Querying approximate shortest paths in anisotropic regions

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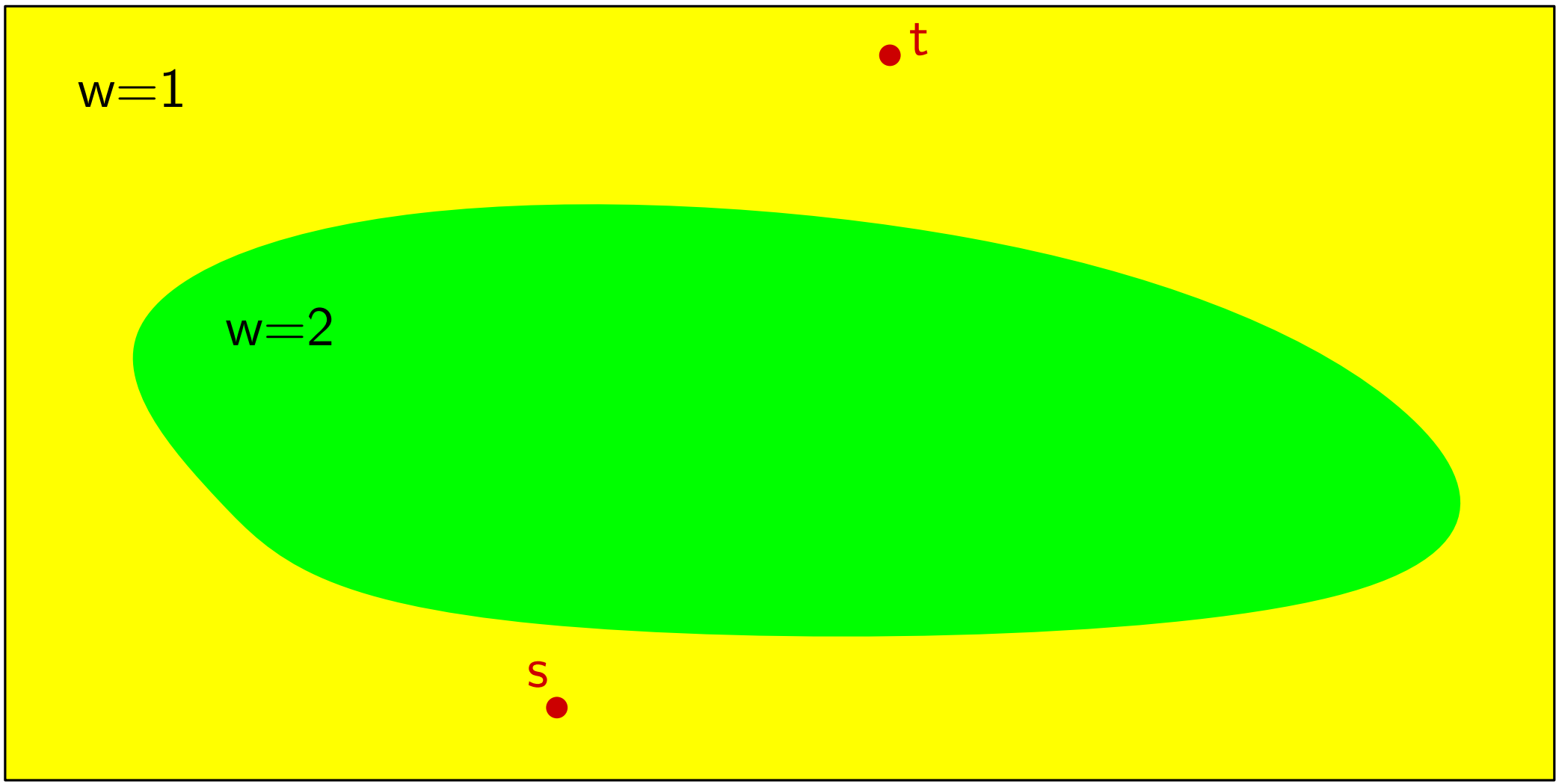
The weighted region problem

cost = weight $\times$ length



Shortest path

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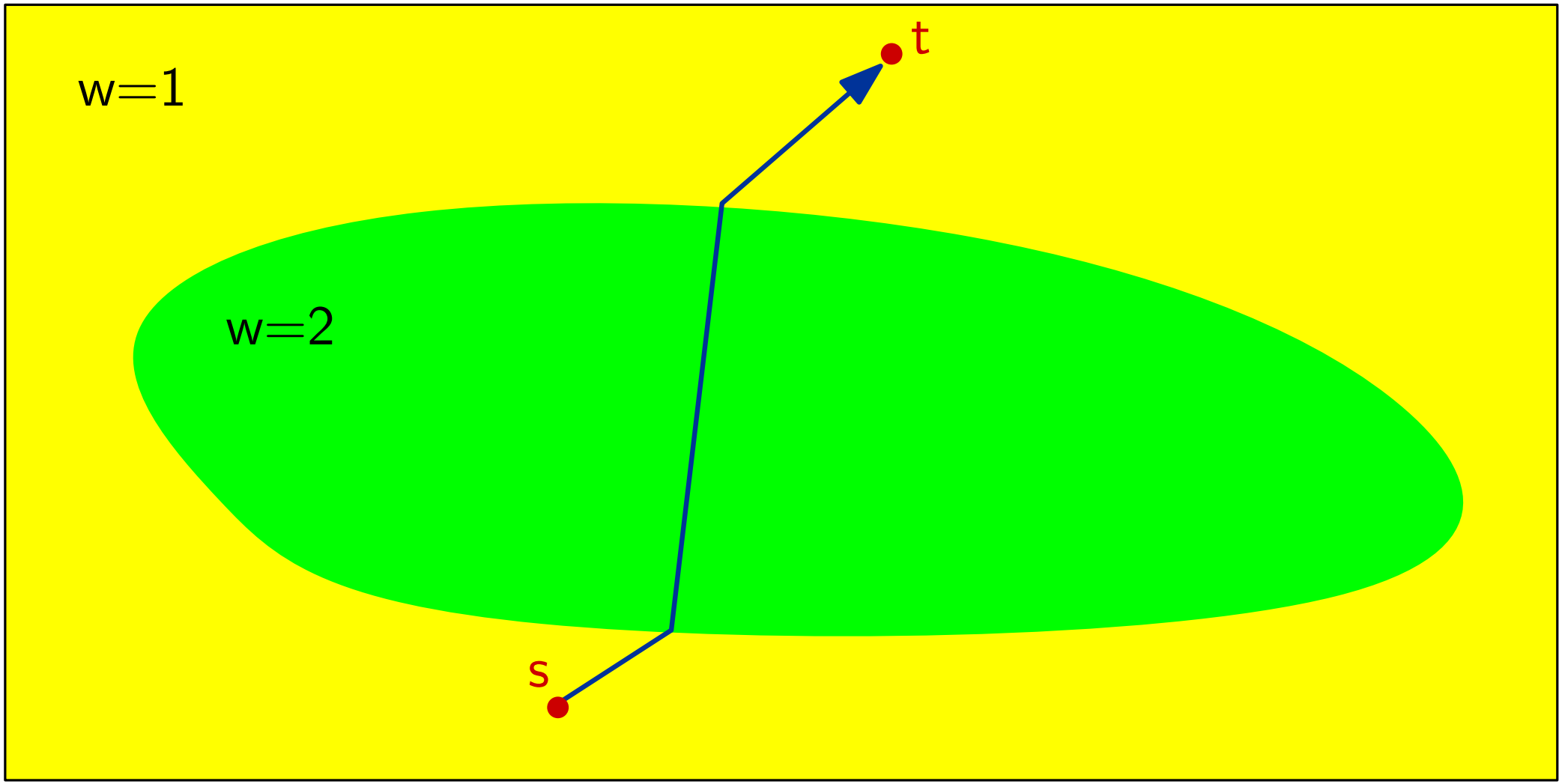
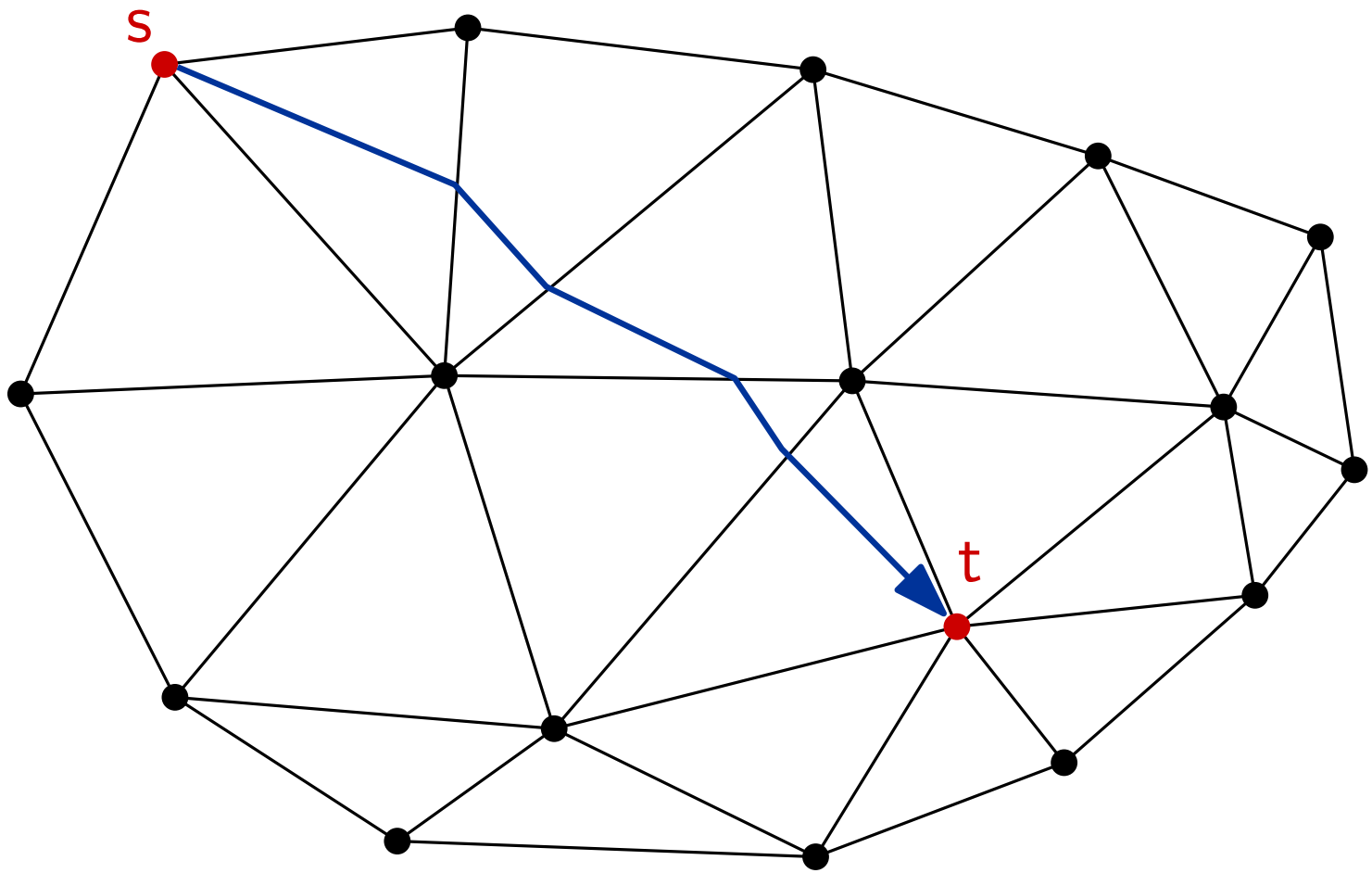


Shortest path

The weighted region problem

- Triangulation with n vertices
- Each face has a weight
- Find an approximate shortest path with cost at most $1 + \epsilon$ times the minimum



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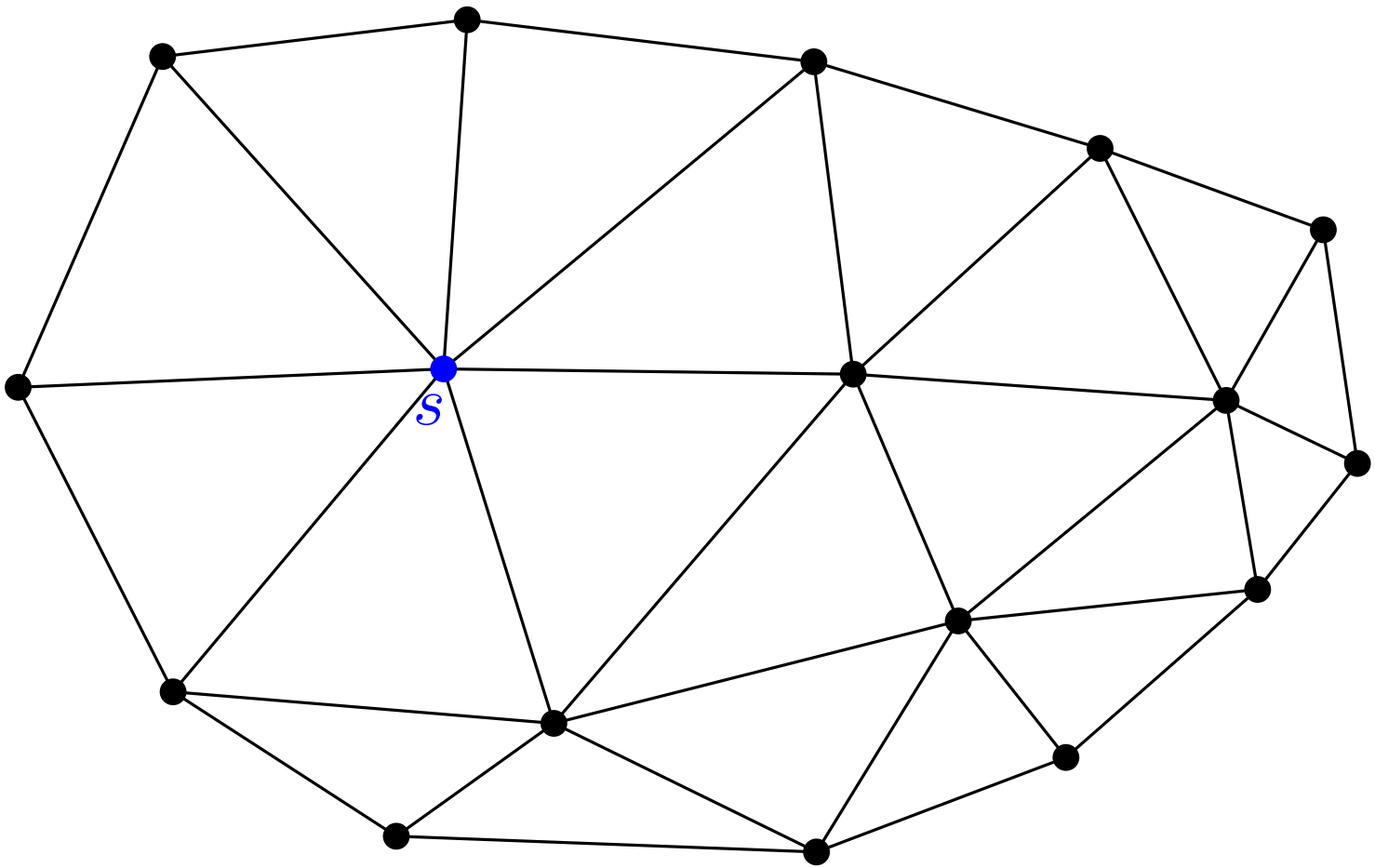
Known results

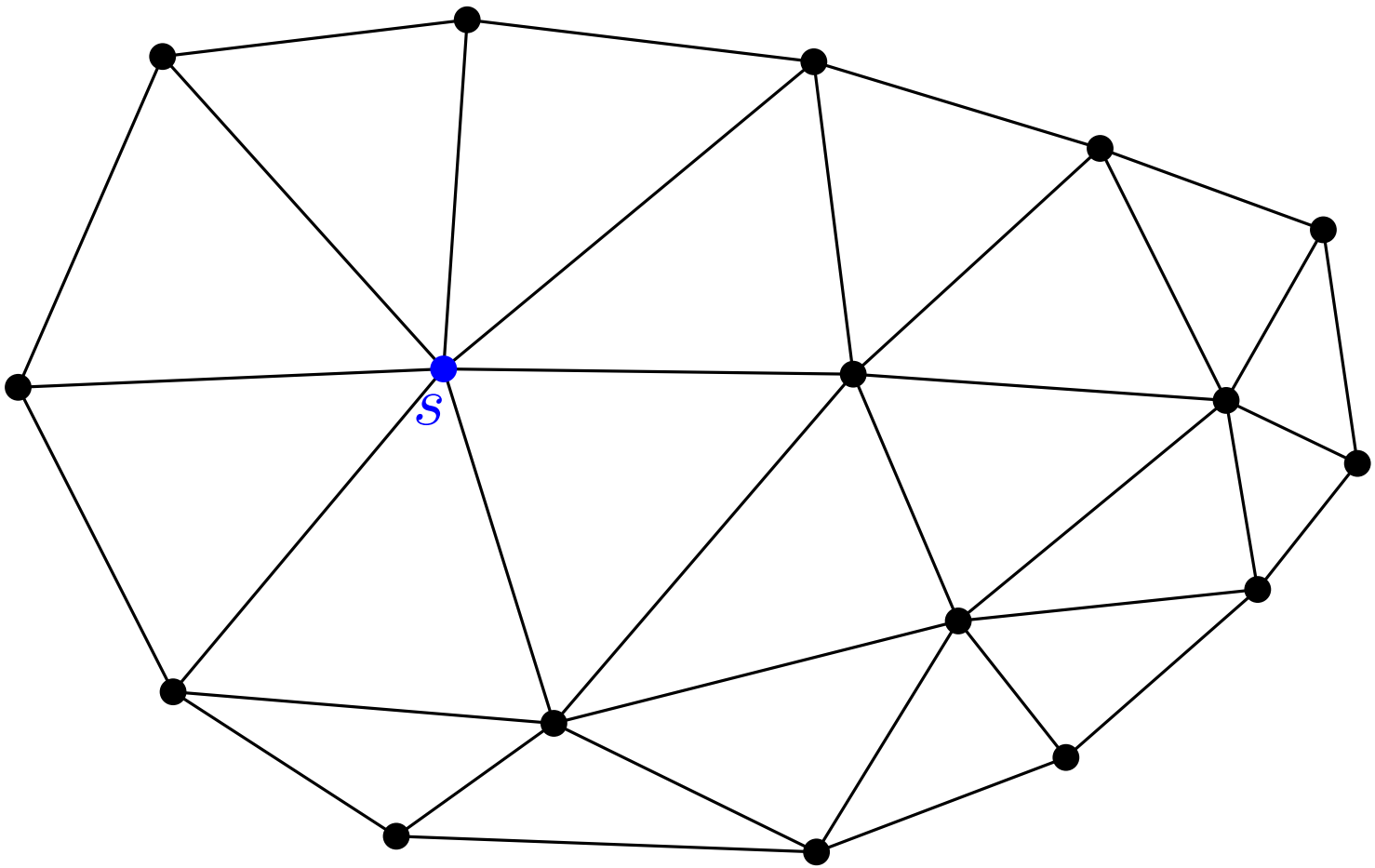
- Mitchell and Papadimitriou (1987): $O(n^8 \log(nN\rho/\epsilon))$
- Aleksandrov, Maheshwari, and Sacks:
  $O(Cn/\sqrt{\epsilon} \times \text{polylog}(n, 1/\epsilon))$,
  C depends on ρ, θ and other parameters.
- Sun and Reif: $O(C'n/\epsilon \times \text{polylog}(n, 1/\epsilon))$,
  C' depends on θ, other parameters, but not ρ.
- Cheng et al. (2007): $O(n^3\rho/\epsilon \times \text{polylog}(n, \rho, 1/\epsilon))$

Query problem



Query problem

Build a data structure



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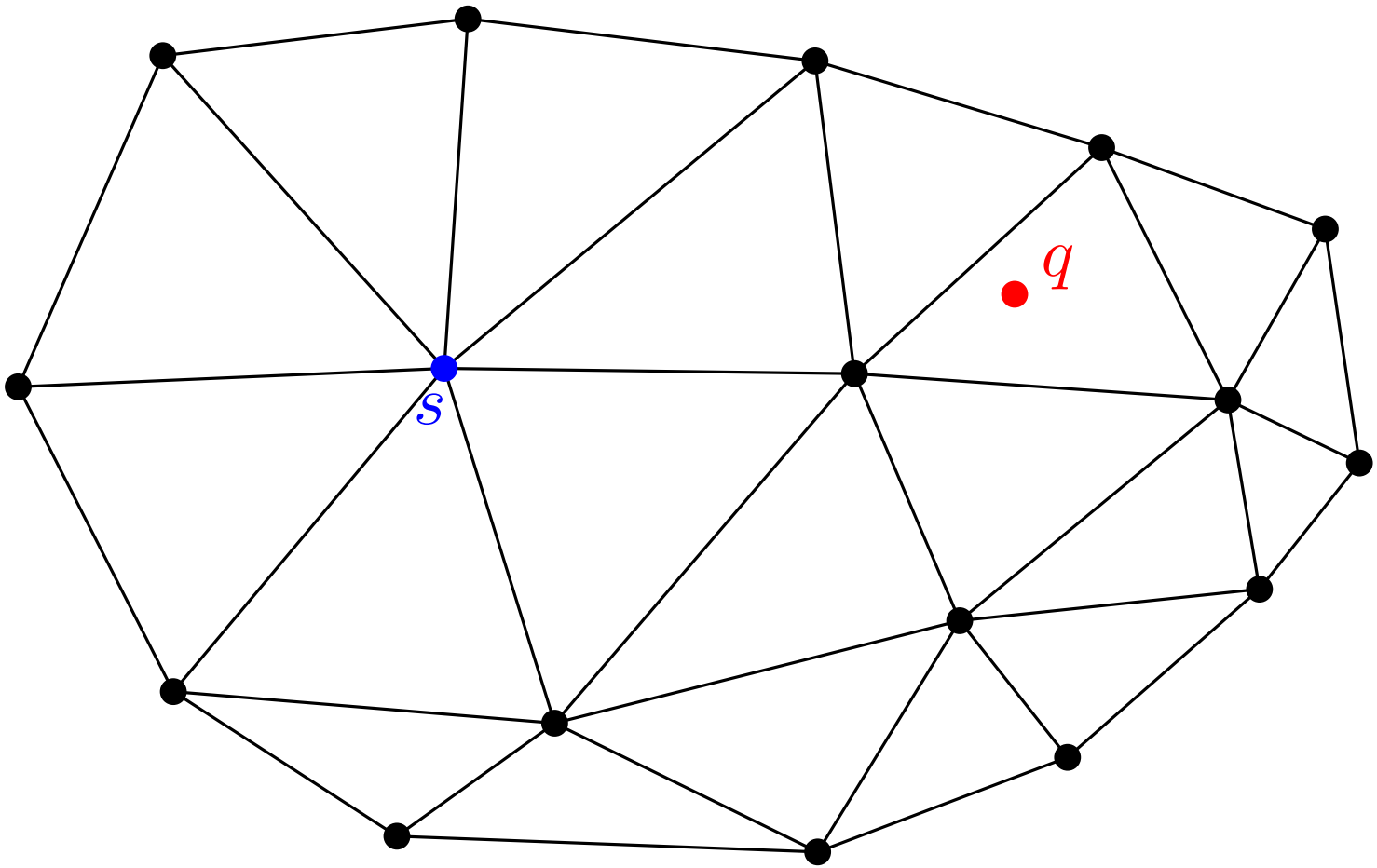

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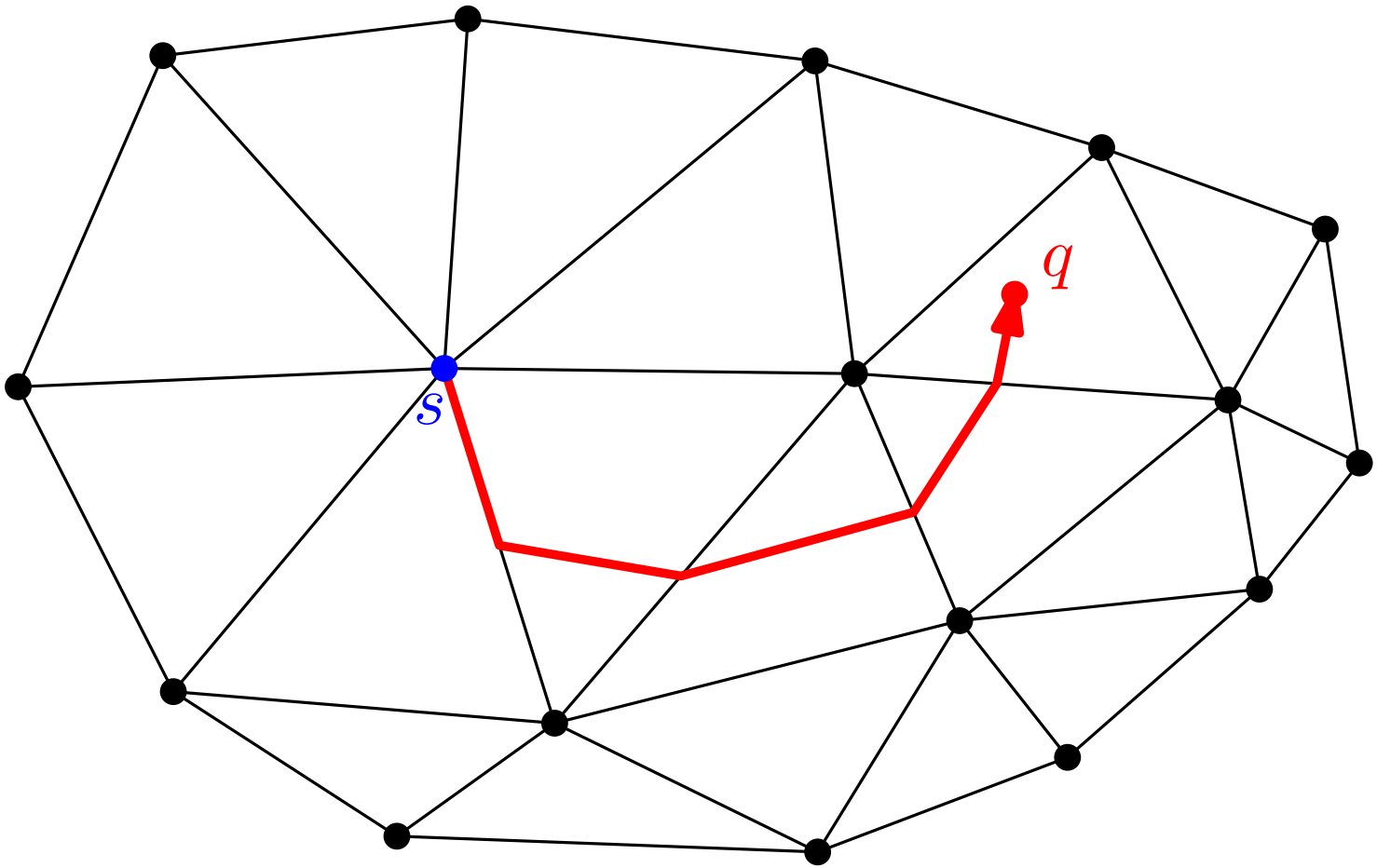

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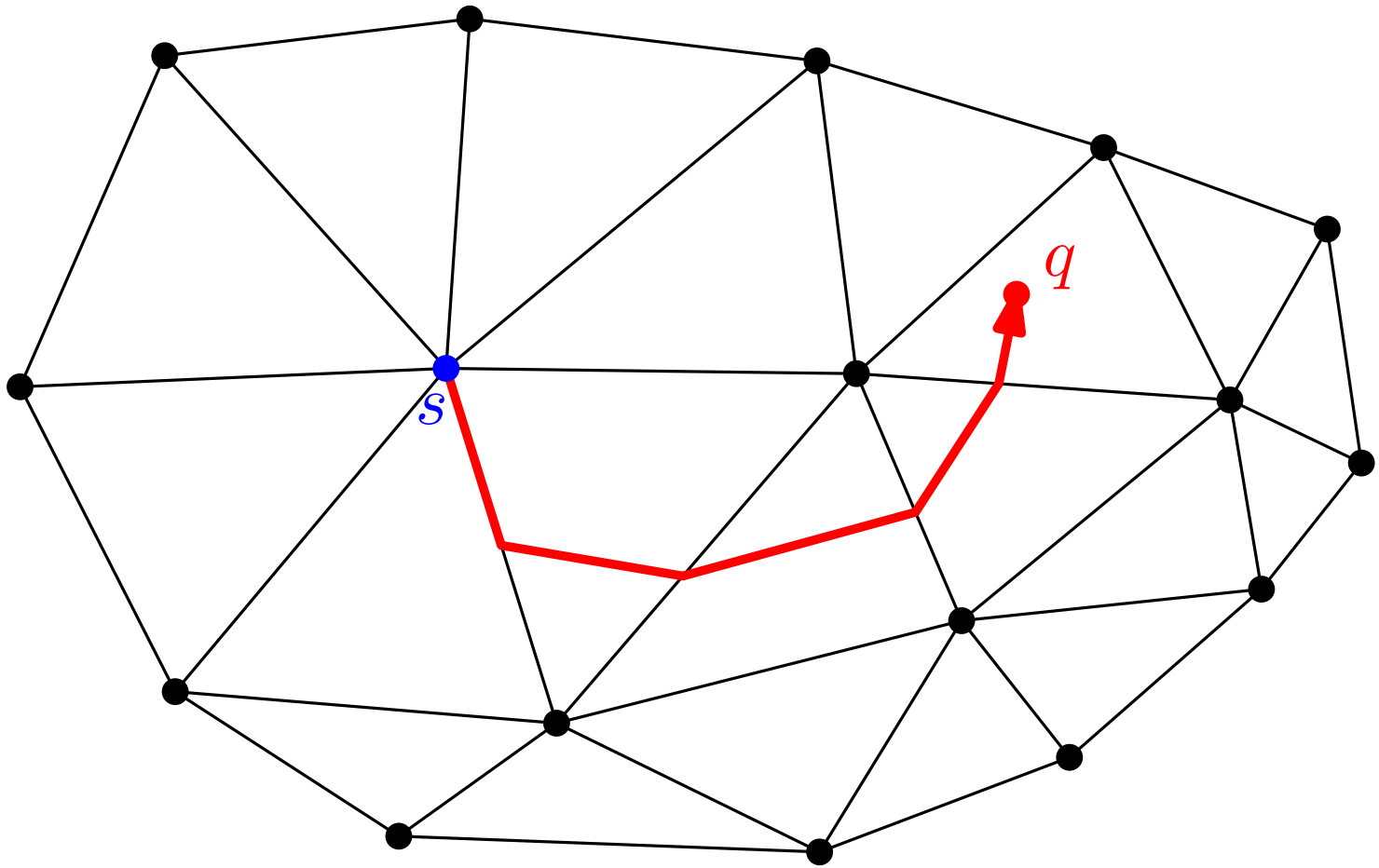

Output an approximate shortest path in time linear in its size

Previous results

Aleksandrov et al.: Space and preprocessing time depend on several parameters including n, ε, ρ and θ.

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Query time: $O(\log(\rho n/\varepsilon))$

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Preprocessing time: $O(\rho^2 n^4/\varepsilon^2 \log^2(\rho n/\varepsilon))$

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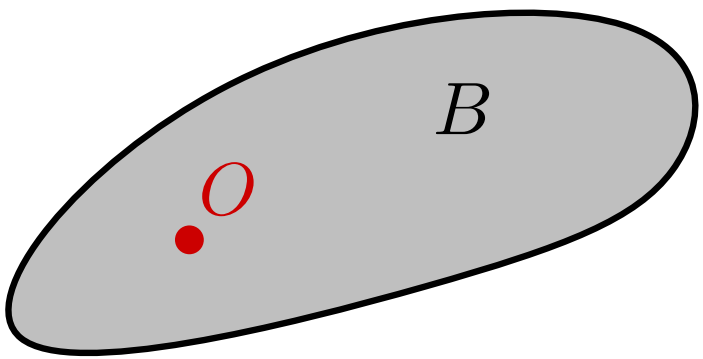
Applies to anisotropic distance functions

Convex distance function

Unit ball B: set of points at distance $\leqslant 1$ from O



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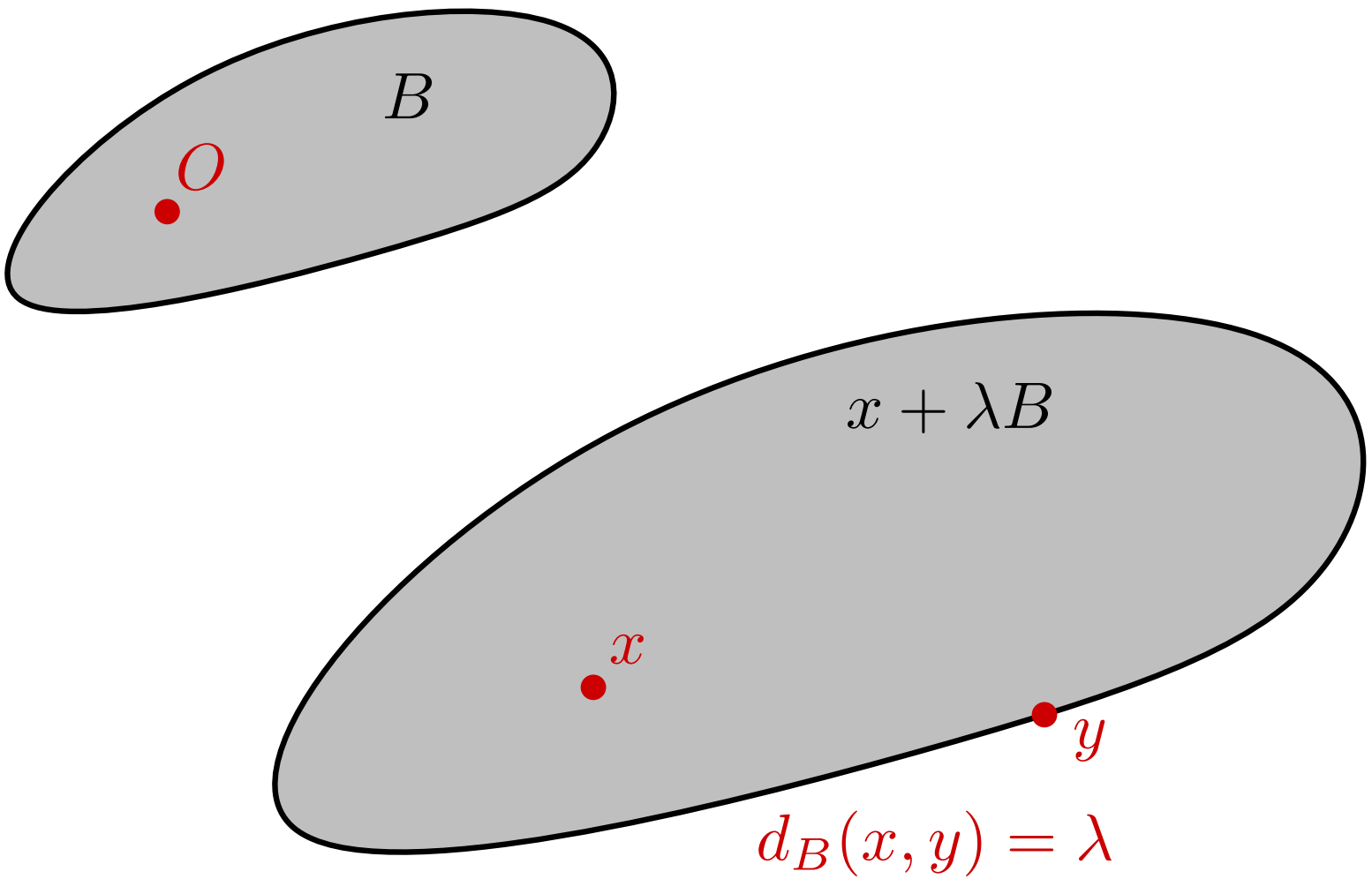

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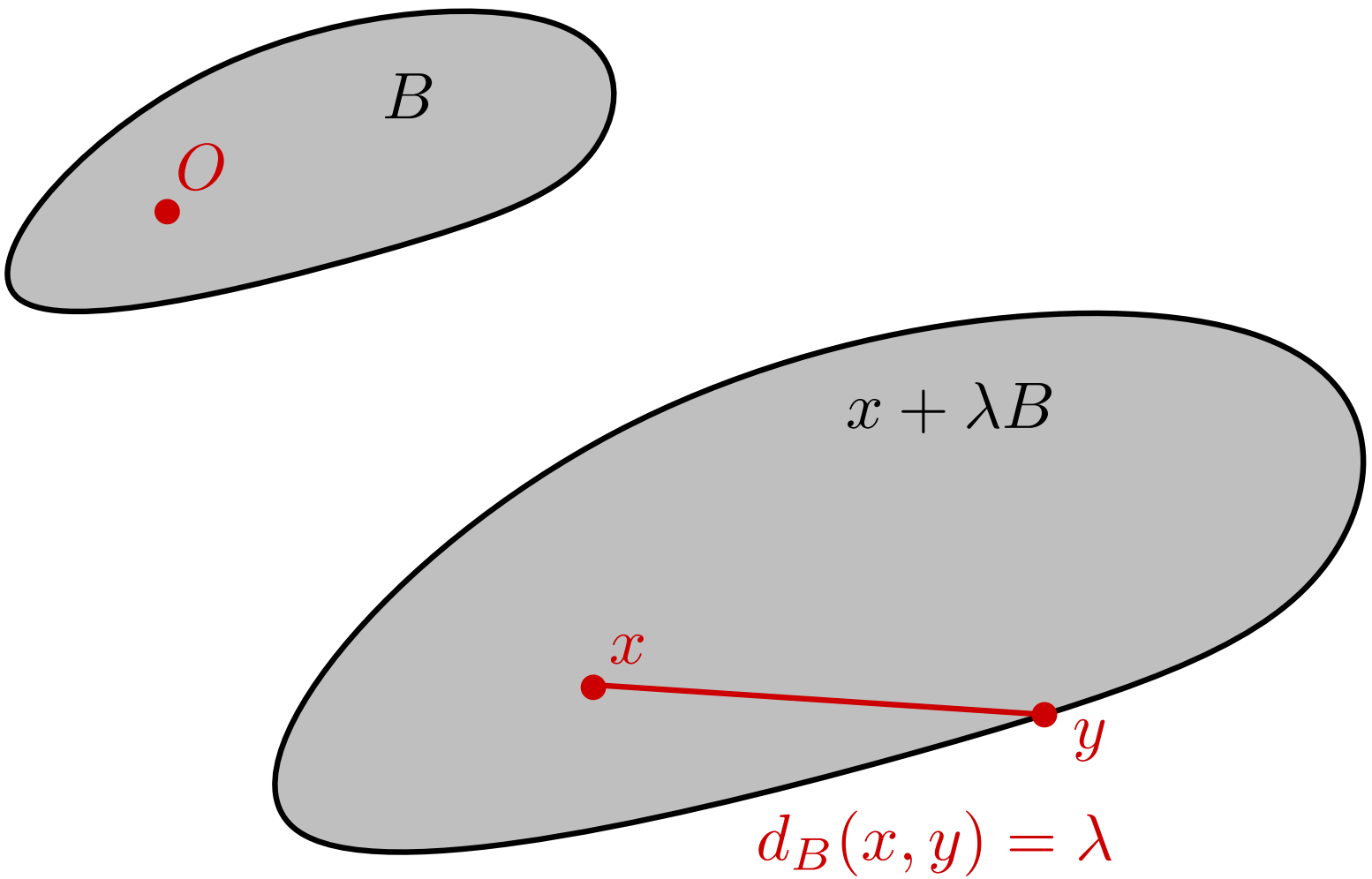
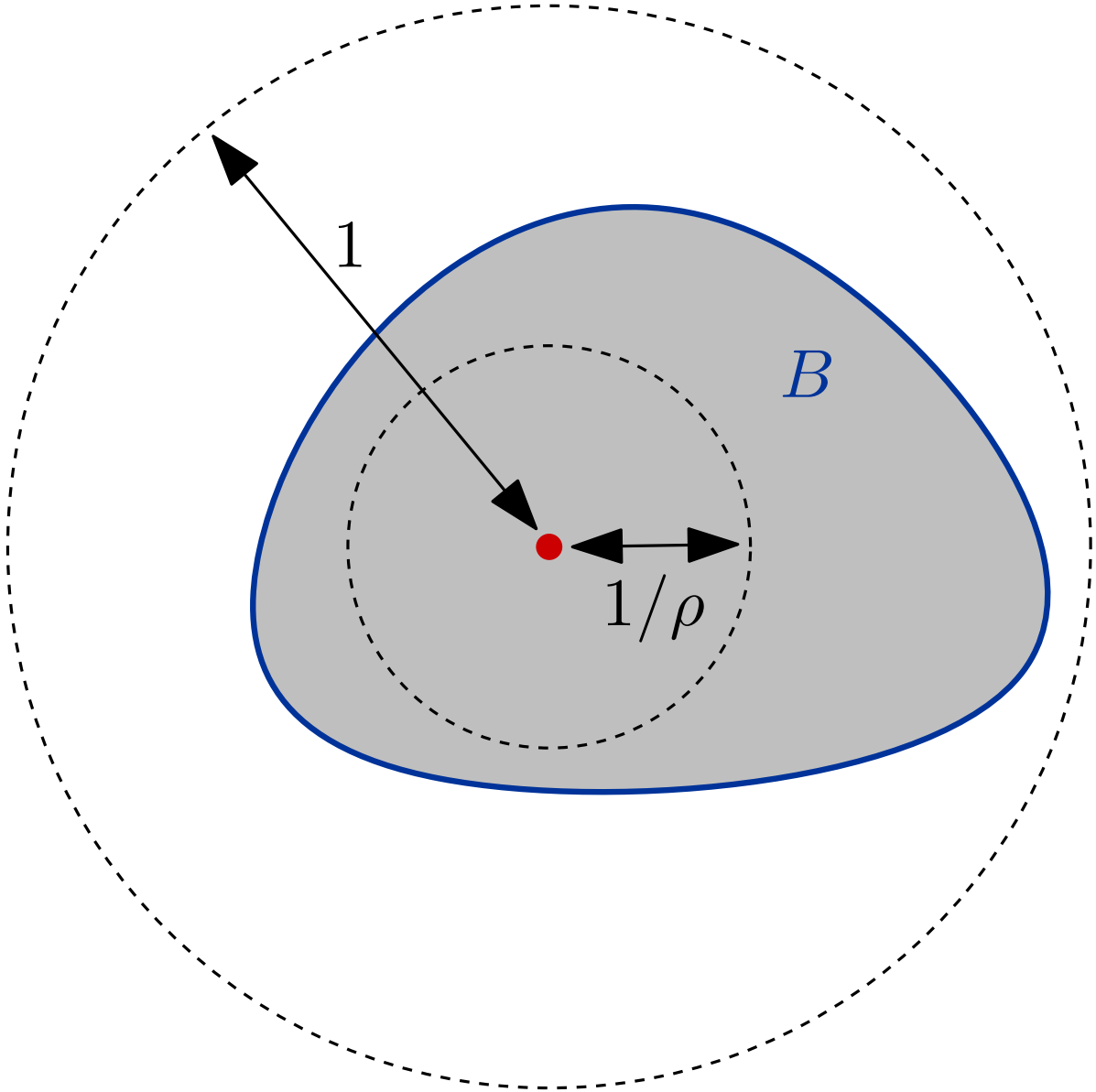


Shortest path is a straight line segment

Model



One convex distance function per face

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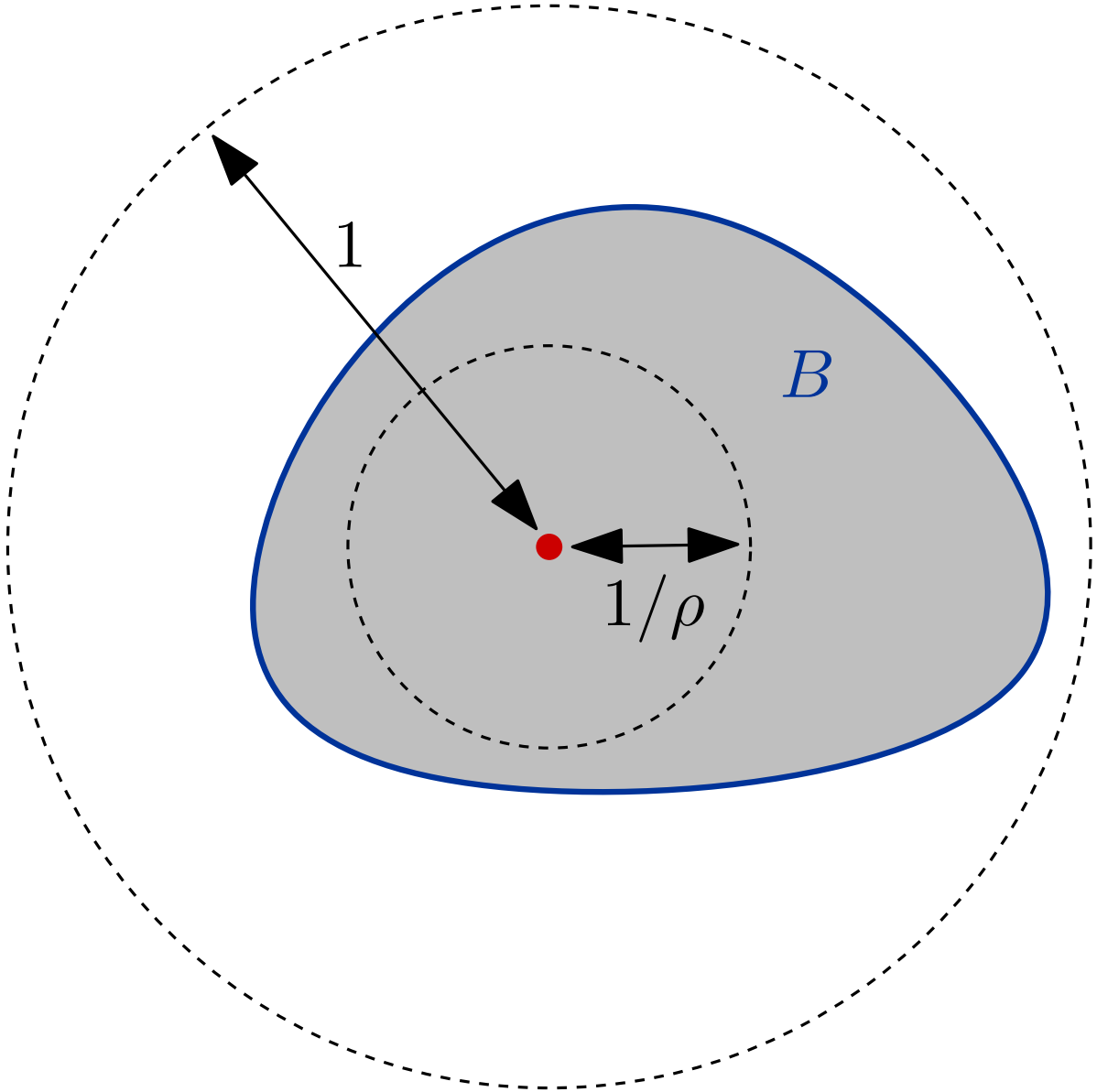

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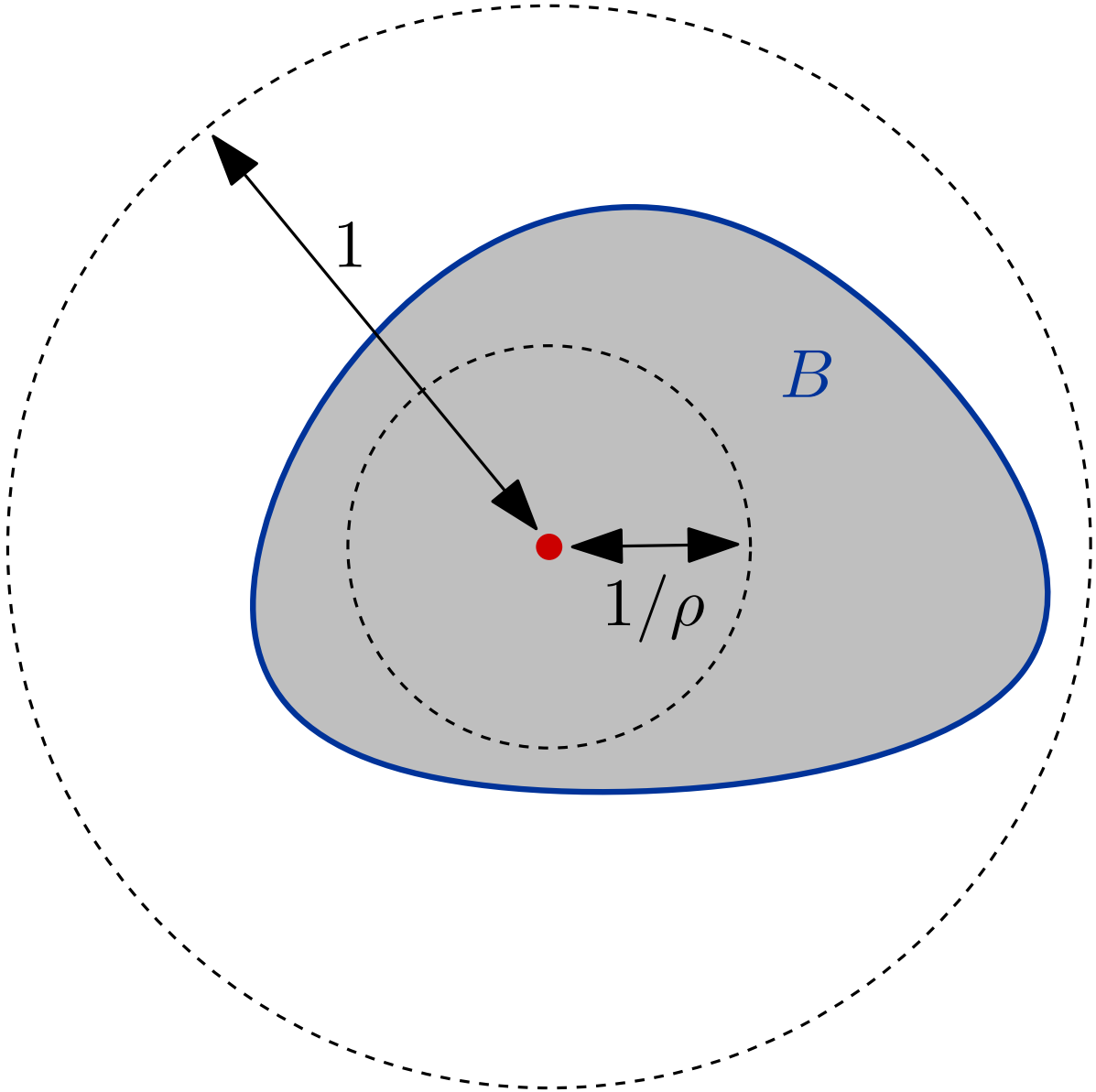
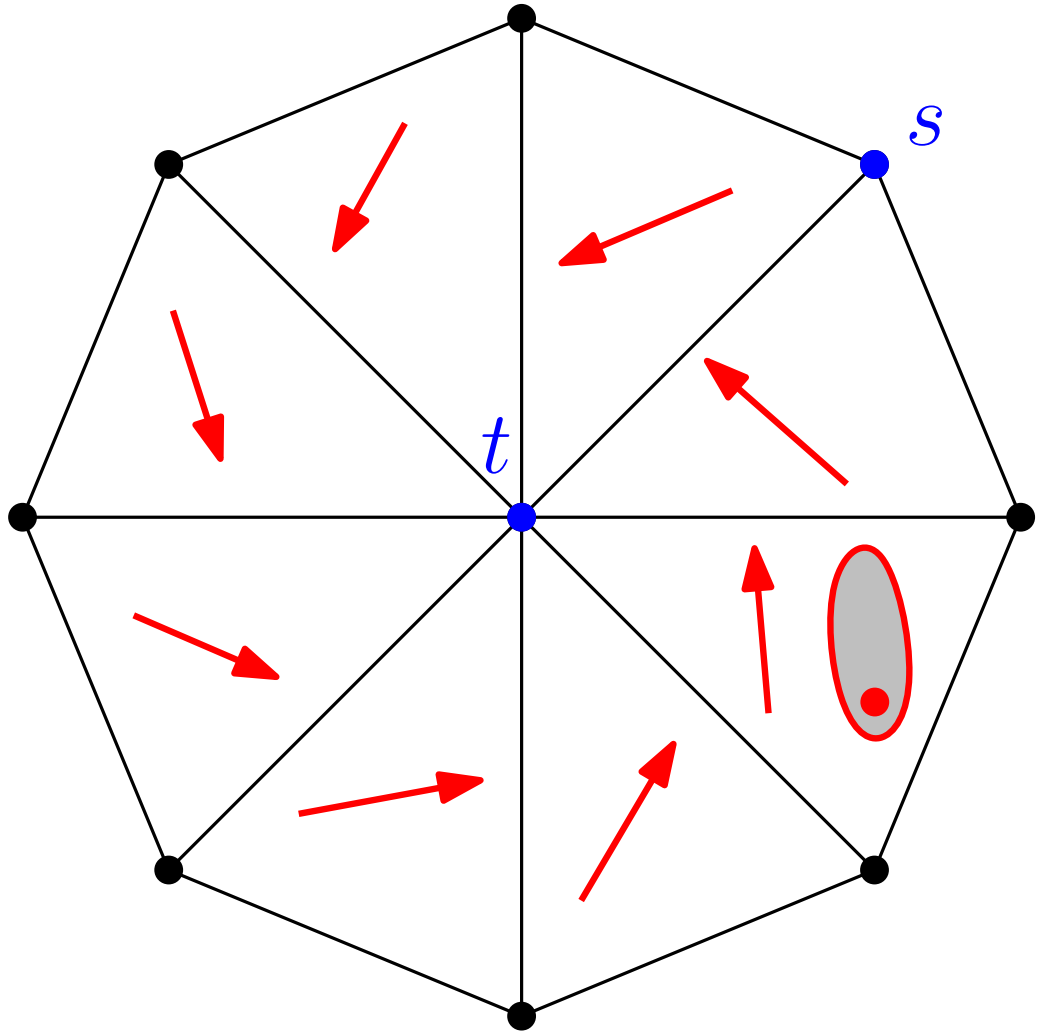


One convex distance function per face

Speed is in interval $[1/\rho, 1]$

Cost of a length ℓ path is in $[\ell, \rho\ell]$.

Exemple of a non-polygonal shortest path



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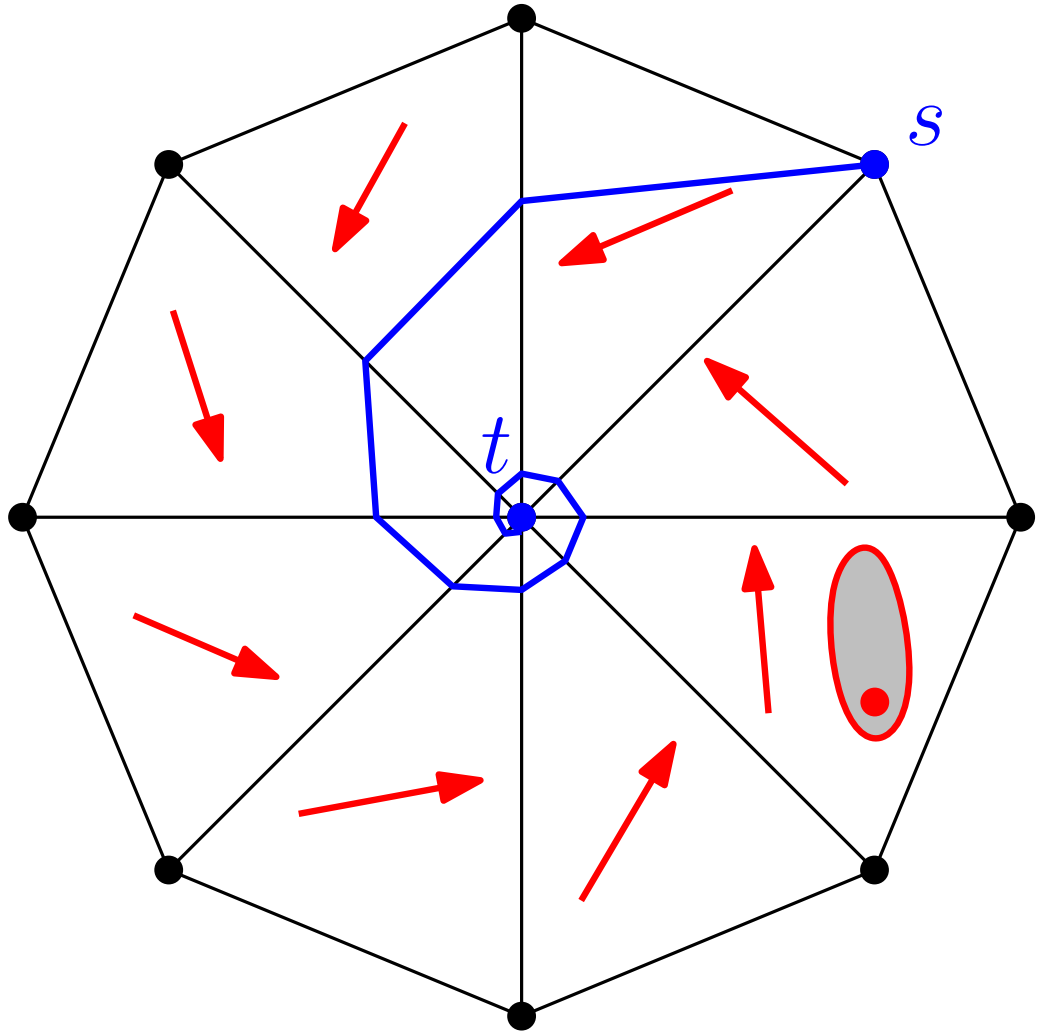

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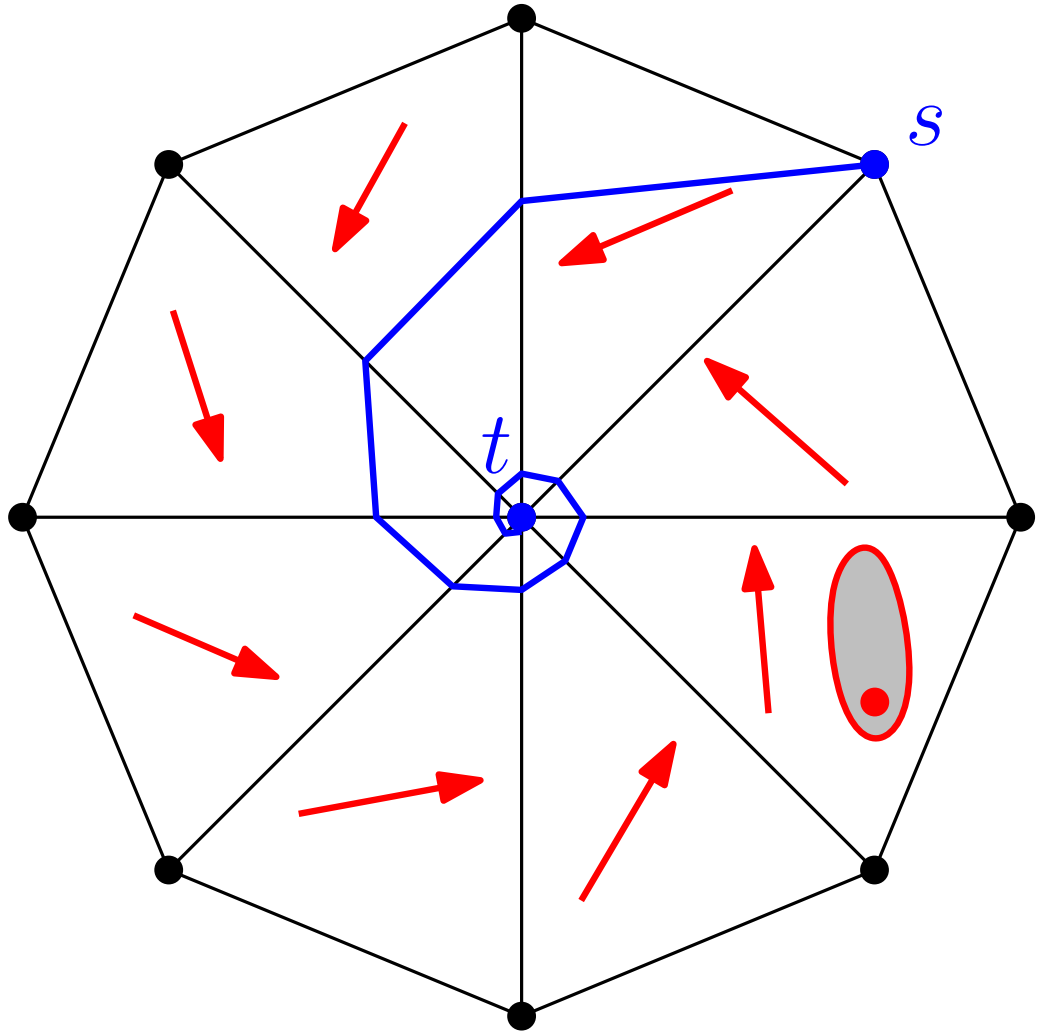

There exists a rectifiable (=finite length) shortest path

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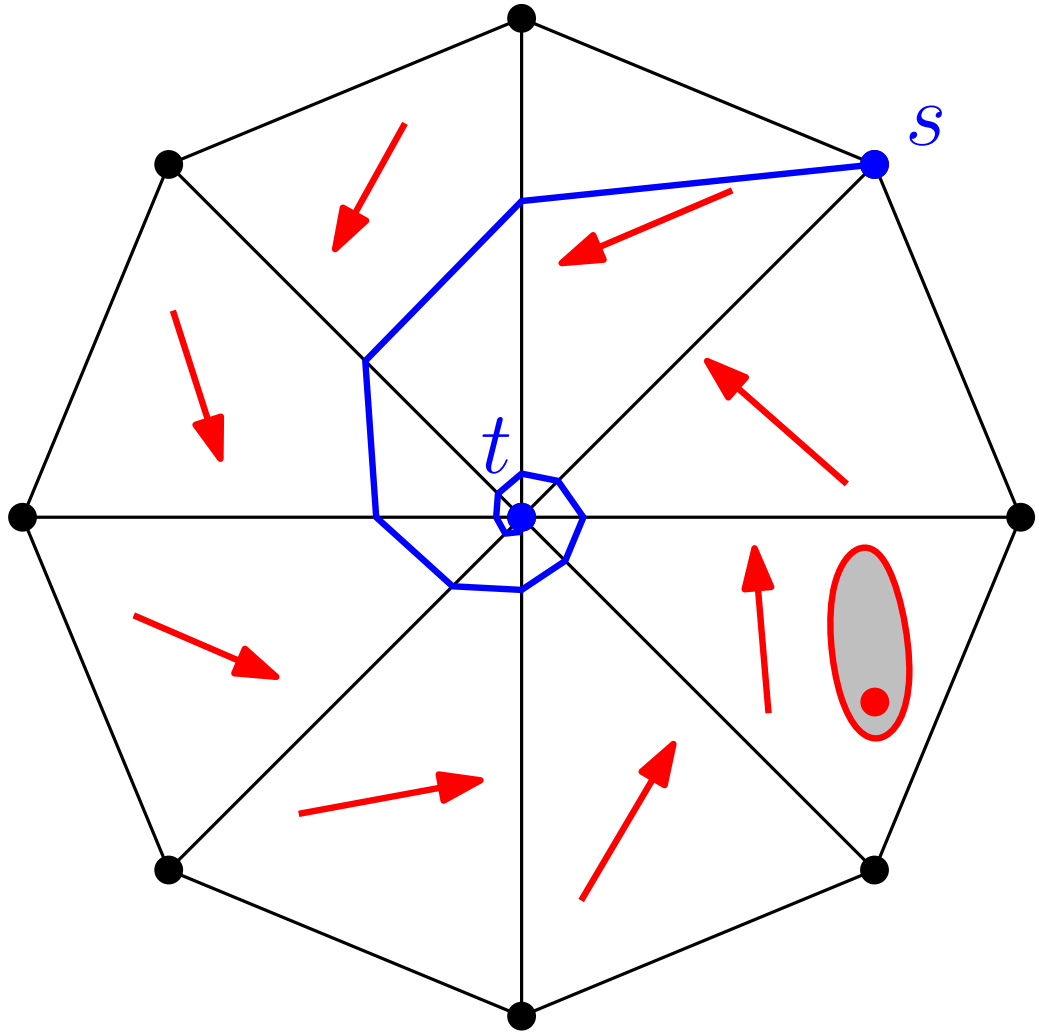
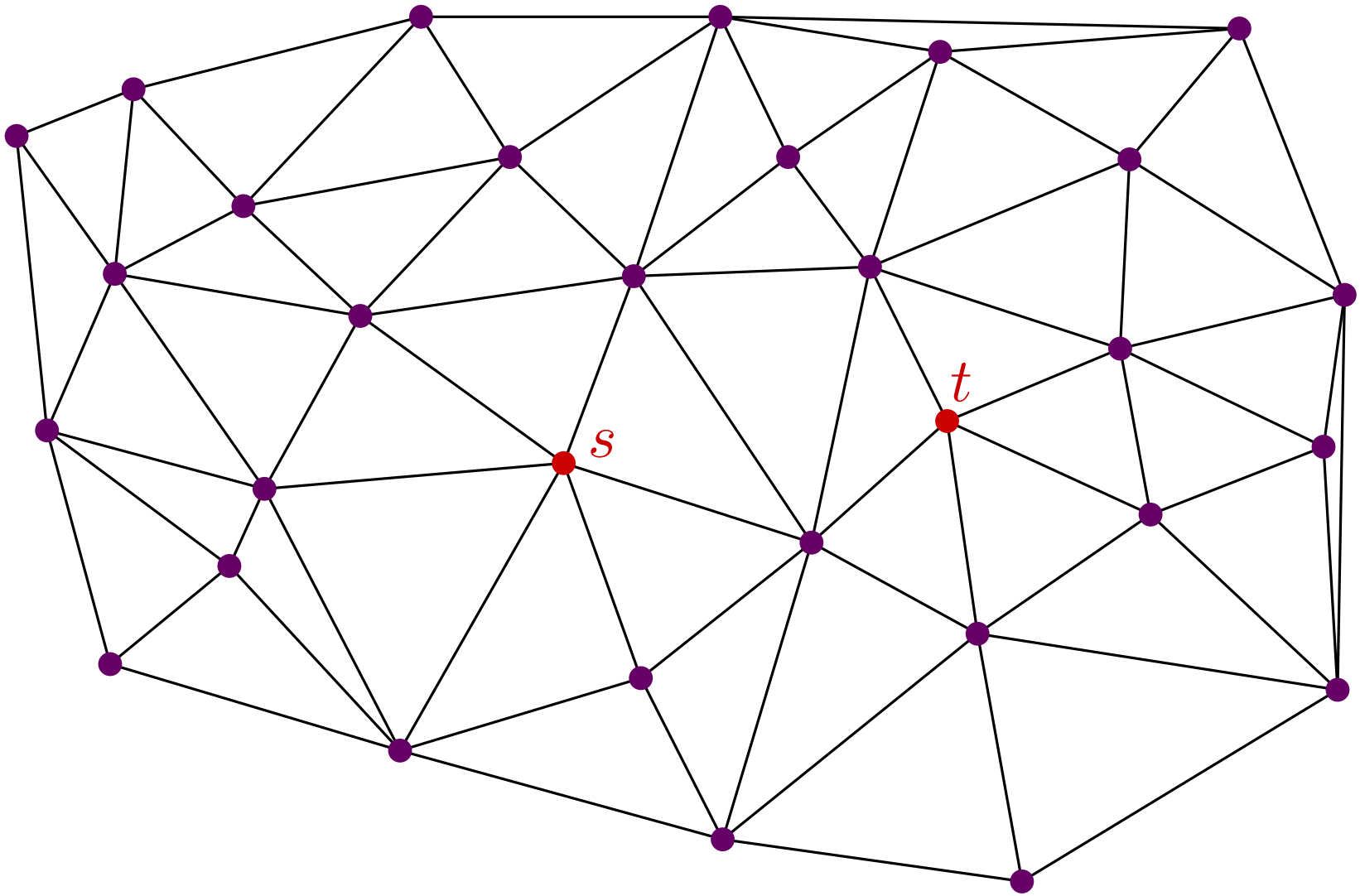


There exists a rectifiable (=finite length) shortest path

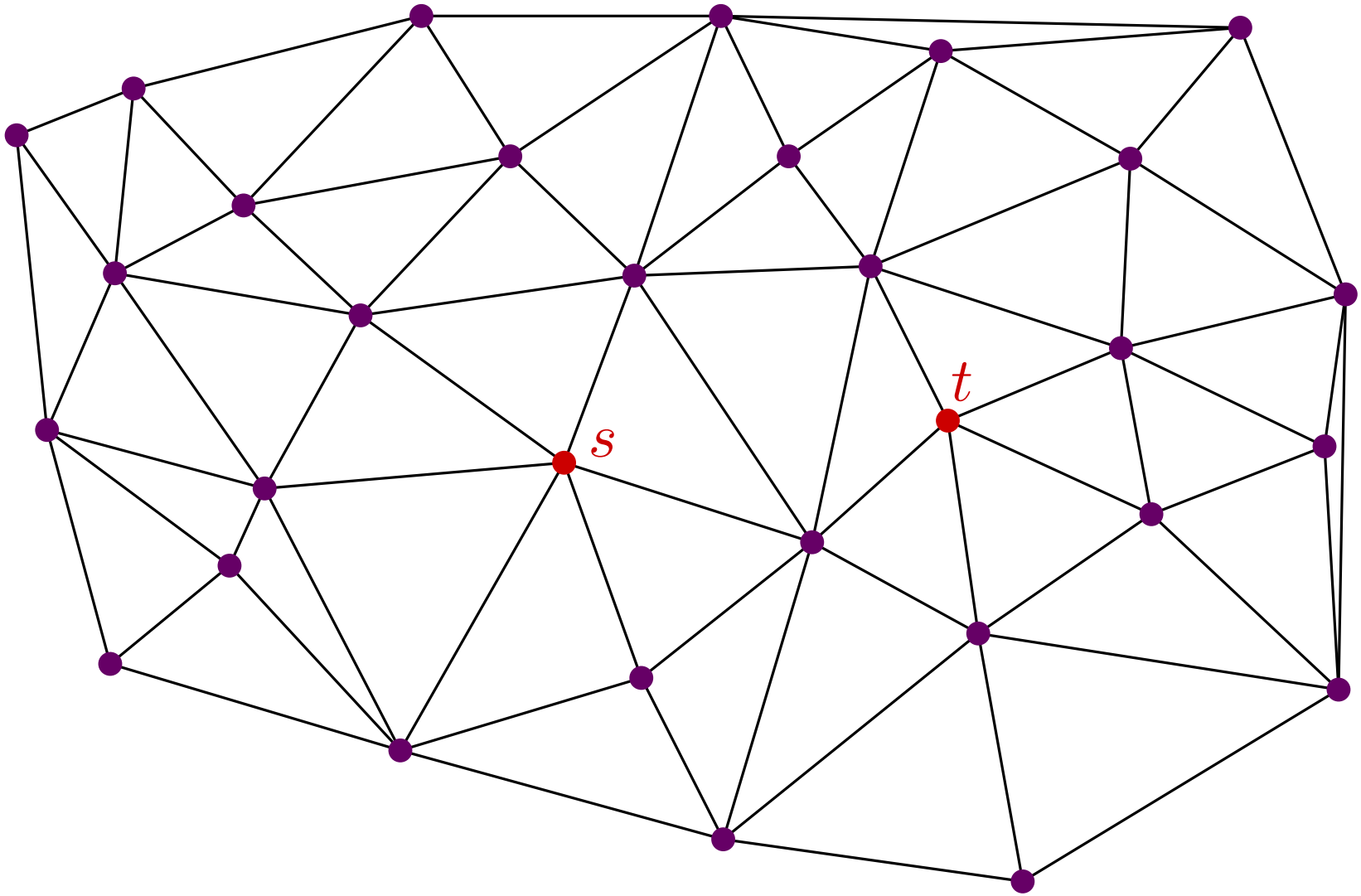
There exists a $(1+\varepsilon)$-approximate shortest path with $O(\rho n^2/\varepsilon)$ edges.

Static (non query) algorithm



Static (non query) algorithm

Circle with radius $\rho\ d(s,t)$ and center s



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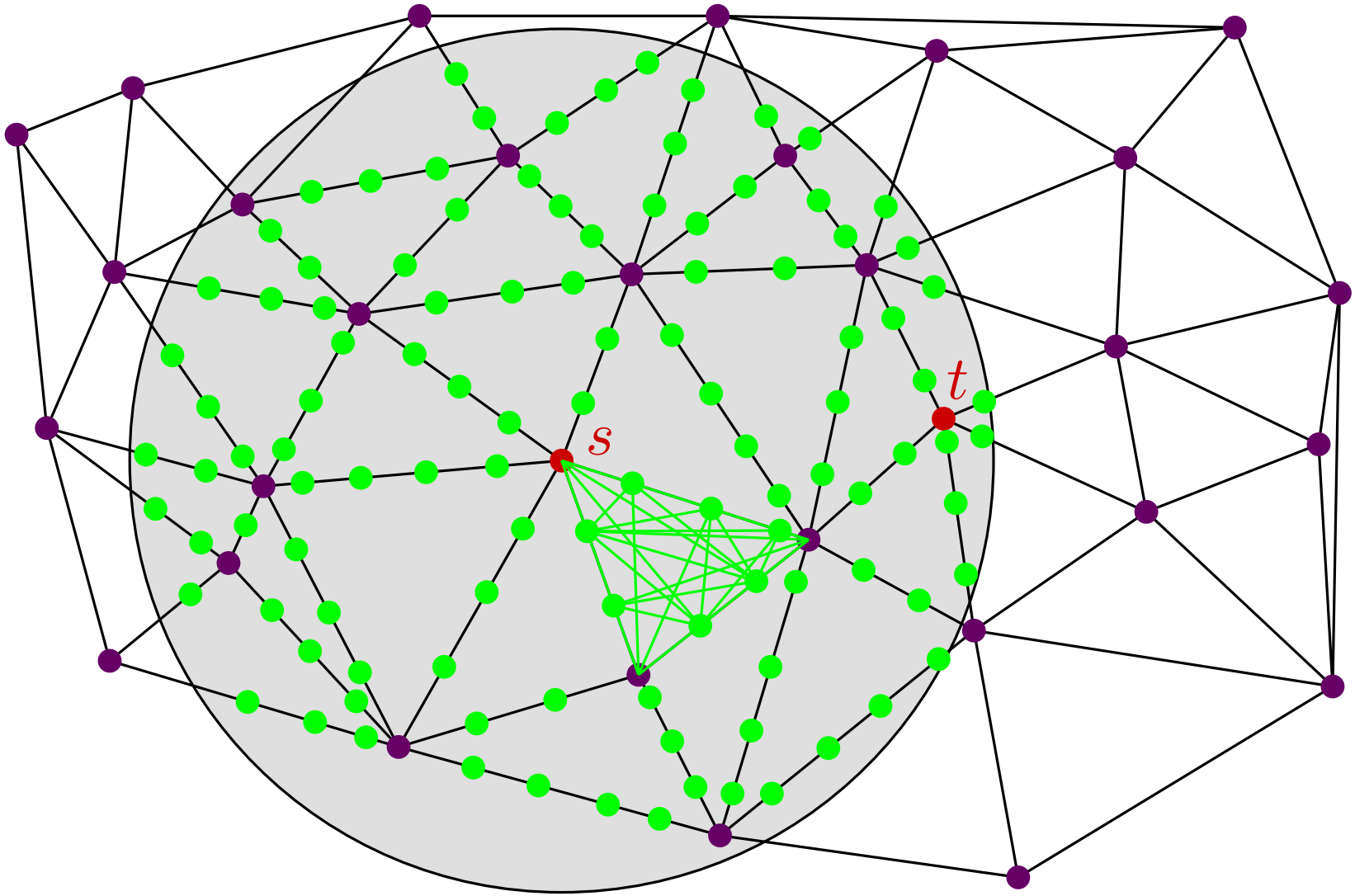

Steiner points with uniform spacing $\delta = 1/\mathrm{poly}(\rho, n, 1/\varepsilon)$

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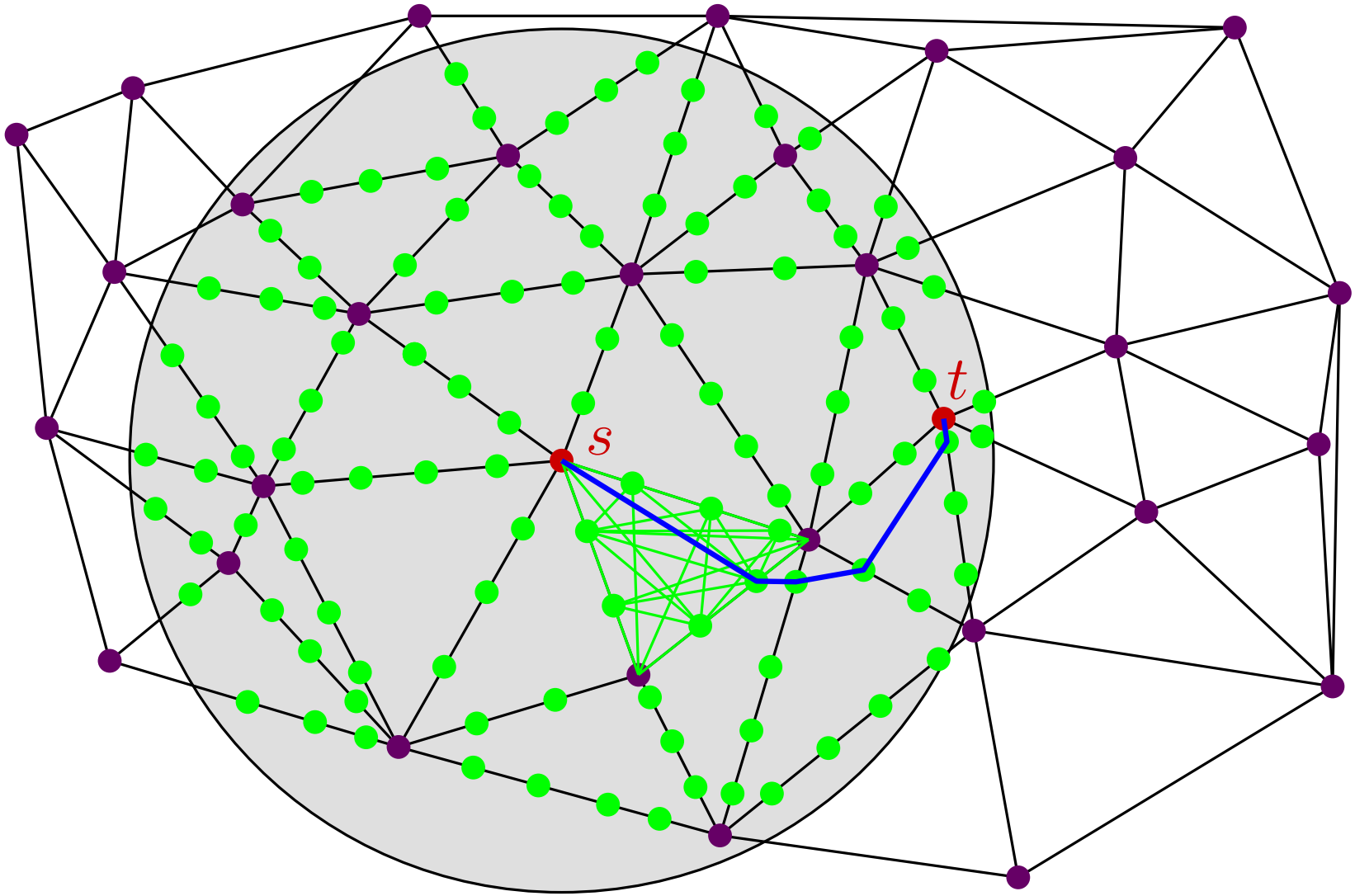

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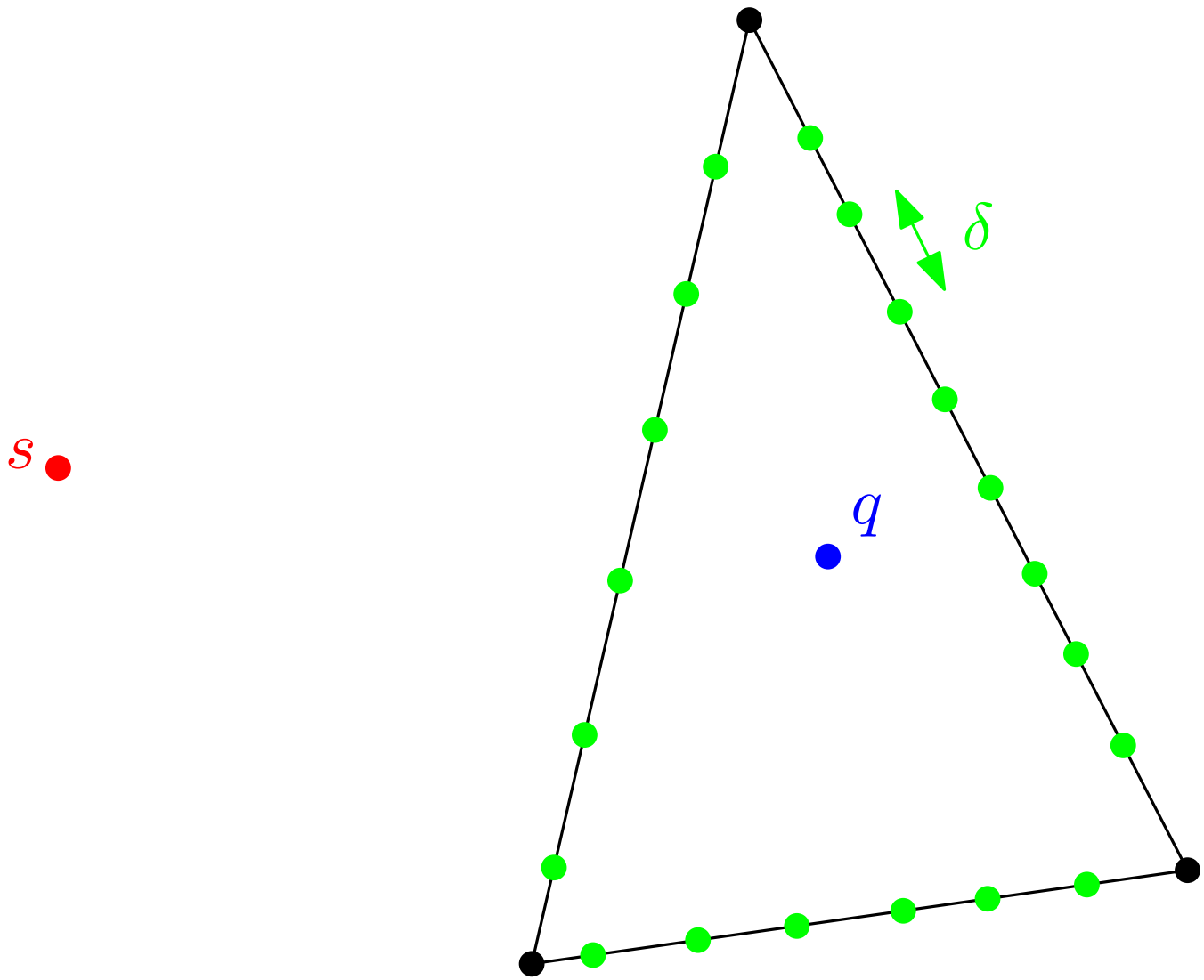
Observation



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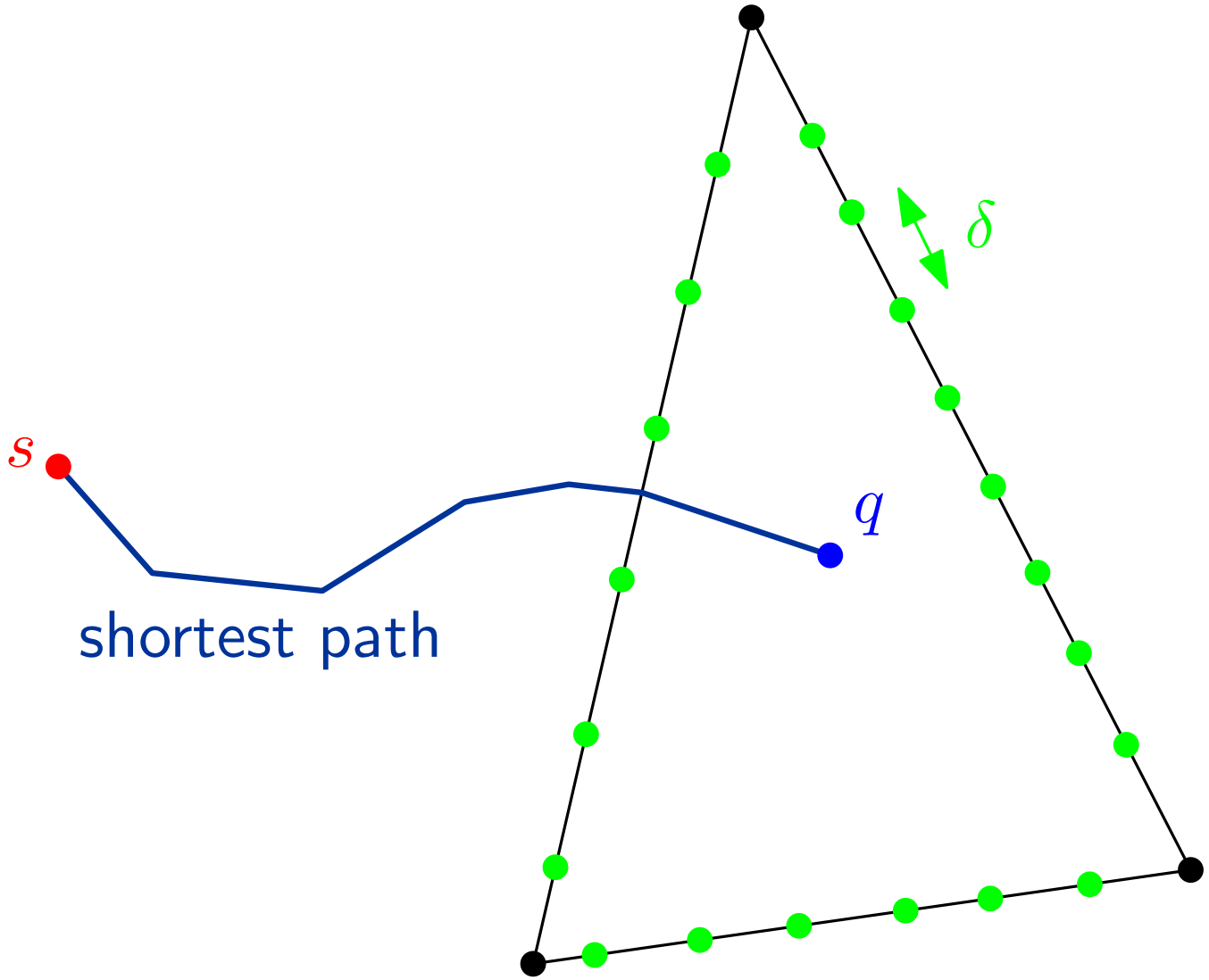

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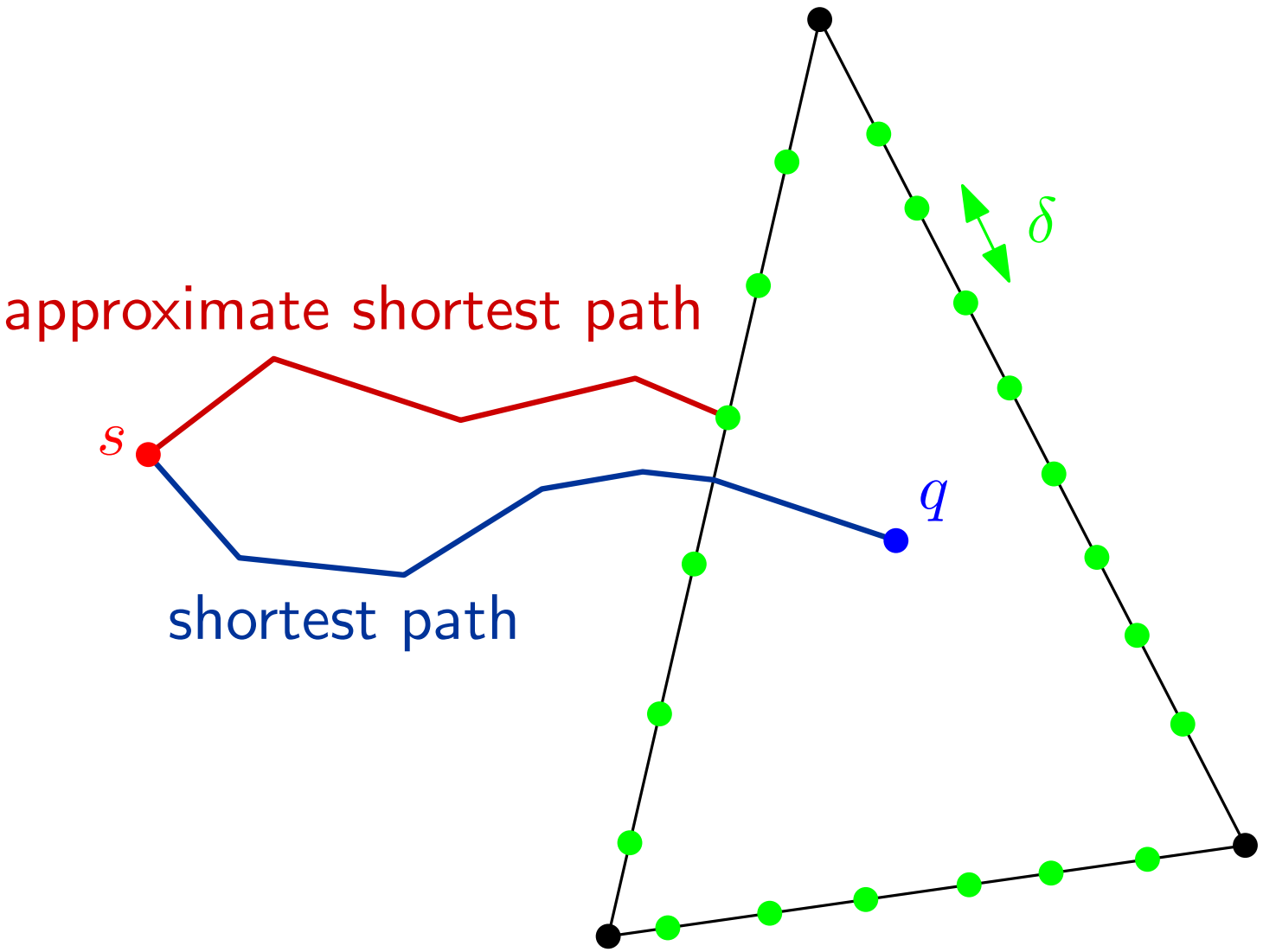

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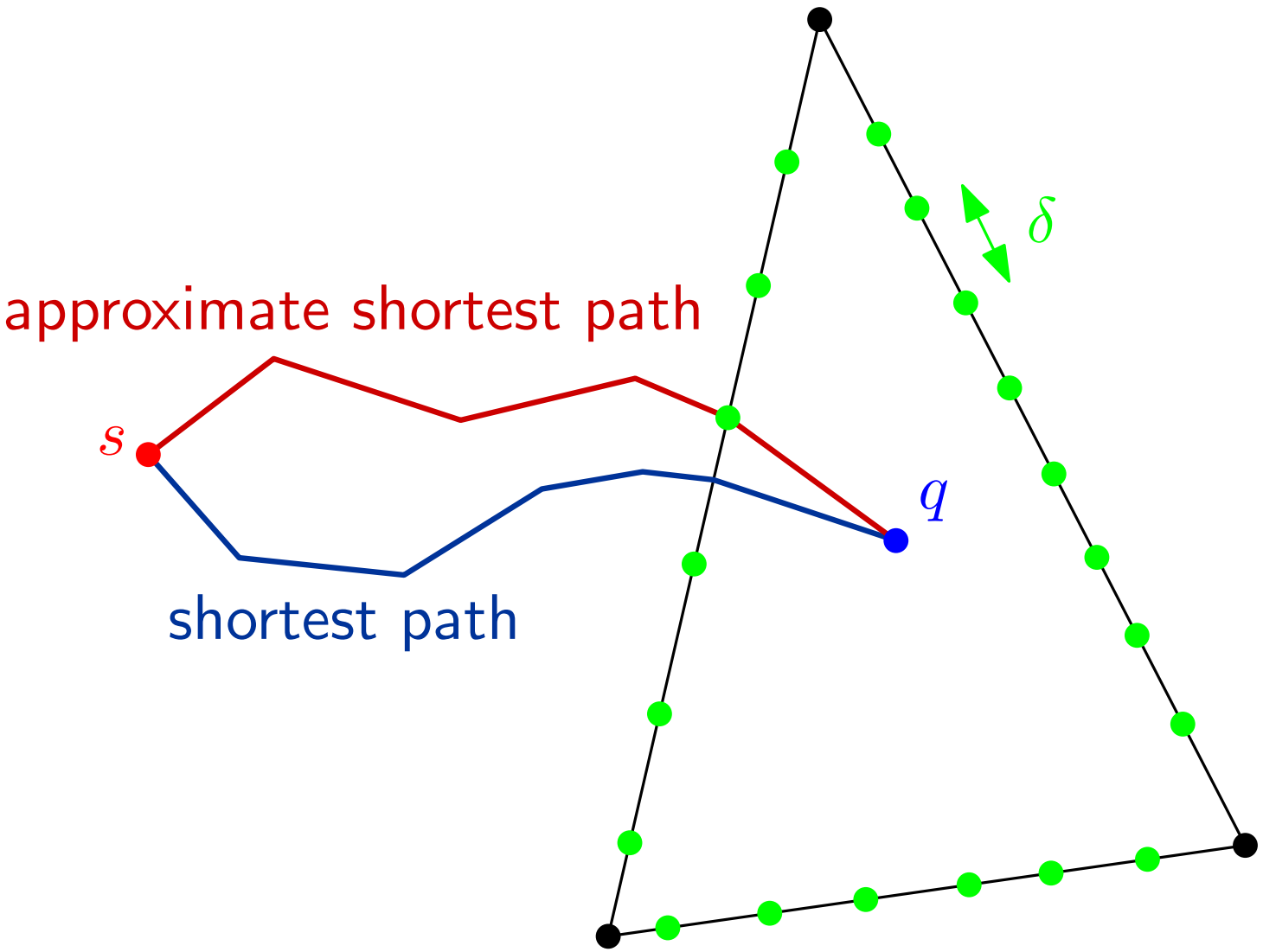

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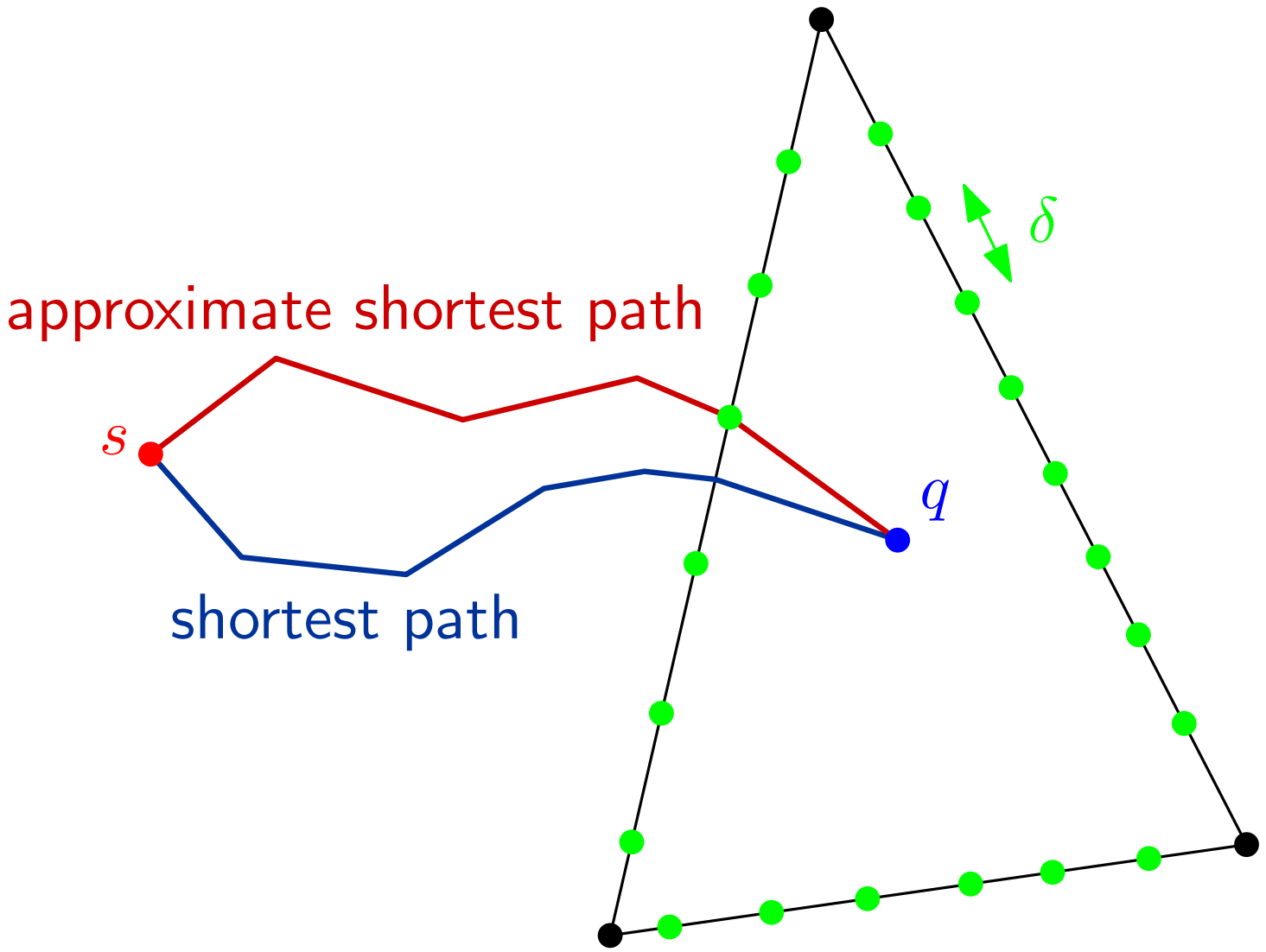
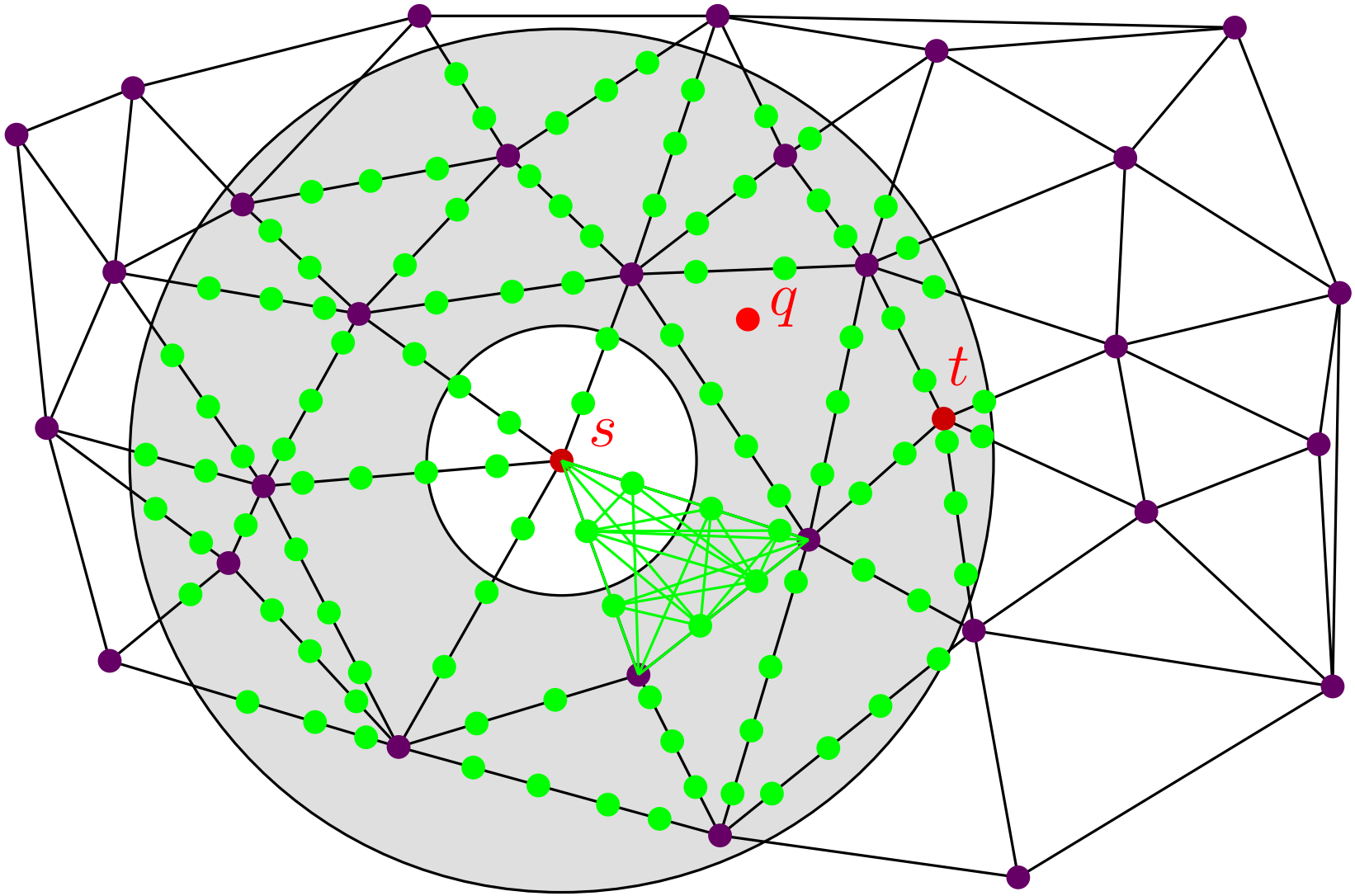


additive error at most δ

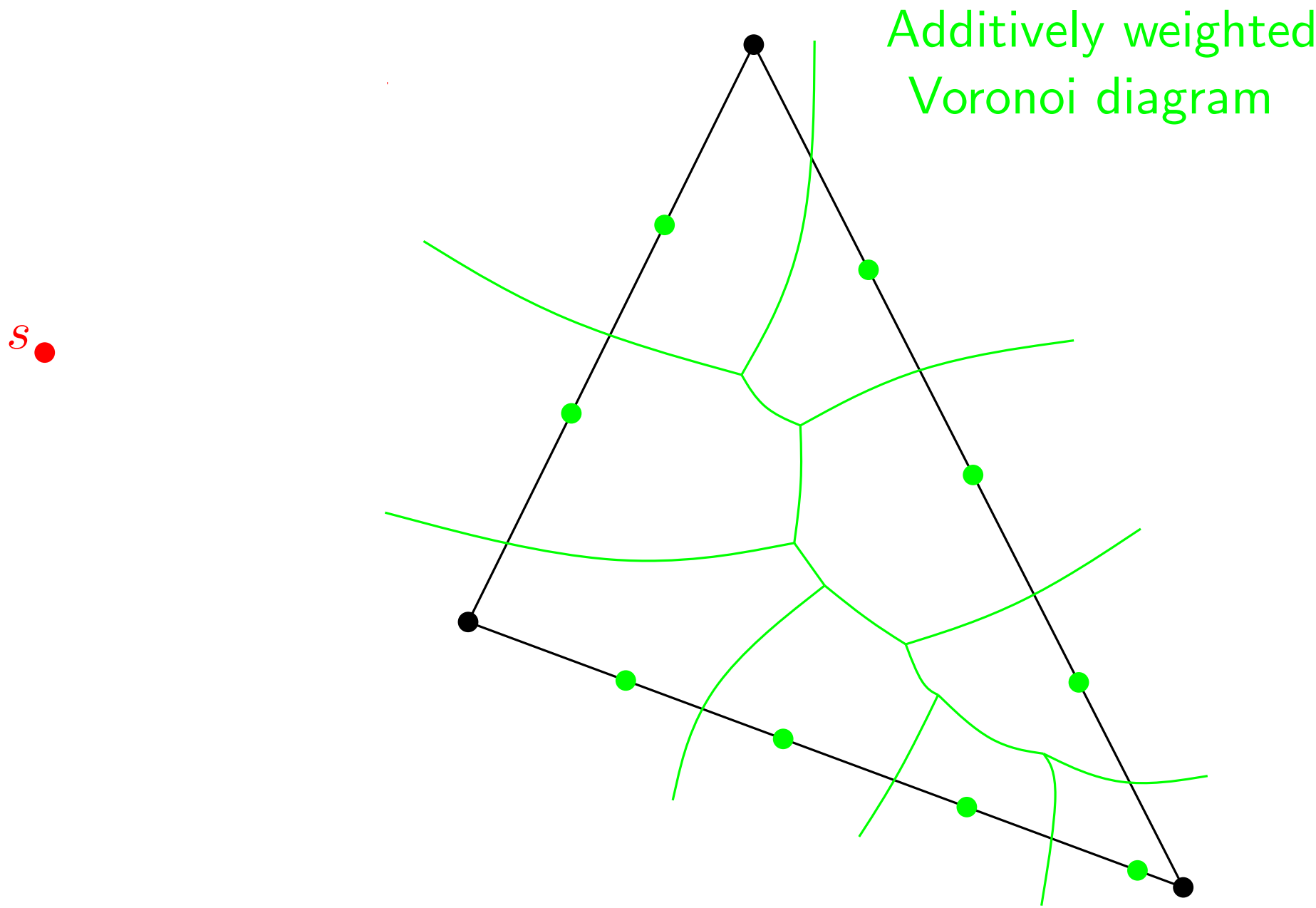
Observation

$(1+\varepsilon)$ approximation when q is in an annulus



radius ratio=poly$(\rho, n, 1/\varepsilon)$

Answering queries within the annulus



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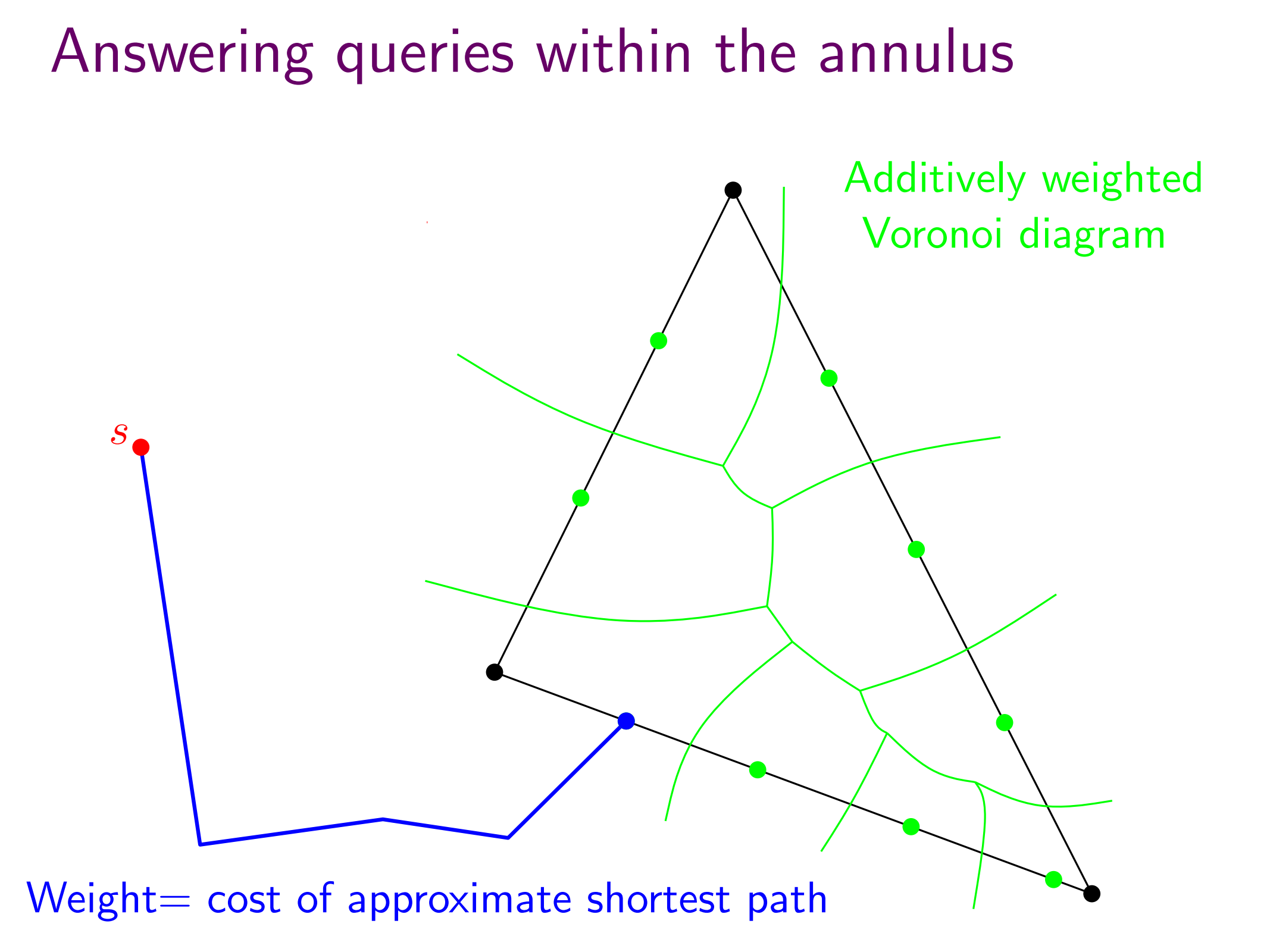

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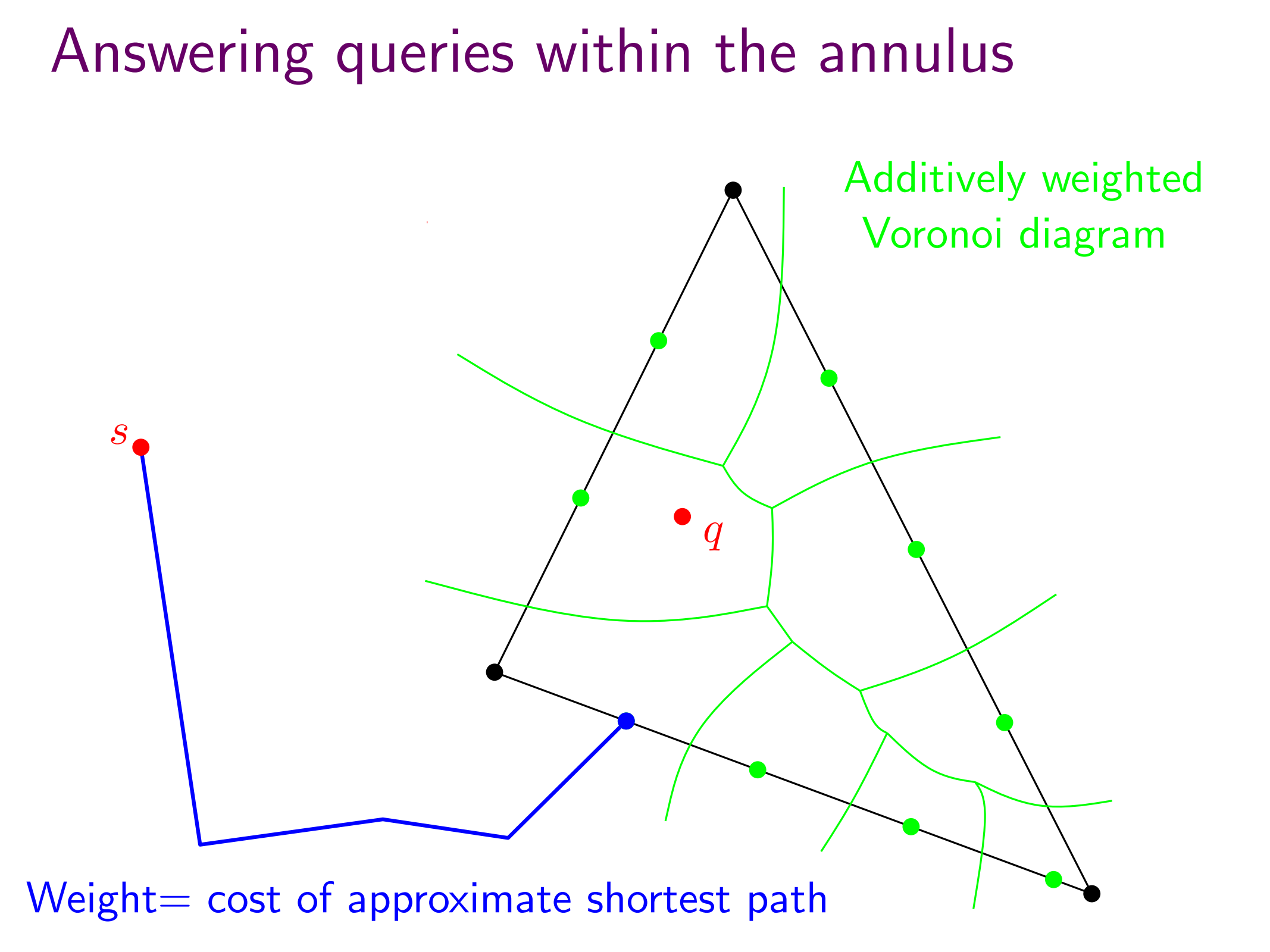

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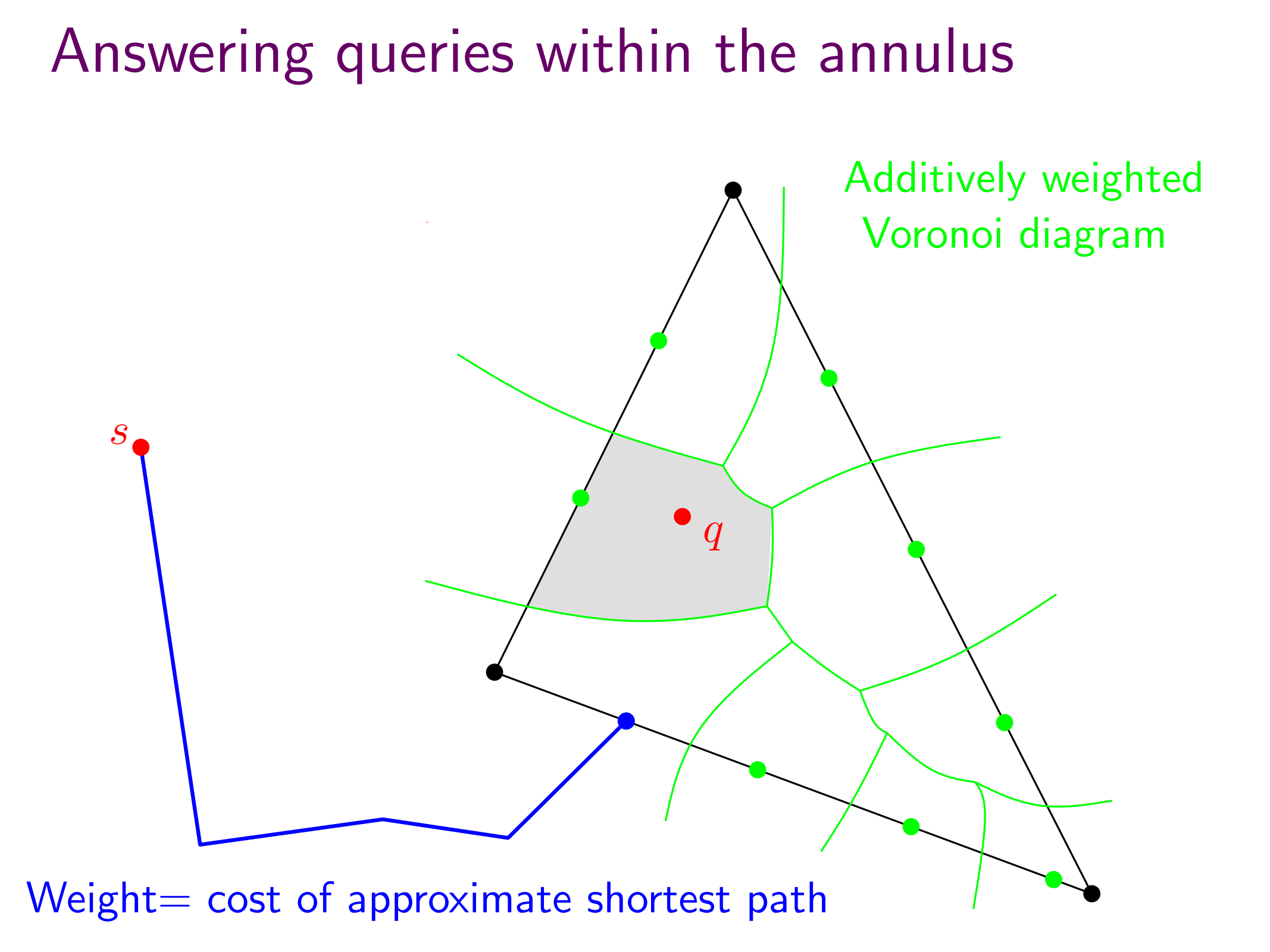

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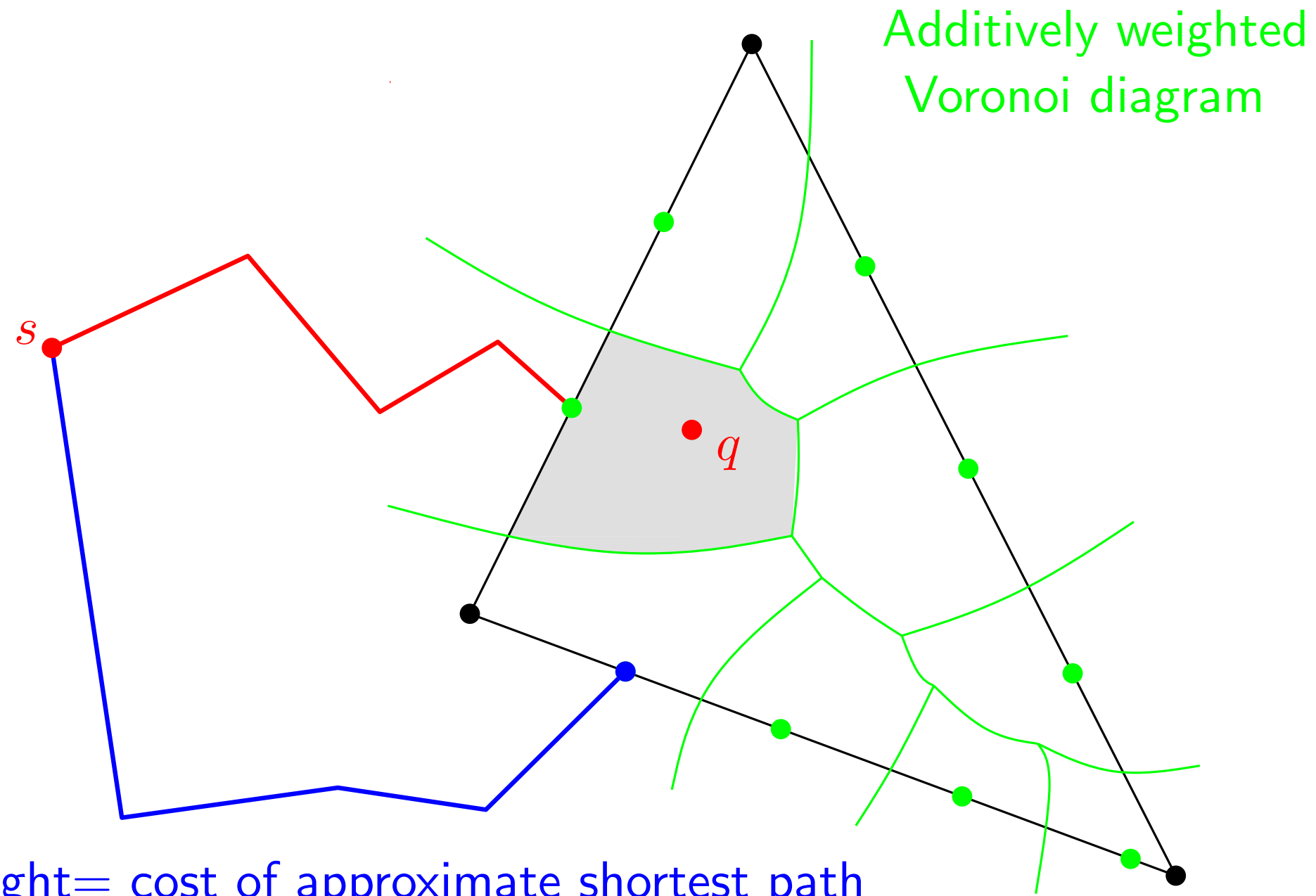

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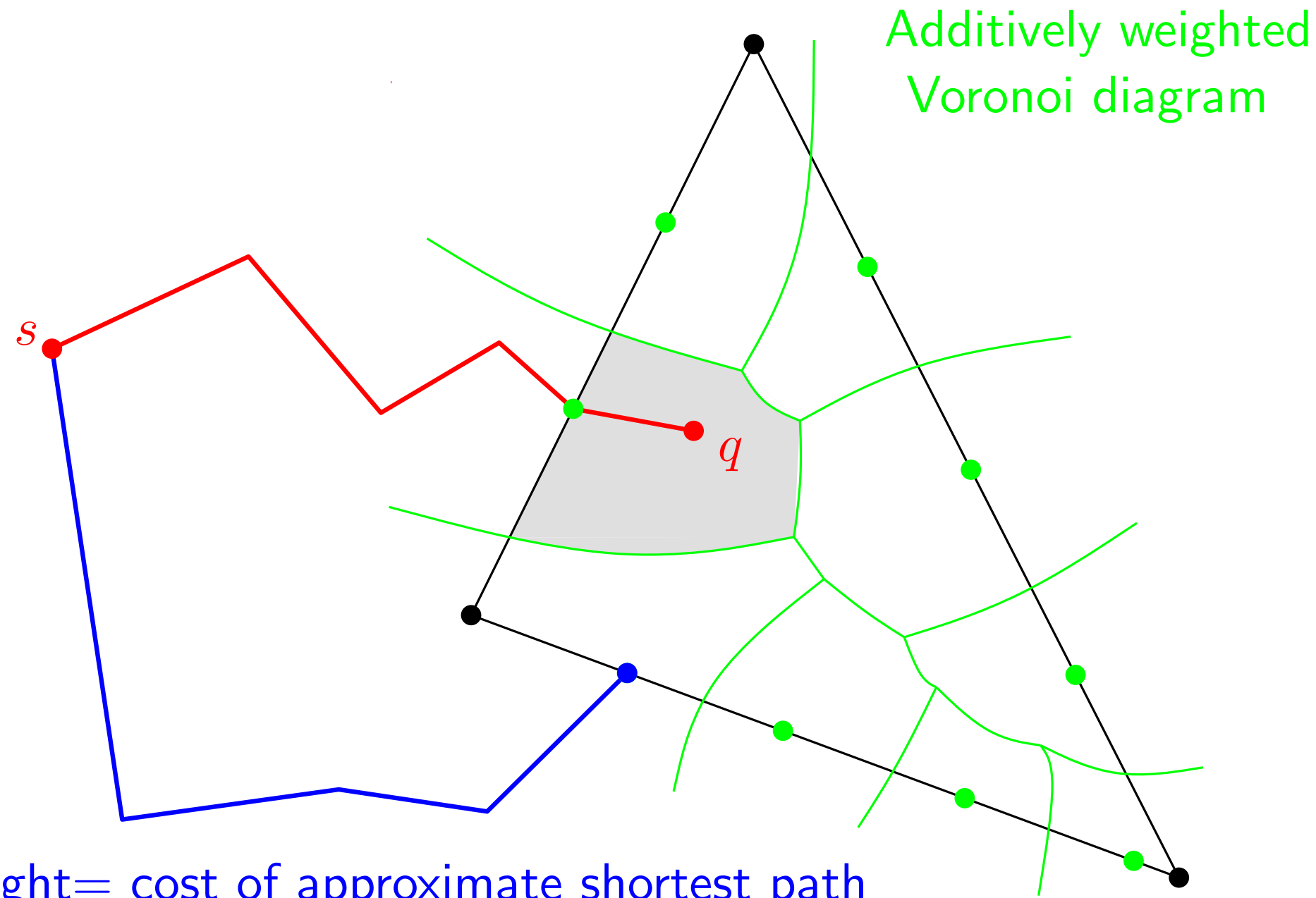

Overview

For each vertex v_i, we construct a data structure for the annulus with radius $r_i = d(s, v_i)/2\rho$ and $R_i = \text{poly}(\rho, n, 1/\varepsilon)d(s, v_i)$

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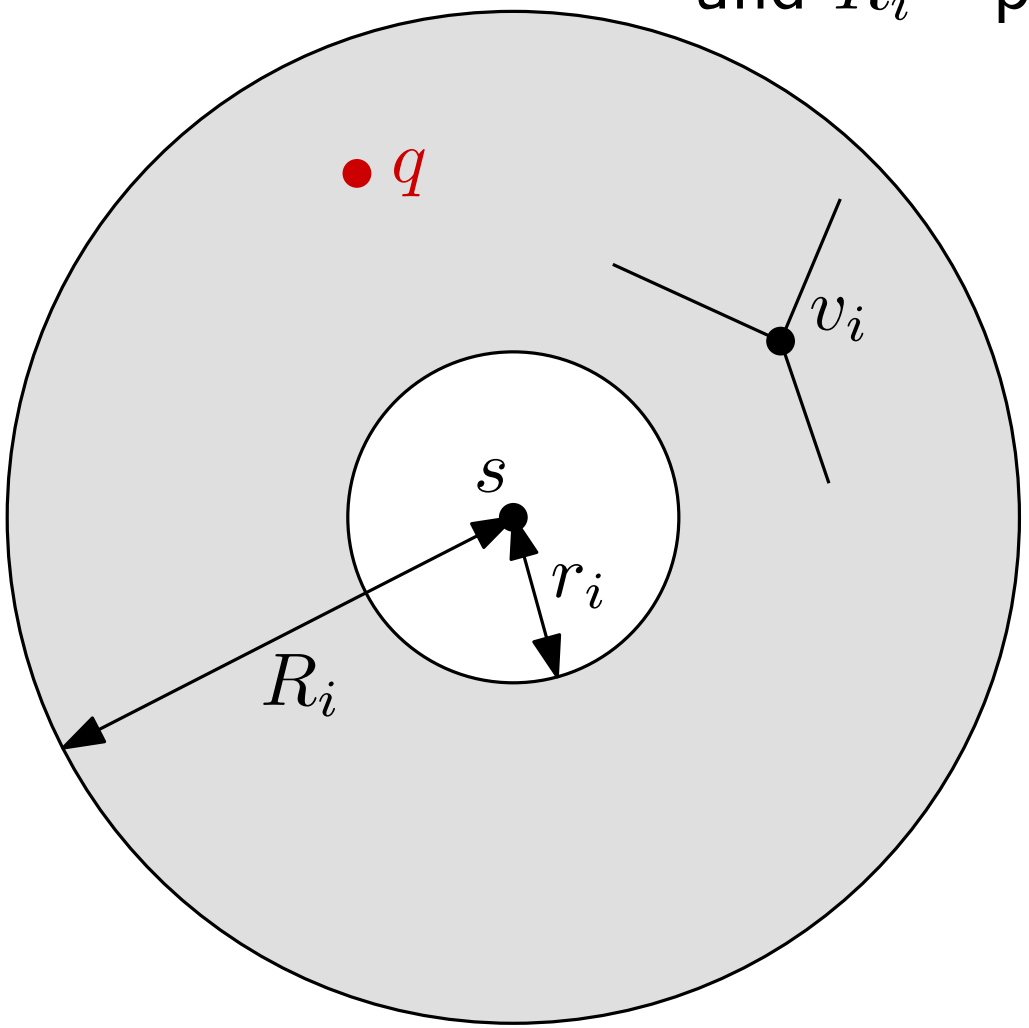

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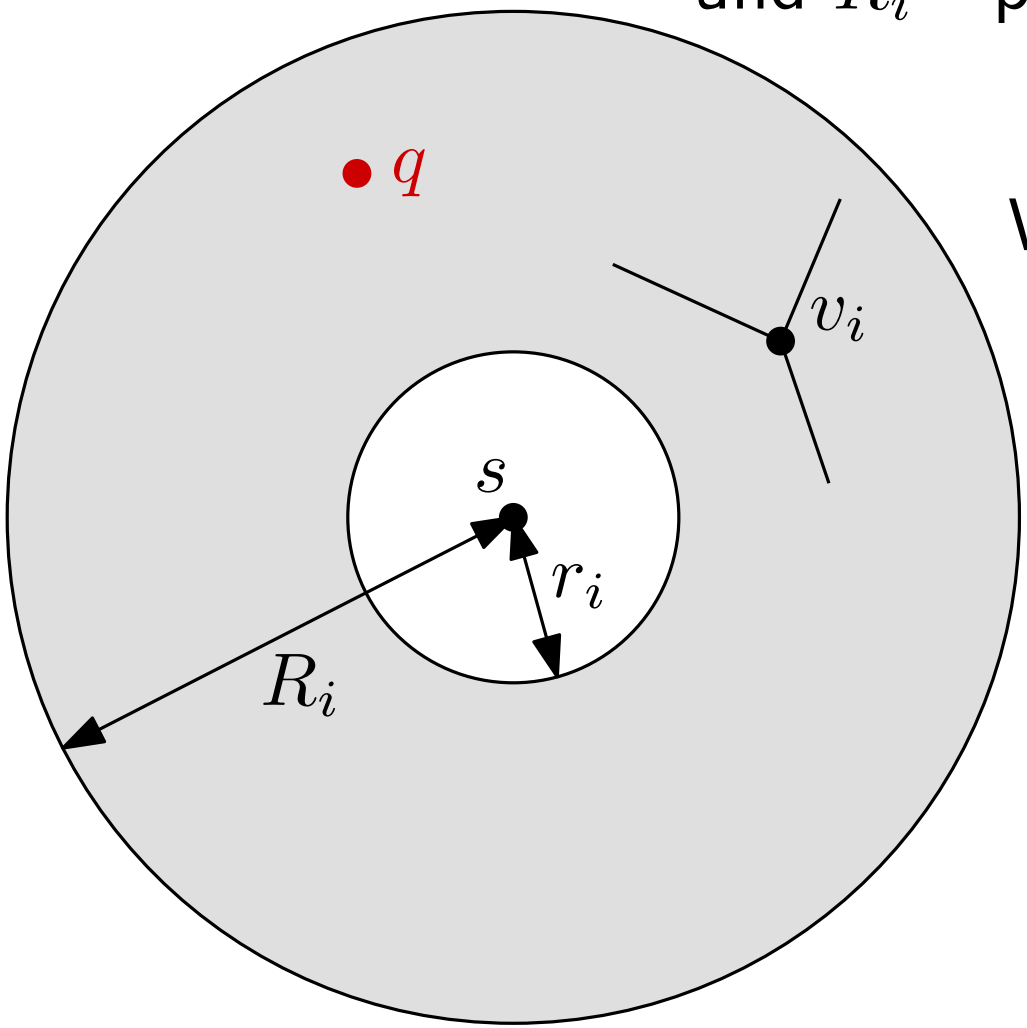

We handle the neighborhood of s through scaling

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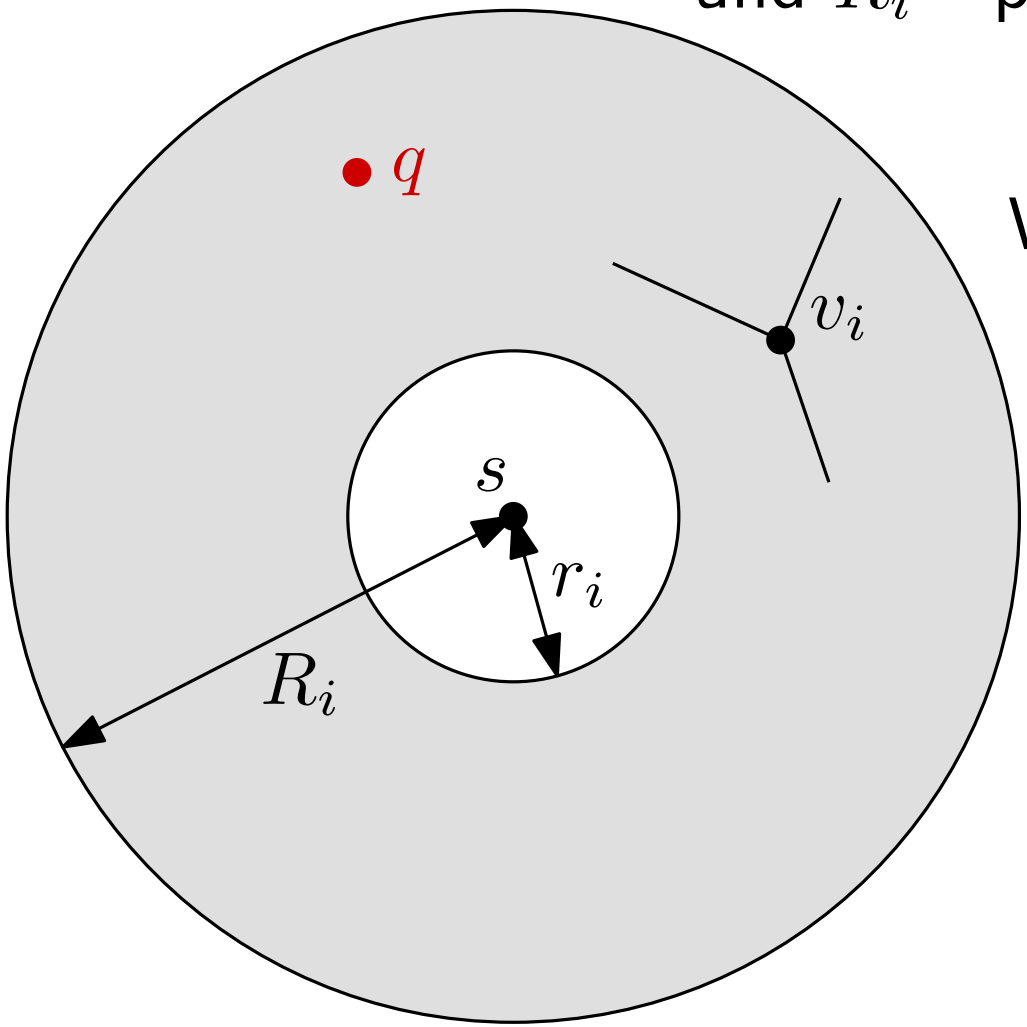
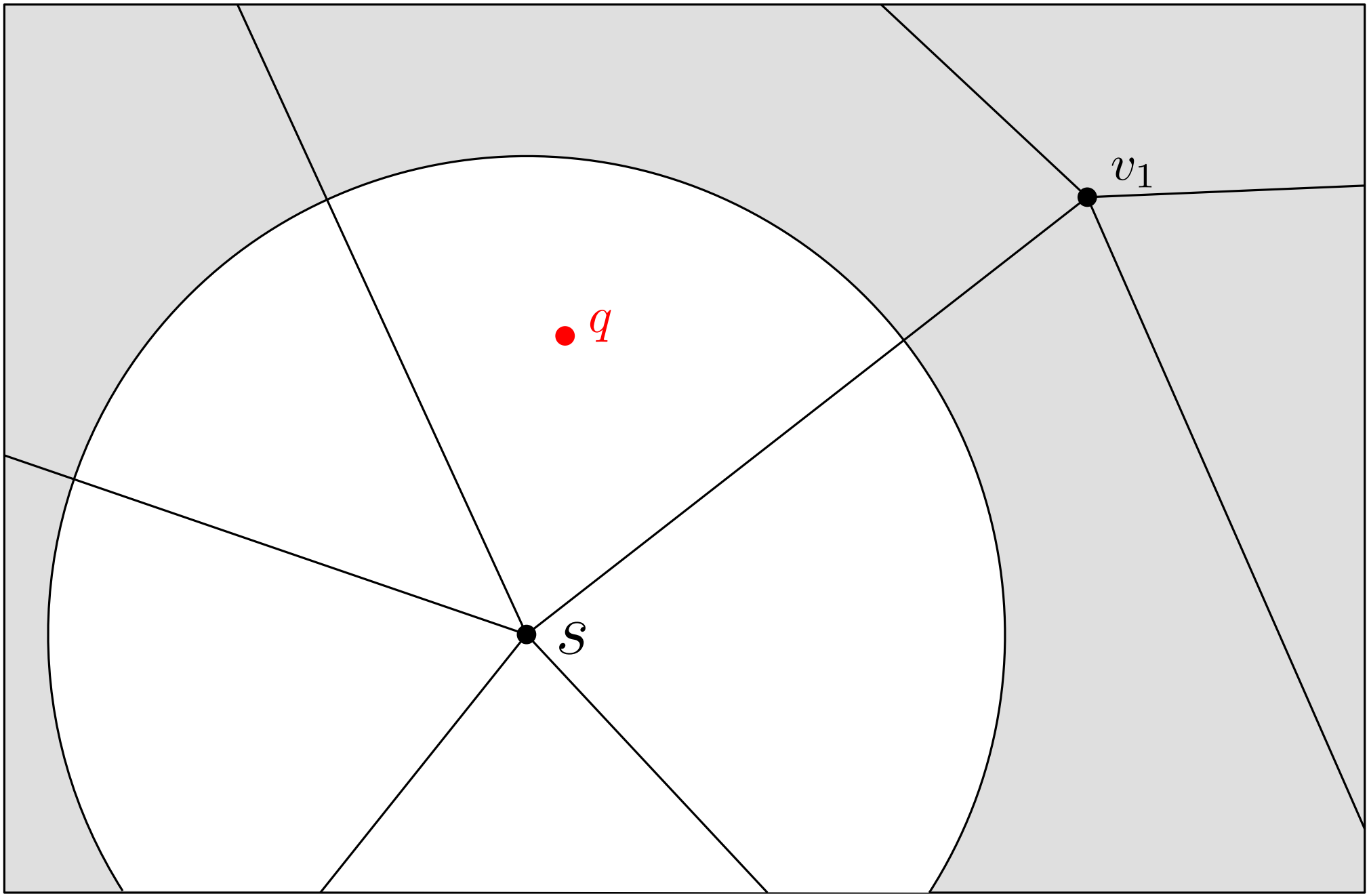


We handle the neighborhood of s through scaling

We handle the empty space (if any) through scaling and perturbation

Close range queries



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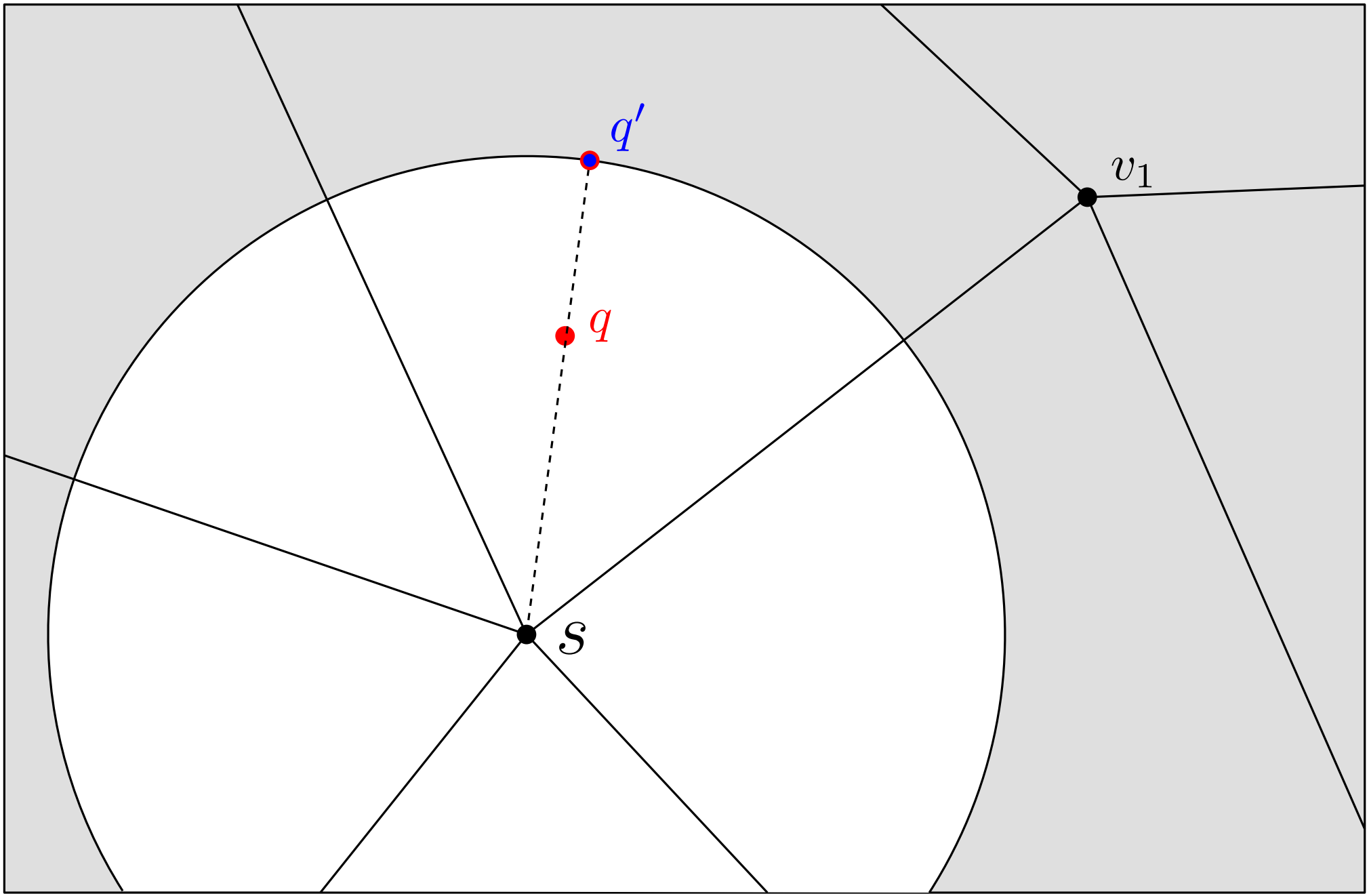

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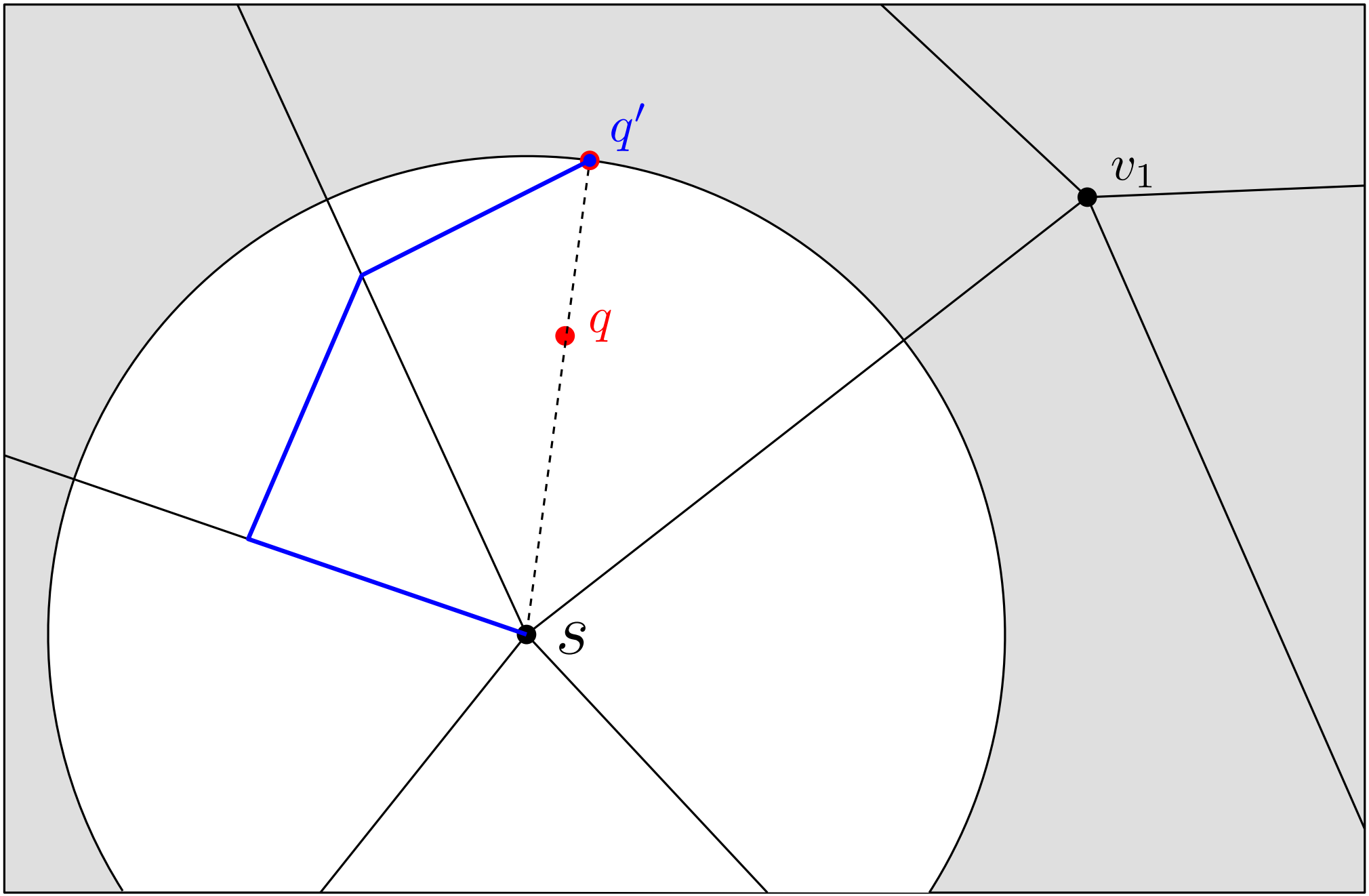

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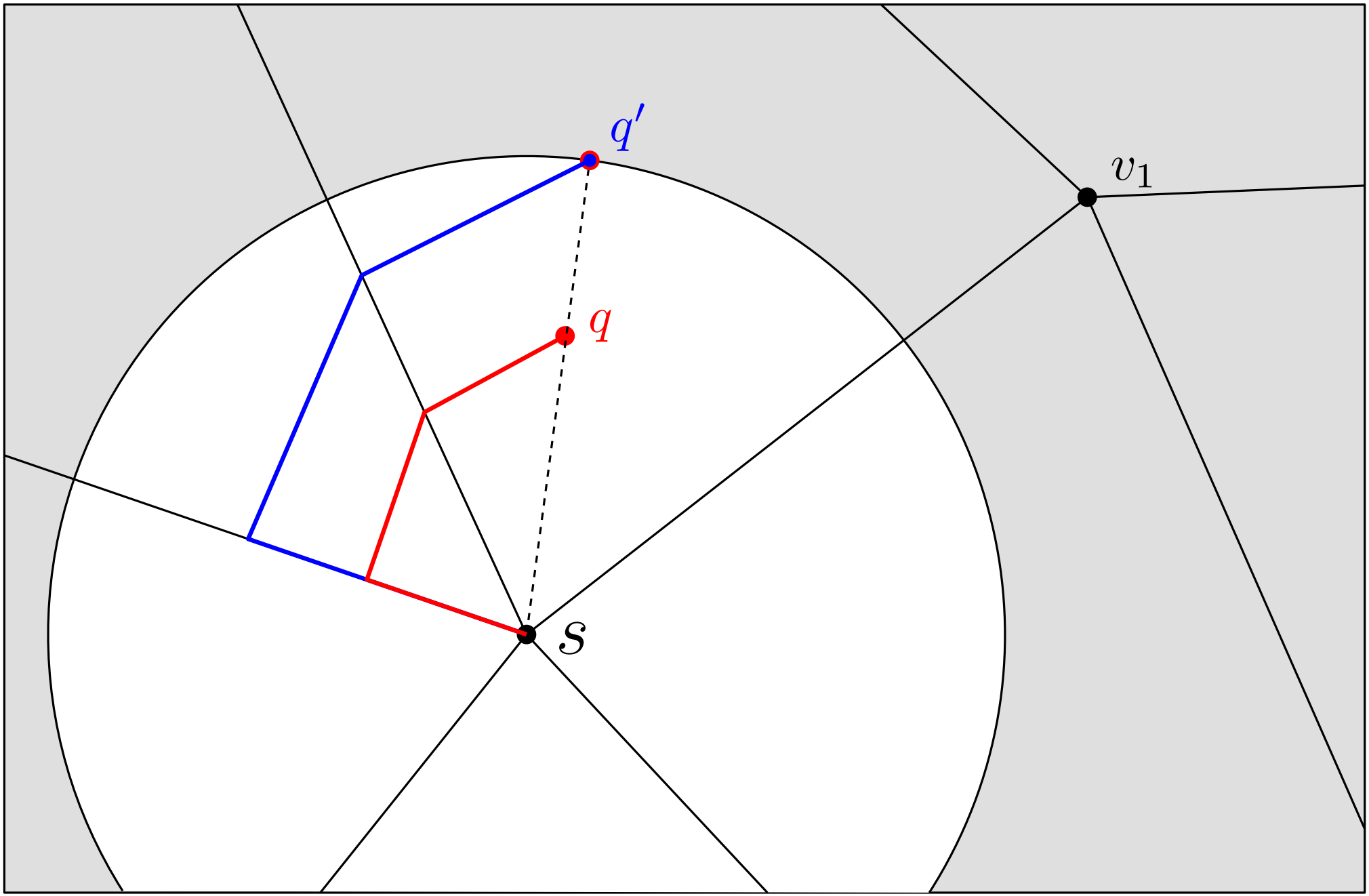
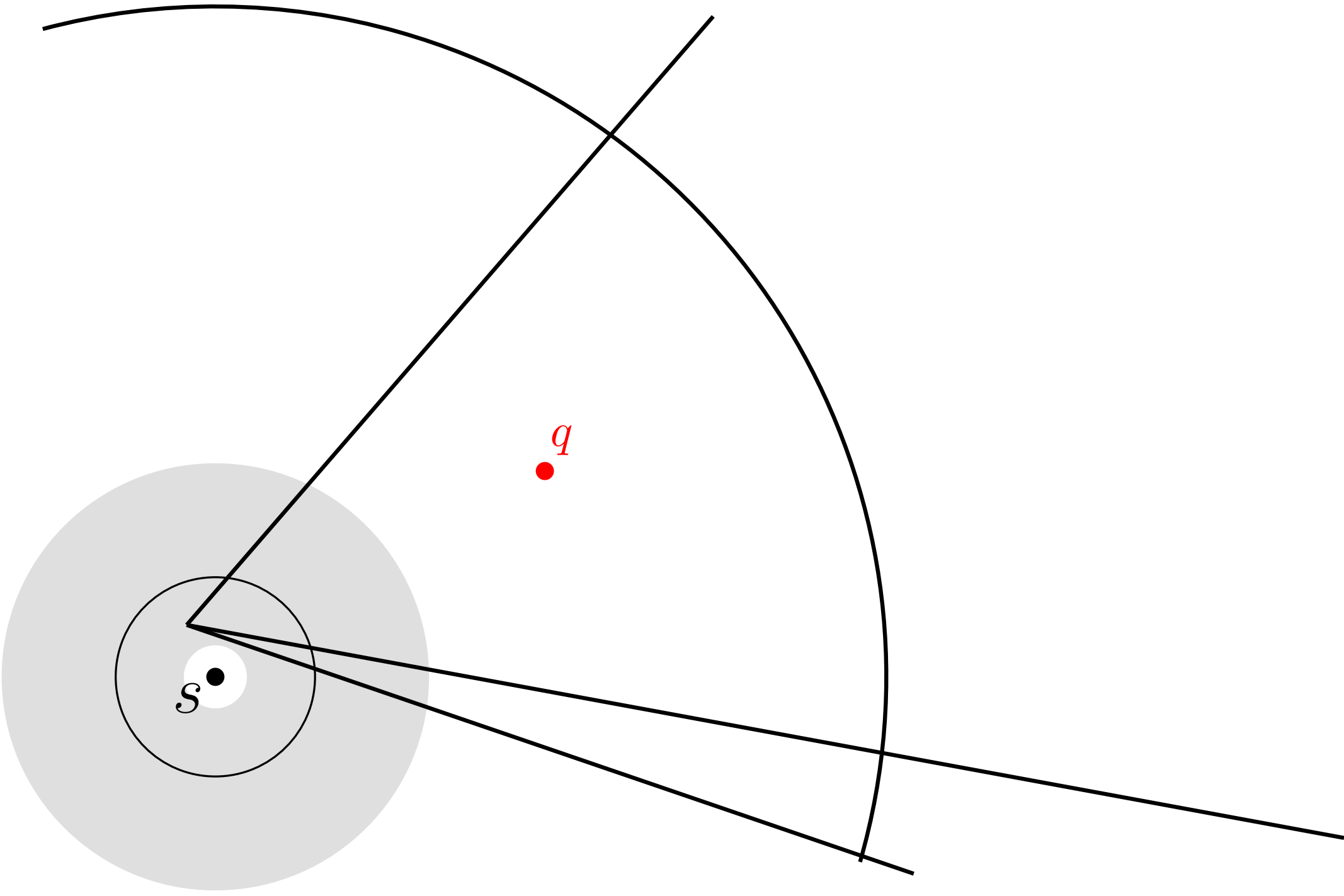


Between two annuli



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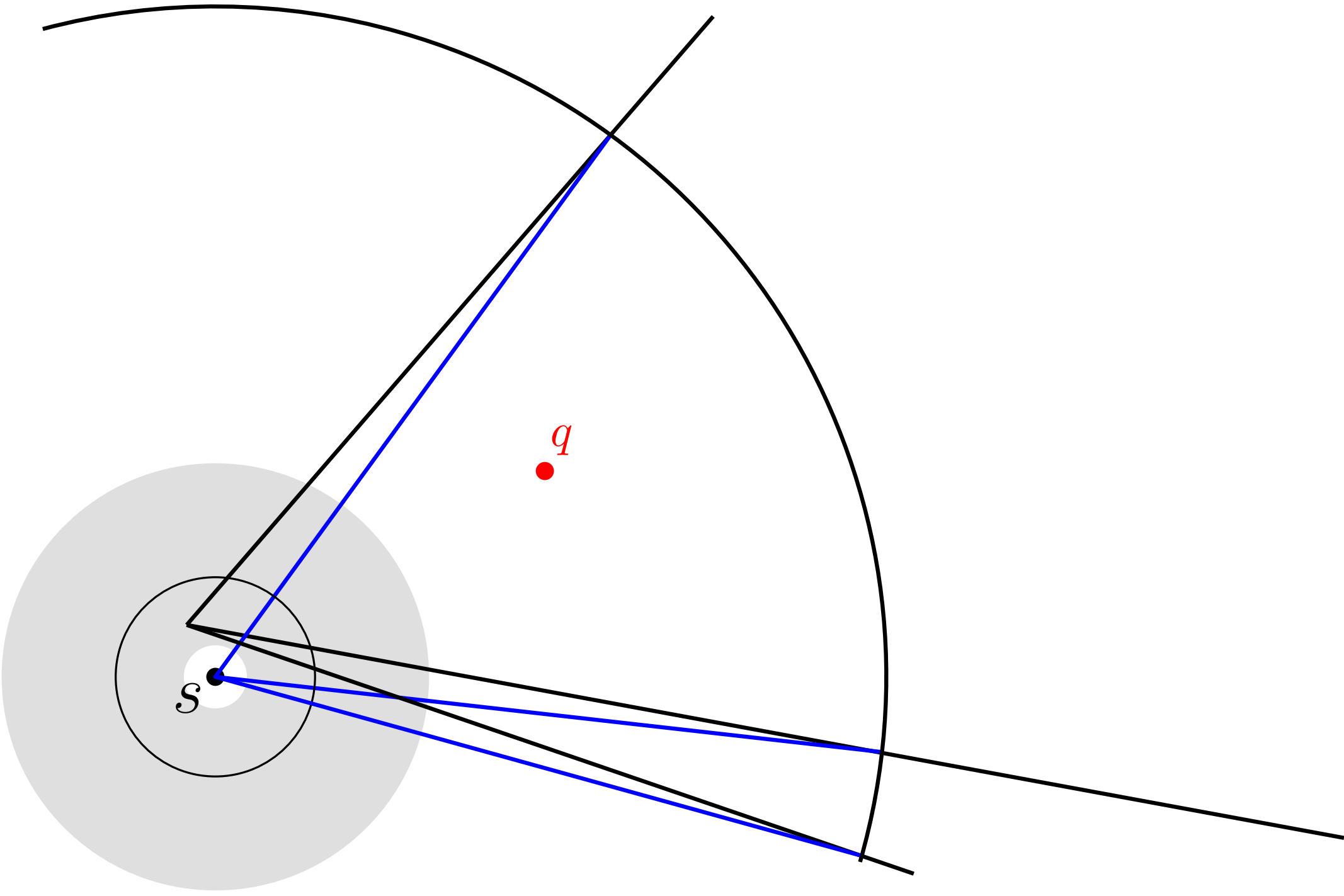

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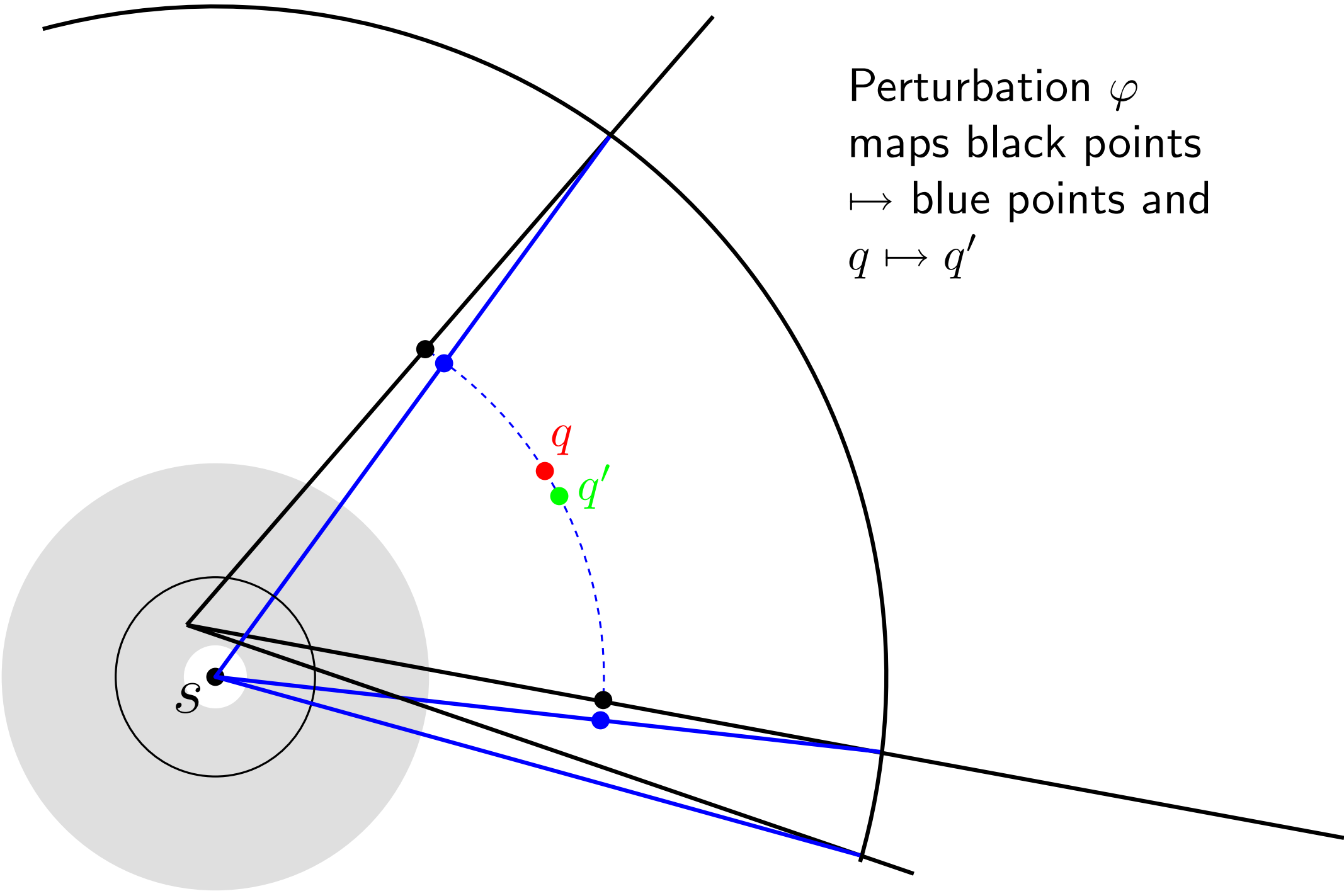
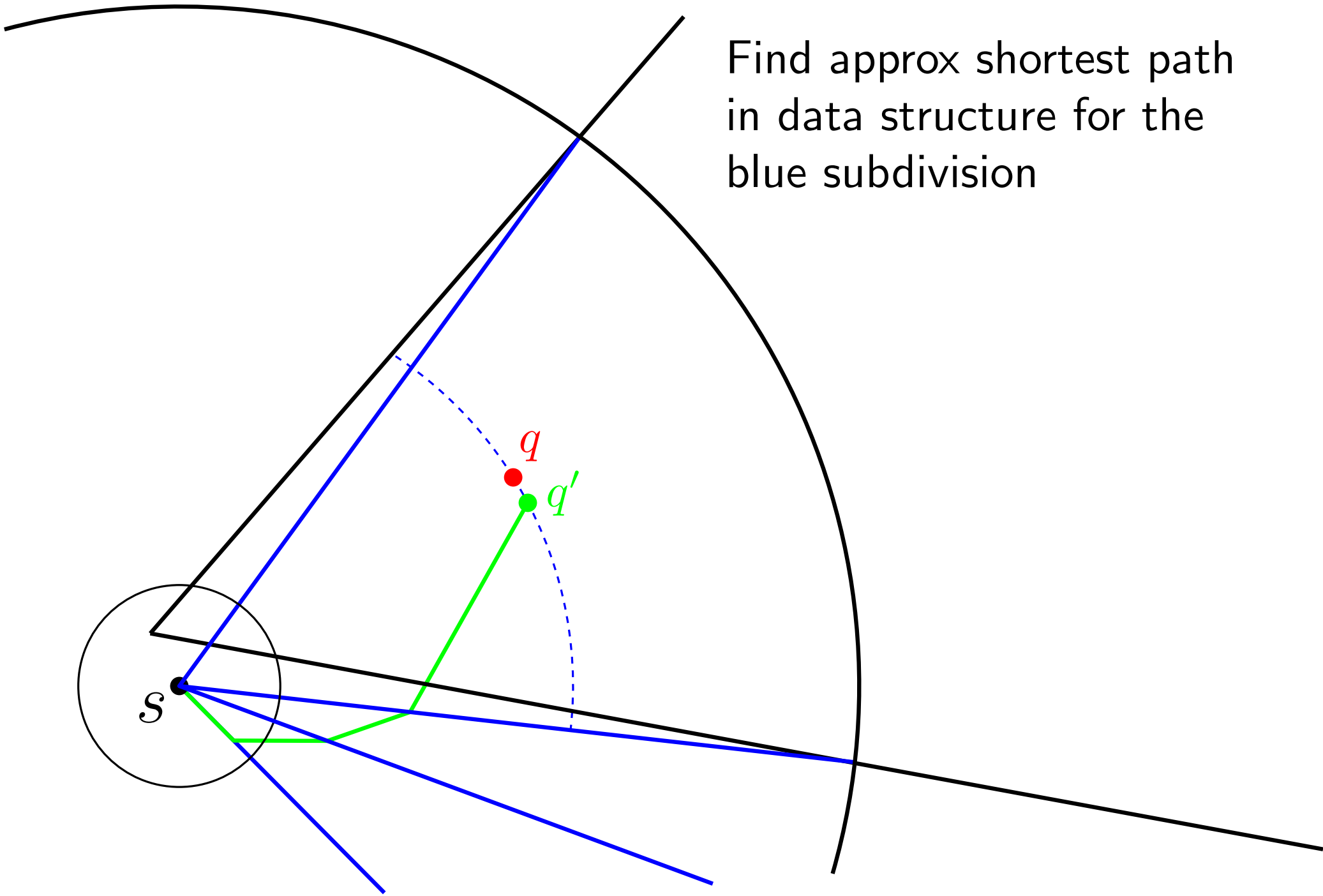


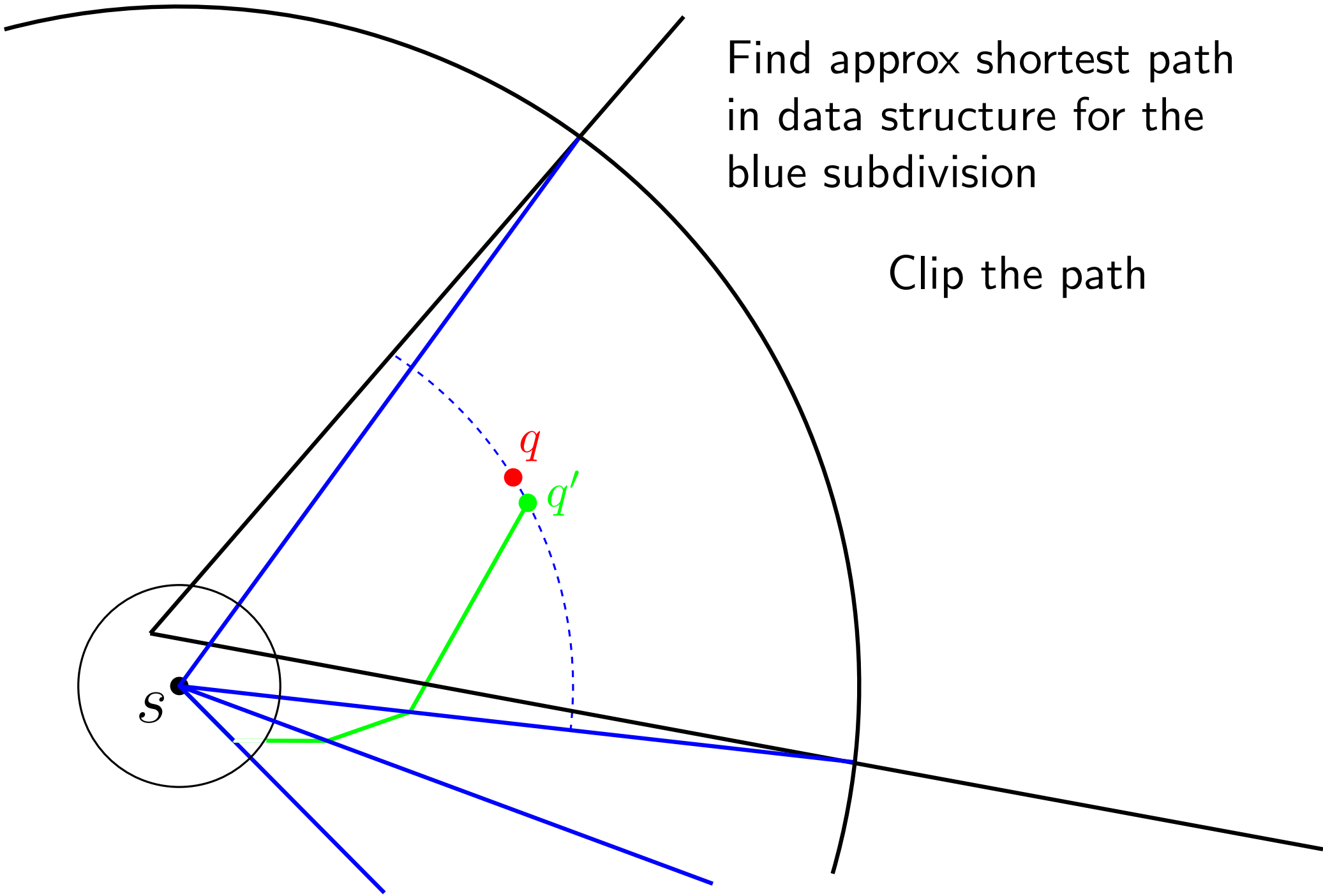
Perturbation φ
maps black points
$\mapsto$ blue points and
$q \mapsto q'$

Between two annuli



Find approx shortest path in data structure for the blue subdivision

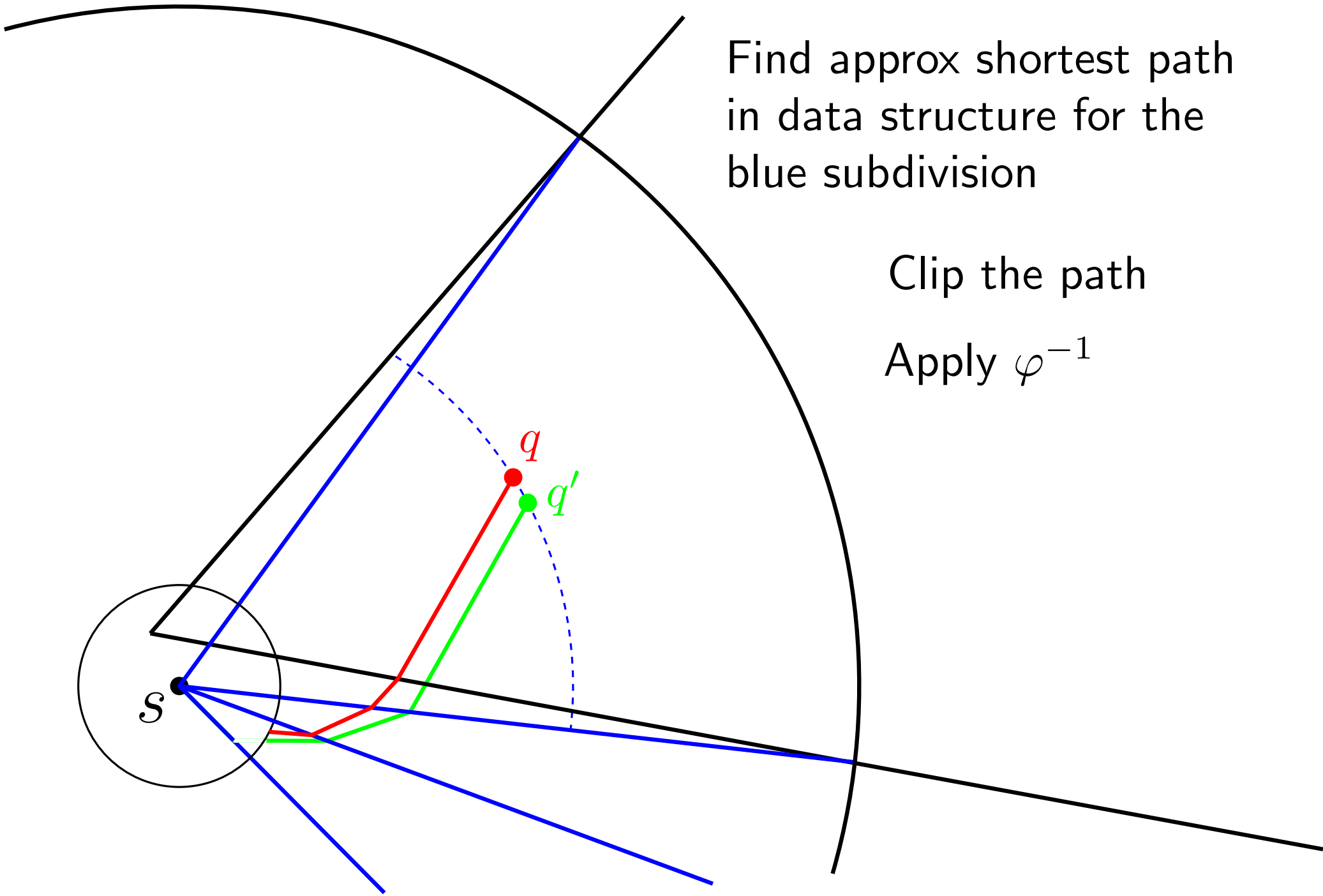
Between two annuli



Find approx shortest path in data structure for the blue subdivision

Clip the path

Between two annuli

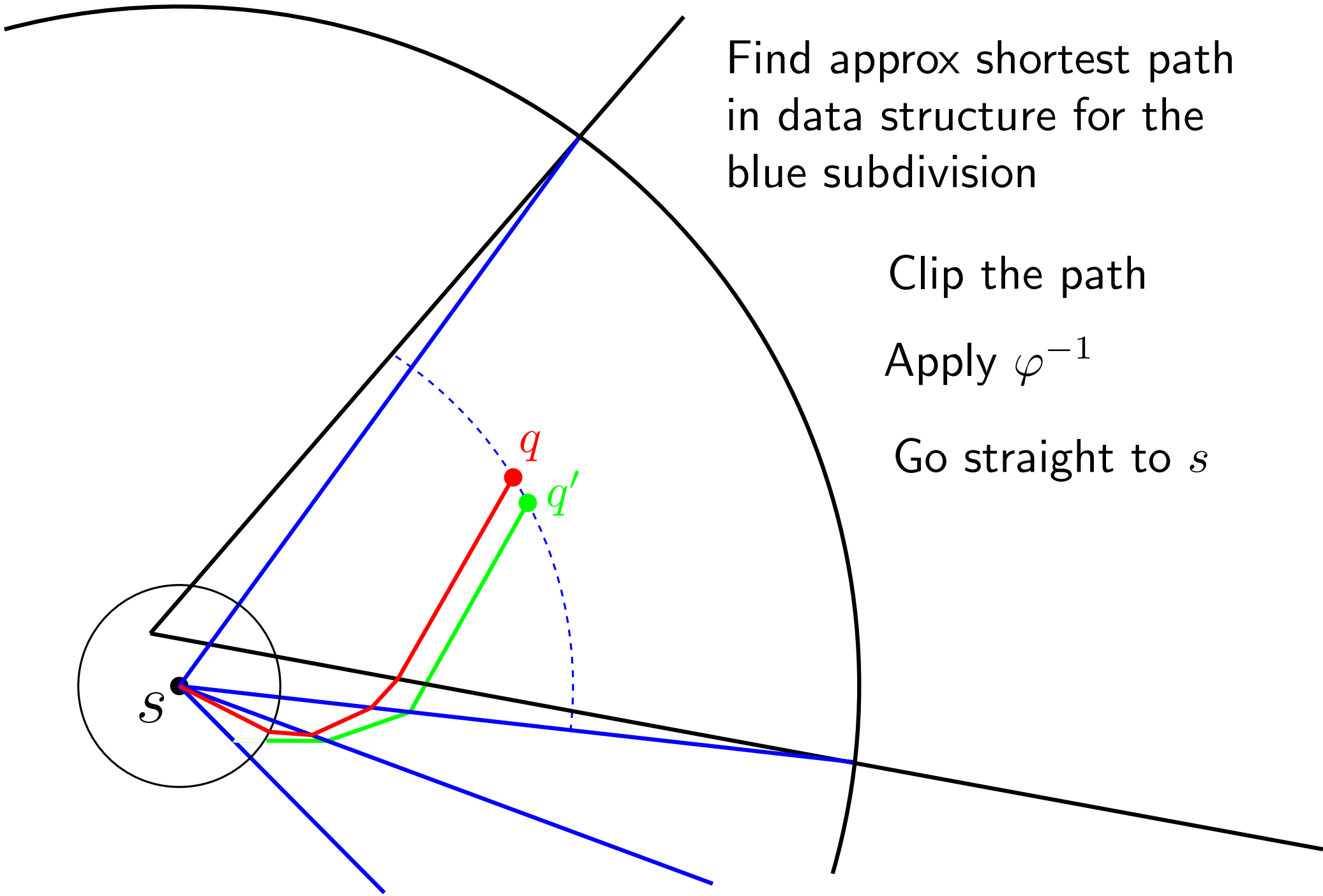


Find approx shortest path in data structure for the blue subdivision

Clip the path

Apply φ^{-1}

Between two annuli



Find approx shortest path in data structure for the blue subdivision

Clip the path

Apply φ^{-1}

Go straight to s

Conclusion

We can also handle obstacles

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2 point queries?

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