpremixed Tubular Flames near Extinction Limit

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ABSTRACT

The characteristics of the diffusive-thermal instability of opposed nonpremixed tubular flames near extinction limit are investigated using the linear stability analysis and 2-D high fidelity numerical simulations. It is confirmed that the diffusive-thermal instability occurs near 1-D extinction limit. In general, the maximum flame temperature increases with decreasing Damköhler number for cases with the same number of cells. However, the representative temperature for the flames with the same number of cells first decreases and then increases again as the Damköhler number approaches the 2-D extinction limit.

Key Words : Diffusive-thermal instability, Nonpremixed tubular flame, Linear stability analysis, Lewis number.

The characteristics of the diffusive-thermal instability of premixed and nonpremixed tubular flames near extinction limits have been extensively investigated by experiments [1-4]. In general, the diffusive-thermal instability of premixed flames occurs near the extinction limit, when the Lewis number of the mixture is less than unity. In nonpremixed flames, however, similar cellular instability can be observed when the Lewis number of fuel is less than unity as shown Fig. 1 [4].

In this study, we investigated the characteristics of the diffusive-thermal instability of opposed tubular nonpremixed flames using the conventional linear stability analysis together with high-fidelity numerical simulations. By assuming 1-step overall reaction, constant density, constant radial velocity without azimuthal direction velocity, the normalized 1-D governing equations for opposed nonpremixed tubular flame can be modeled as follows:

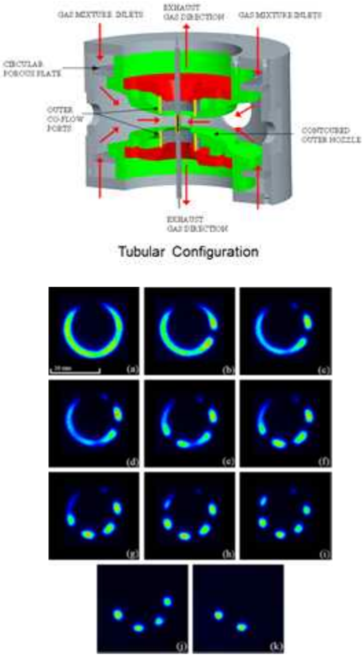

Fig. 1 Schematic of tubular nonpremixed flame burner (top, by courtesy of Prof. R. Pitz) and cell formation with different stretch rates (bottom) from Ref. [4].

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$$\tilde{u}_r \frac{d}{d\tilde{r}} \begin{pmatrix} \tilde{T} \\ \tilde{Y}_F \\ \tilde{Y}_O \end{pmatrix} = \frac{1}{\tilde{r}} \frac{d}{d\tilde{r}} \left(\tilde{r} \frac{d}{d\tilde{r}} \right) \begin{pmatrix} \tilde{T} \\ \tilde{Y}_F / L_F \\ \tilde{Y}_O / L_O \end{pmatrix} + Da\, \tilde{Y}_F \tilde{Y}_O e^{-\tilde{T}_a/\tilde{T}} \begin{pmatrix} q \\ -\alpha_F Y_{O,2} \\ -\alpha_O Y_{F,1} \end{pmatrix} \tag{1}$$

$$\begin{aligned} &\tilde{T} = \tilde{T}_2,\ \tilde{Y}_F = 0,\ \tilde{Y}_O = 1,\ \tilde{u}_r = -1, \quad \text{at} \quad \tilde{r} = \tilde{r}_2, \\ &\tilde{T} = \tilde{T}_1,\ \tilde{Y}_F = 1,\ \tilde{Y}_O = 0,\ \tilde{u}_r = \tilde{u}_{r,1}, \quad \text{at} \quad \tilde{r} = \tilde{r}_1, \end{aligned} \tag{2}$$

where T is the temperature, Y_F is the fuel mass fraction, Y_O is the oxidizer mass fraction, Da is the Damköhler number, q is the heat release rate, T_a is the activation energy, L_F is the Lewis number of the fuel, and L_O is the Lewis number of the oxidizer. The fuel and oxidizer issue from inner and outer nozzles, respectively, and nonpremixed tubular flame is formed between the nozzles. The subscripts 1 and 2 denote the fuel and oxidizer boundaries, respectively. In the present study, we adopted the following specific values: $q=1.2$, $\tilde{T}_a=8$, $L_F=0.3$, $L_O=1.0$, $\alpha_F Y_{O,2}=1.0$, $\alpha_O Y_{F,1}=0.36$, $\tilde{r}_1=20$, $\tilde{r}_2=100$, $\tilde{T}_1=\tilde{T}_2=0.2$, and $\tilde{u}_{r,1}$.The fuel Lewis number of 0.3 is adopted to describe the high diffusivity of hydrogen, which may lead to diffusive-thermal instability of tubular flames near the extinction limit.

The 1-D governing equations were solved by the finite difference method with the continuation method. Figure 2 shows the maximum flame temperature as a function of Da, which is a typical “C-curve” representing well-burning nonpremixed flames and extinction limit of the 1-D tubular flame. Note that the lower part of the C-curve is unphysical such that in real situations, one cannot observe such flames. The extinction Damköhler numbe, Da_E, is found to be approximately 13,894 and the corresponding maximum flame temperature is 0.9916.

Following the typical linear stability analysis [5], the solution variables of the 1-D tubular flame are perturbed in the radial and azimuthal directions, and time as in Eqs. (4)~(6). The bar denotes the mean value of each variable, ϵ a small quantity related to perturbation, λ the amplification factor in time, and k the wavenumber in the azimuthal direction.

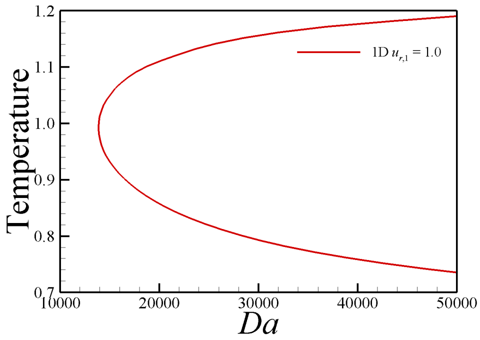

Fig. 2 Maximum flame temperature as a function of the Damköhler number.

By substituting Eqs. (4)~(6) into 2-D governing equations including the azimuthal direction terms, one can obtain the linear stability equations of $O(\epsilon)$ as in Eq. (7). Note that the equation of $O(1)$ satisfies Eq. (1) such that $O(\epsilon)$ is the leading order to determine the linear stability of the nonpremixed tubular flames.

$$\tilde{T} = \overline{T}(r) + \epsilon T'(r) e^{ik\theta + \lambda\tilde{t}} \tag{4}$$

$$\tilde{Y}_F = \overline{Y_F}(r) + \epsilon Y_F'(r) e^{ik\theta + \lambda\tilde{t}} \tag{5}$$

$$\tilde{Y}_O = \overline{Y_O}(r) + \epsilon Y_O'(r) e^{ik\theta + \lambda\tilde{t}} \tag{6}$$

$$\begin{aligned} &\frac{1}{\tilde{r}} \frac{d}{d\tilde{r}} \left(\tilde{r} \frac{d}{d\tilde{r}} \right) \begin{pmatrix} T' \\ Y_F'/L_F \\ Y_O'/L_O \end{pmatrix} - \tilde{u}_r \frac{d}{d\tilde{r}} \begin{pmatrix} T' \\ Y_F' \\ Y_O' \end{pmatrix} \\ &- \begin{pmatrix} k^2/\tilde{r}^2 + \lambda \\ k^2/\tilde{r}^2 L_F + \lambda \\ k^2/\tilde{r}^2 L_{O+}\lambda \end{pmatrix} \begin{pmatrix} T' \\ Y_F' \\ Y_O' \end{pmatrix} \\ &+ Da \left(\overline{Y_F} Y_O' + Y_F' \overline{Y_O} + \frac{\overline{Y_F}\,\overline{Y_O}\,\tilde{T}_a T'}{\overline{T}^2} \right) \\ &\times \exp\left(-\frac{\tilde{T}_a}{\overline{T}} \right) \begin{pmatrix} q \\ -\alpha_F Y_{O,2} \\ -\alpha_O Y_{F,1} \end{pmatrix} = 0. \end{aligned} \tag{7}$$

Introducing the finite difference approximation into Eq. (7) yields an eigenvalue problem, $\mathrm{A}x = \lambda \mathrm{B}x$, where x is the solution vector including T', Y_F', and Y_O'. The eigenvalue, λ, can be found by solving $\mathrm{A}x = \lambda \mathrm{B}x$ for different wavenumber, k.

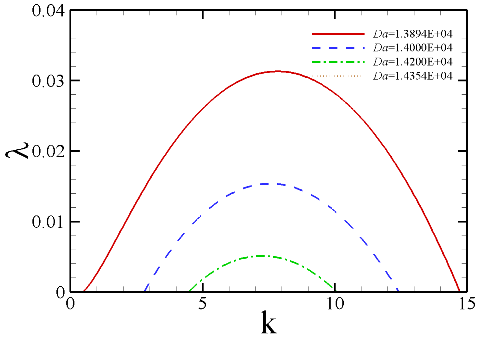

Fig. 3 Largest $\mathrm{Re}(\lambda)$ as a function of wavenumber, *k*, for different *Da*.

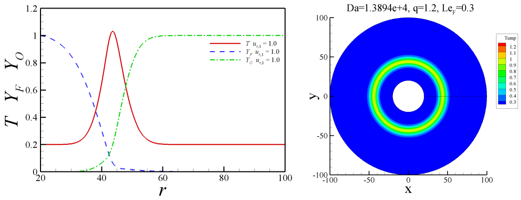

Fig. 4 1-D profiles of temperature, fuel mass fraction, and oxidizer mass fraction, and temperature isocontours of the initial condition for 2-D simulation for *Da* = 13,894.

Figure 3 shows the largest real eigenvalue as a function of the wavenumber for different *Da*. For large *Da*, the nonpremixed tubular flames are stable because $\mathrm{Re}(\lambda)$ is always less than zero for all wavenumbers. However, the tubular flames becomes unstable as *Da* approaches the extinction Da_E (= 13,894) since the corresponding $\mathrm{Re}(\lambda)$ has positive value in some range of *k* as shown in Fig. 3. The largest *Da* of which the largest $\mathrm{Re}(\lambda)$ is positive is found to be approximately 14,354. It is of interest to note that the range of wavenumber exhibiting positive largest $\mathrm{Re}(\lambda)$ is increased with decreasing *Da* down to Da_E.

Based on the results of the linear stability analysis and 1-D solutions for different *Da*, the high-fidelity numerical simulations of 2-D tubular flames are performed. For 2-D simulations, the 8^{th}-order central difference scheme for spatial derivatives and the 4^{th}-order Runge-Kutta method for time integration are used with the massage passing interface (MPI) for parallel computing [6].

Figure 4 shows the 1-D profiles of the solution variables which are used to generate

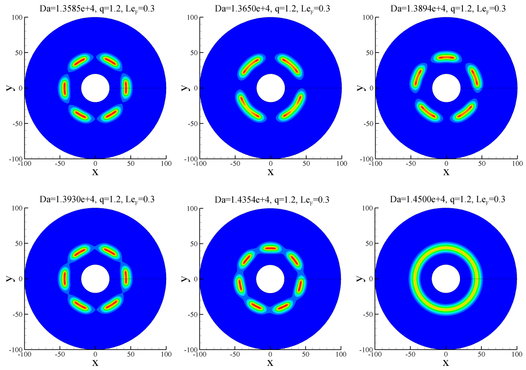

Fig. 5 Temperature isocontours for 2-D cases with different *Da*.

2-D initial conditions for a given *Da*. The 2-D initial temperature is specified by perturbating the corresponding 1-D solution using:

$$\widetilde{T}_{\mathrm{2D,init}}(r,\theta) = \widetilde{T}_1 + (1 + A\sin\theta)(\widetilde{T}(r) - \widetilde{T}_1) \quad (8)$$

where the perturbation amplitude, *A*, is chosen as 0.01 in the present study.

Figure 5 shows the temperature isocontours for 2-D cases with different *Da*. It is readily observed that the maximum temperature is increased with decreasing *Da* and the number of flame cells varies with *Da*. The onset of the cellular instability occurs approximately at *Da* = 14,354. It is well-known that the 2-D cellular flames survive beyond the 1-D extinction limit and the maximum temperatures of the 2-D cellular flames are greater than the maximum temperatures of the corresponding 1-D flames due to the focusing effect of fuel at the flame edges [5].

Figure 6 shows the maximum temperature of 2-D cases and the number of flame cells as a function of *Da*. The maximum temperatures the 2-D tubular flames in the stable region (*Da* > 14,354) coincide with those of corresponding 1-D flames. The maximum temperature, however, deviates significantly from that of 1-D flame in the unstable region (13,582 ≲ *Da* ≲ 14,354), which is attributed to the diffusive-thermal instability. It is also found that the maximum temperature is increased with decreasing *Da* for the cases with the same number of flame cells.

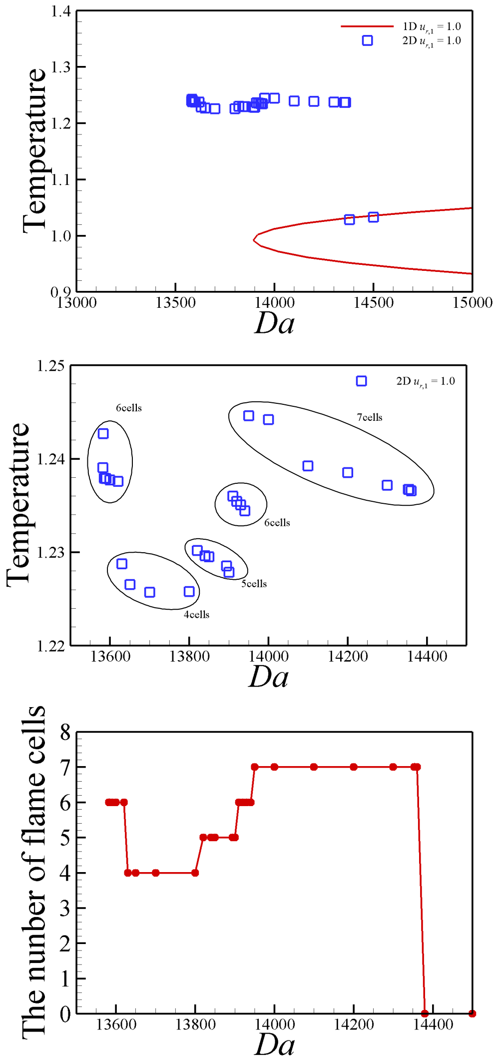

Fig. 6 Maximum temperature vs. *Da* for 1-D and 2-D solutions (top and middle). The number of flame cells vs. *Da* for 2-D cases.

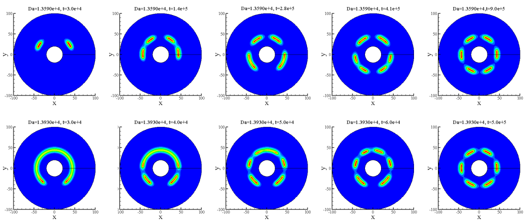

Fig. 7 The temporal evolution of flame cells for different *Da*: *Da* = 13,590 (top) and *Da* = 13,930 (bottom). Time increases from left to right.

Figure 7 shows the temporal evolution of the flame cells for two different *Da*. It is readily observed that even if the number of flame cells are the same, the cell formation procedures and the steady maximum temperatures are different from each other.

The effect of *Da* on the formation of diffusive-thermal instability of opposed nonpremixed tubular flames near extinction limit was investigated numerically. From the linear stability analysis, the onset of the diffusive-thermal instability was identified. From the 2-D simulations, it was found that the number of flame cells and maximum flame temperatures are highly dependent on *Da*; the number of flame cells initially decreases and then increases again with decreasing *Da*. It was also found that the maximum flame temperature is increased with decreasing *Da*. for the cases with the same number of flame cells. However, the representative maximum flame temperature for a group of flames with the same number of cells decreases first and then increases as *Da* approaches the 2-D extinction limit.

Acknowledgments

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References

[1] S. Hu, P. Wang, R. W. Pitz, *Proc. Combust. Inst.*, 31, 2007, 1093-1099.
[2] S. Hu, R. W. Pitz, *Combust. Flame* 156, 2009, 51-61.
[3] S. Hu, R. W. Pitz, Y. Wang, *Combust. Flame* 156, 2009, 90-98.
[4] S. W. Shopoff, P. Wang, R. W. Pitz, *Combust. Flame* 158, 2011, 876-884, 2165-2177.
[5] M. Short, J. Buckmaster, S. Kochevets, *Combust. Flame* 125, 2001, 893-905.
[6] C. S. Yoo, J. H. Frank, J. H. Chen, *Combust. Flame* 156, 2009, 140-151.