Lecture 6
Synchrotron Radiation and Free Electron Lasers

(Ch. 3, 8 and 10 of UP-ALP)

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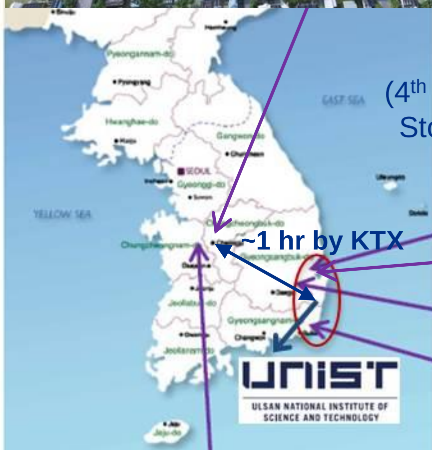
Half of the large accelerators are ineed light sources



4GSR

(4th Generation Storage Ring)

PLS II

(Pohang Light Source)

PAL XFEL

(Pohang Accelerator Lab. X-ray Free Electron Laser)

RISP

(Rare Isotope Science Project)

KHIMA

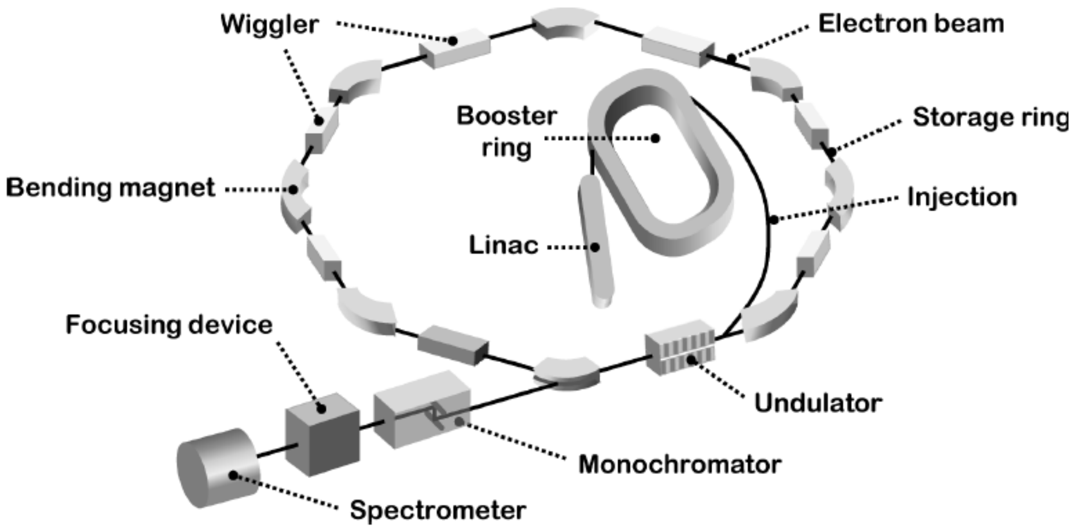
(Korea Heavy-Ion Medical Accelerator)

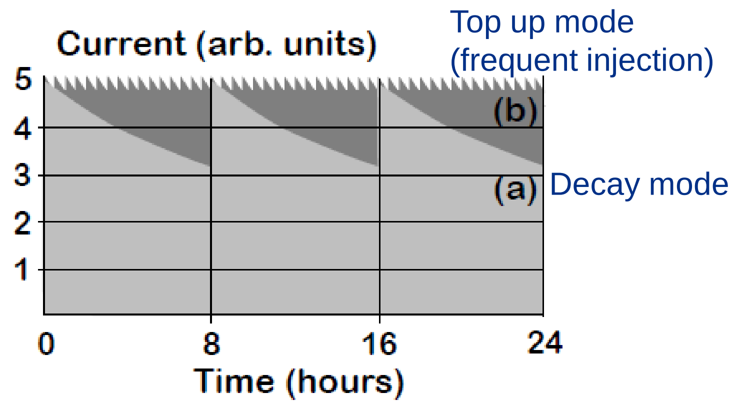
KOMAC

(Korea Multi-purpose Accelerator Complex)

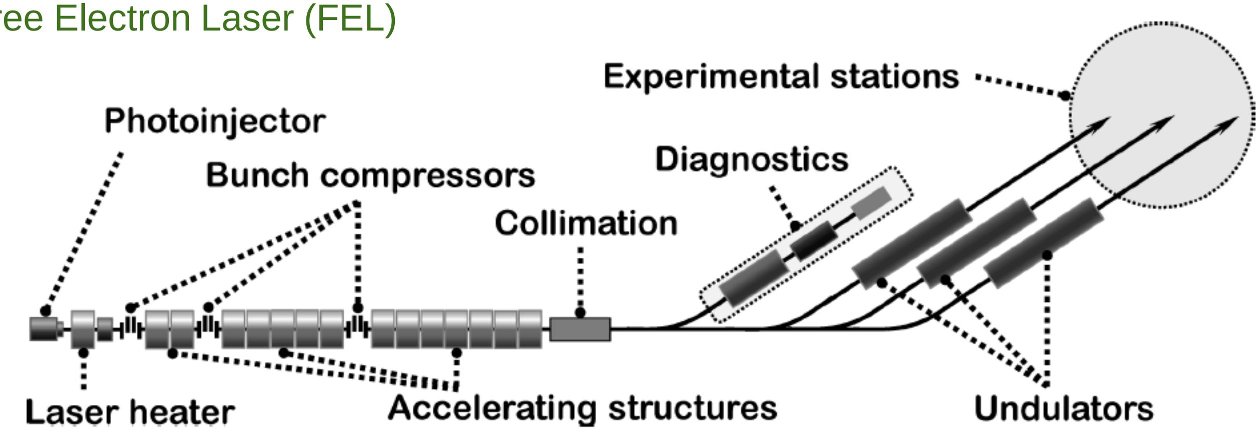
Synchrotron Light Source VS Free Electron Laser

A genertic Synchrotron Radiation (SR) light source





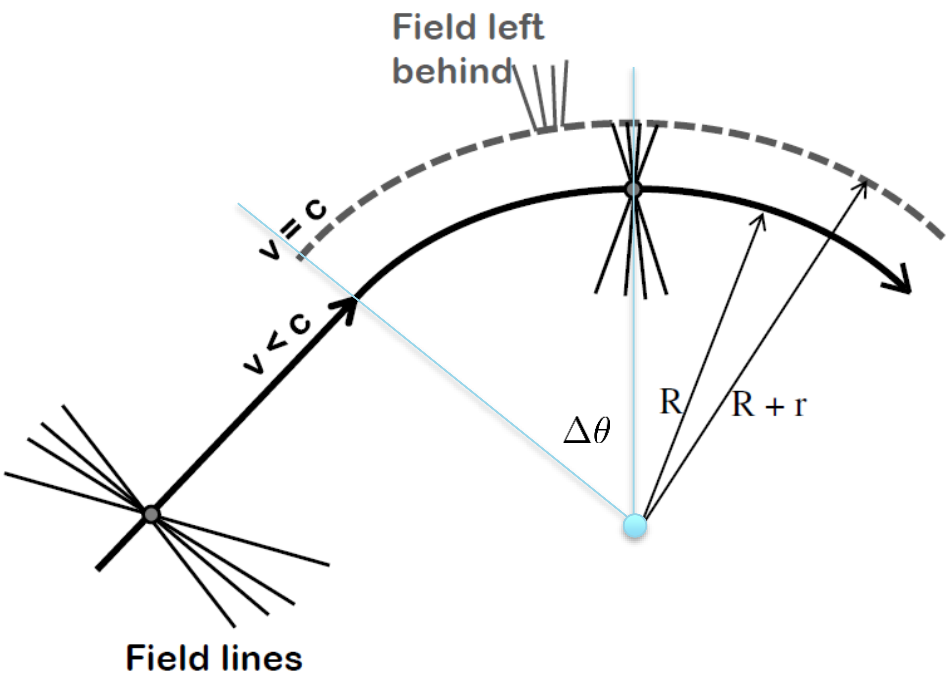
A genertic Free Electron Laser (FEL)



Synchrotron Radiation

3.1.1 SR power loss

- The synchrotron radiation is the result of the charged particle leaving part of its fields behind (as the field cannot catch up with the motion of the particles)



$$\frac{(R+r)\Delta\theta}{c} > \frac{R\Delta\theta}{v}$$

- The part of field moving further away (r) would be left behind. For $\gamma \gg 1, \beta \approx 1$,

$$r = R\left(\frac{c}{v} - 1\right) = R\frac{1-\beta}{\beta} = R\frac{(1-\beta)(1+\beta)}{\beta(1+\beta)} \approx R\frac{1-\beta^2}{2} = \frac{R}{2\gamma^2}$$

3.1.1 SR power loss (back-of-envelope estimation)

- Energy in the field that is left behind (i.e., radiated)

$$W \propto \int E^2 dV$$

- The field E can be estimated as the field on the radius r:

$$E \propto \frac{e}{r^2}$$

- The characteristic volume can be estimated

$$\int dV \sim \int_0^{2\pi} \int_0^{\pi} \int_r^{\infty} r^2 \sin\theta dr d\theta d\phi \sim 2 \int_0^{2\pi r} \int_r^{\infty} r dr ds$$

- The energy loss per unit length:

$$r \approx \frac{R}{2\gamma^2}$$

$$\frac{dW}{ds} \propto 2 \int_r^{\infty} E^2 r dr \sim 2 \int_r^{\infty} \frac{e^2}{r^4} r dr \sim \frac{e^2}{r^2} \sim \frac{e^2 \gamma^4}{R^2}$$

- Compare with the exact formula:

$$\frac{dW}{ds} = \frac{2}{3} \frac{e^2 \gamma^4}{R^2} \quad [\mathrm{CGS}]$$

3.1.2 Cooling time

- Let's rewrite the exact formula for power loss per unit length in the way that does not depend on the systems of units:

$$\frac{dW}{ds} = \frac{2}{3}\frac{r_e \gamma^4}{R^2} mc^2$$

Classical radius of an electron

- SR energy loss of a particle per turn is:

$$U_0 = \frac{dW}{ds} \times (2\pi R) = \frac{4\pi}{3}\frac{r_e \gamma^4}{R} mc^2$$

- Cooling time of a particle with energy $E_0 = \gamma mc^2$ cycling with revolution period $T_0 = 2\pi R/c$:

$$\tau = \frac{E_0}{U_0/T_0} \approx \frac{2\pi R}{c}\frac{\gamma mc^2}{U_0}$$

  – Or, inverse cooling time can be written as

$$\tau^{-1} \approx \frac{2}{3}\frac{c r_e \gamma^3}{R^2}$$

- The beam of particles can have a range of angles within the beam.
- The momentum of radiation is in the same direction as the momentum of the particle.
- RF cavity restores only longitudinal momentum, thus emittances of other degrees of freedom are reduced (cooling).

3.1.3 Cooling time and partition

- Traditionally, there is a factor of 2 in the definition in the cooling time:

$$\tau = \frac{2E_0}{U_0/T_0} \longrightarrow \tau^{-1} = \frac{1}{3}\frac{cr_e\gamma^3}{R^2}$$

- We can express the evolution of the beam emittance (indeed, cooling is not about the energy itself but about temperature-its spread) under the influence of an SR damping as

$$\varepsilon(t) = \varepsilon_0 \exp(-2t/\tau)$$

- Both transverse planes, as well as the longitudinal motion in rings, are usually coupled. Thus, we can expect the damping will be distributed between these degrees of freedom in some proportion depending on details of the optics.
- Distribution of cooling between the DOF is defined by partition numbers:

$$\tau_i = \frac{\tau}{J_i}$$

- The total radiated power (U_0) due to SR is fixed, therefore

$$\sum_{i=x,y,E} \tau_i^{-1} = const. \longrightarrow \sum_{i=x,y,E} J_i = 4$$

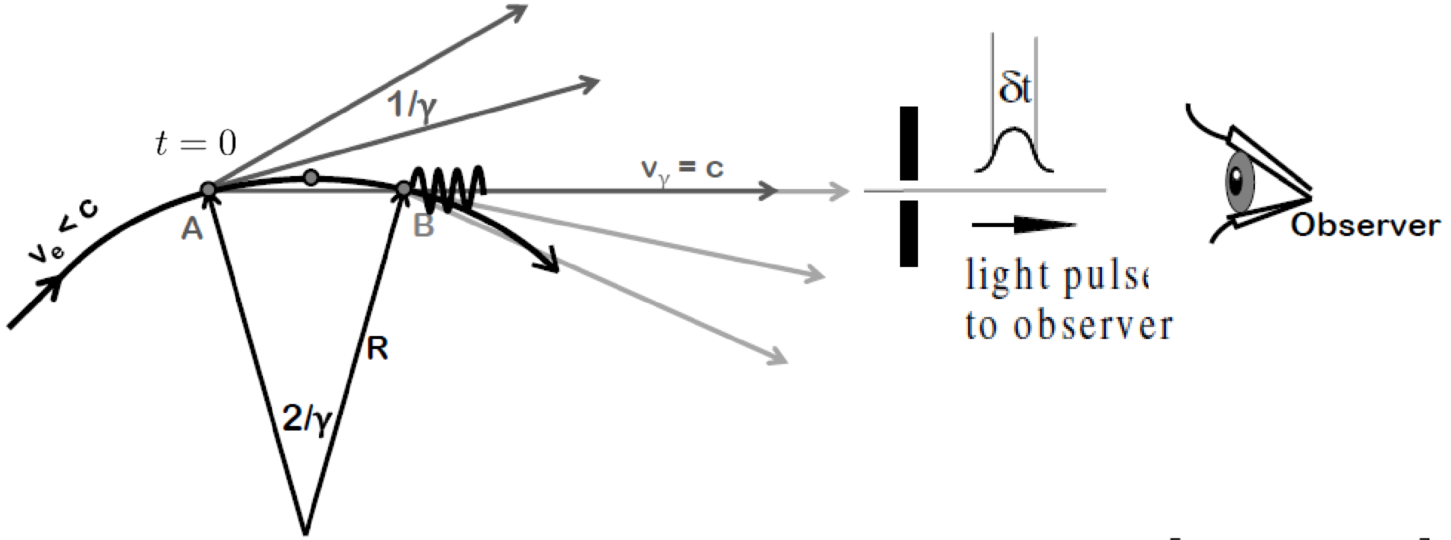
Robinson damping theorem or Partition theorem

  - For a typical accelerator

$$J_x \approx 1, \quad J_y \approx 1, \quad J_E \approx 2$$

3.1.4 SR Photon energy

- Based on the relativistic kinematics, we note that the radiation of relativistic particles is emitted into a cone with angular spread of $1/\gamma$.



Time for first photon to arrive at point B: $t_1 = \dfrac{2R\sin(1/\gamma)}{c}$

Time for the particle to arrive at point B: $t_2 = \dfrac{2R}{\gamma v}$

$$\delta t = t_2 - t_1 = \frac{2R}{v}\left[\frac{1}{\gamma} - \beta\sin(1/\gamma)\right]$$
$$\approx \frac{2R}{v\gamma}(1-\beta) \approx \frac{R}{c\gamma^3}$$

- Estimate the characteristic frequency (or, critical frequency: SR spectrum reaches a maximum near here and drops exponential zero above this) of emitted photons as the inverse of the time duration of the light pulse.

$$\omega_c \approx \frac{1}{\delta t} \approx \frac{c\gamma^3}{R} \longrightarrow \frac{3}{2}\frac{c\gamma^3}{R}$$

3.1.5 Number of photons

- The photon energy is

$$\omega_c \approx \frac{c\gamma^3}{R}$$

$$\epsilon_c = \hbar\omega_c \approx \frac{\gamma^3 \hbar c}{R}$$

- The number of photons emitted per unit length can be obtained by dividing the energy loss per unit length by the energy of the photons:

$$\frac{dW}{ds} \approx \frac{e^2\gamma^4}{R^2} \quad [\mathrm{CGS}]$$

$$\frac{dN}{ds} \approx \frac{1}{\epsilon_c}\frac{dW}{ds} \approx \frac{\alpha\gamma}{R}$$

$$\text{Fine structure constant: } \alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} \approx 1/137 \quad [\text{unitless, indepent of units}]$$

- The number of photons emitted per unit of the bending angle θ:

$$\frac{dN}{ds} = \frac{dN}{Rd\theta} \approx \frac{\alpha\gamma}{R} \longrightarrow N \approx \alpha\gamma\theta$$

3.2.1 SR-induced energy spread

- The energy spread $\Delta E/E$ will grow due to statistical fluctuations $\sqrt{N}$ of the number of emitted SR photons

$$\frac{d\left((\Delta E/E)^2\right)}{ds} \approx \frac{dN}{ds}\frac{\epsilon_c^2}{(\gamma mc^2)^2}$$

- Using

(reduced) Compton wavelength

$$\frac{dN}{ds} \approx \frac{\alpha\gamma}{R} \qquad \epsilon_c \approx \frac{\gamma^3 \hbar c}{R} \qquad \alpha = \frac{e^2}{\hbar c}, \quad r_e = \frac{e^2}{mc^2}, \quad \lambda_e = \frac{\hbar}{mc} = \frac{r_e}{\alpha}$$

Fine structure constant [CGS] Classical electron radius [CGS]

- We have

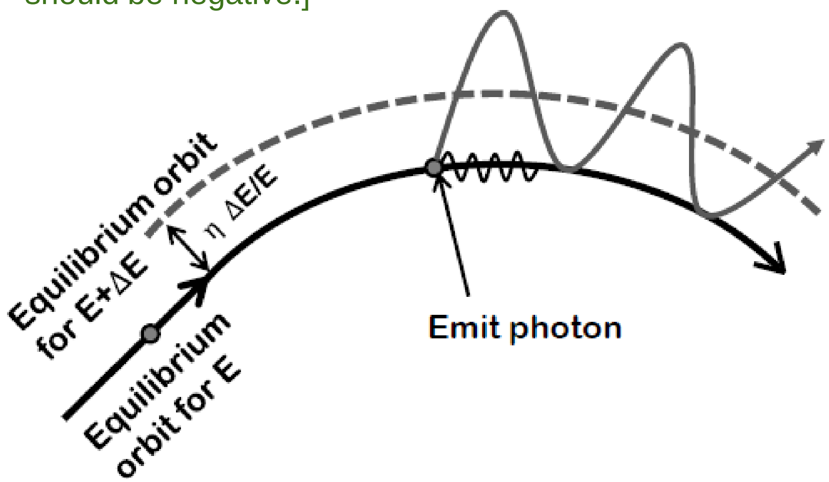
$$\frac{d\left((\Delta E/E)^2\right)}{ds} \approx \frac{r_e \lambda_e \gamma^5}{R^3}$$

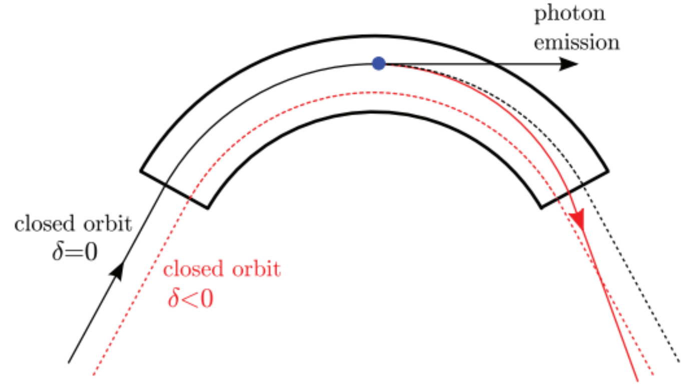
$$\frac{d\left((\Delta E/E)^2\right)}{ds} = \frac{55}{24\sqrt{3}}\frac{r_e \lambda_e \gamma^5}{R^3} \approx 1.32\frac{r_e \lambda_e \gamma^5}{R^3}$$ [Exact formula]

3.2.2 SR-induced emittance growth

- When a photon is emitted and the energy of the particle becomes equal to $E + \Delta E$ (where $\Delta E < 0$, the particle starts to oscillate around a new equilibrium orbit.

[Cartoon in the textbook seems unphysical because ΔE should be negative.]





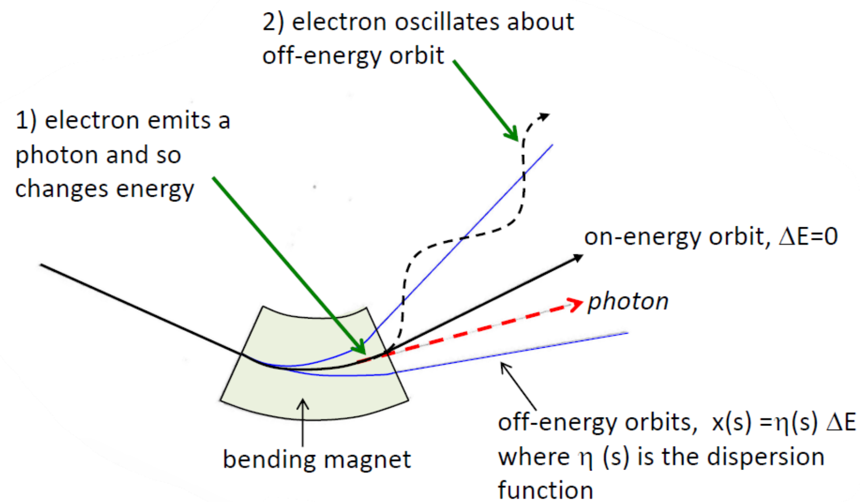
- The amplitude of the oscillation:

$$\Delta x \approx \eta \frac{\Delta E}{E}$$

$$\sigma_x = (\varepsilon_x \beta_x)^{1/2}$$

- Estimation for the emittance grwoth:

$$\Delta \varepsilon_x \approx \frac{\Delta x^2}{\beta_x}$$



3.2.2 SR-induced emittance growth (cont'd)

- By using the previous equations, we obtain an estimation for the emittance growth:

$$\frac{d\varepsilon_x}{ds} \approx \frac{\eta^2}{\beta_x}\frac{d\left((\Delta E/E)^2\right)}{ds} \approx \frac{\eta^2}{\beta_x}\frac{r_e\lambda_e\gamma^5}{R^3}$$

- In the above estimation we ignored the dependence of β_x and η on s; however, these dependences can alter the results.

$$\frac{d\varepsilon_x}{ds} \approx \frac{\left[\eta^2 + (\beta_x\eta' - \beta_x'\eta/2)^2\right]}{\beta_x}\frac{55}{24\sqrt{3}}\frac{r_e\lambda_e\gamma^5}{R^3}$$

- Here, we define dipersion $\mathcal{H}$–function (sometimes called the 'curly-H function'):

$$\begin{aligned}
\mathcal{H} &= \frac{\left[\eta^2 + (\beta_x\eta' - \beta_x'\eta/2)^2\right]}{\beta_x} \\
&= \frac{\left[\eta^2 + (\beta_x\eta' + \alpha_x\eta)^2\right]}{\beta_x} \\
&= \frac{(1+\alpha_x^2)}{\beta_x}\eta^2 + 2\alpha_x\eta\eta' + \beta_x\eta'^2 \\
&= \gamma_x\eta^2 + 2\alpha_x\eta\eta' + \beta_x\eta'^2
\end{aligned}$$

3.2.3 Equilibrium emittance

- Let's look at the estimated rate of emittance growth:

$$\frac{d\varepsilon_x}{ds} \approx \frac{\eta^2}{\beta_x}\frac{d\left((\Delta E/E)^2\right)}{ds} \approx \frac{\eta^2}{\beta_x}\frac{r_e\lambda_e\gamma^5}{R^3}$$

- The evolution of the beam emittance under the influence of an SR damping is

$$\varepsilon(t) = \varepsilon_0 \exp(-2t/\tau) \longrightarrow \frac{d\varepsilon}{ds} = -\frac{2}{c\tau}\varepsilon$$

$$\tau^{-1} = \frac{1}{3}\frac{cr_e\gamma^3}{R^2}$$

- At equilibrium,

$$\frac{d\varepsilon_x}{ds} + \frac{d\varepsilon}{ds} = 0 \longrightarrow \varepsilon_{x0} \approx \frac{3}{2}\frac{\eta^2}{\beta_x}\frac{\lambda_e\gamma^2}{R}$$

[Note] In the equation above, we ignored the dependence of R on longitudinal coordinate s. In order to obtain more accurate formulas, one needs to use the values $\langle 1/R^2\rangle$ and $\langle 1/R^3\rangle$, which are averaged over the orbit period.

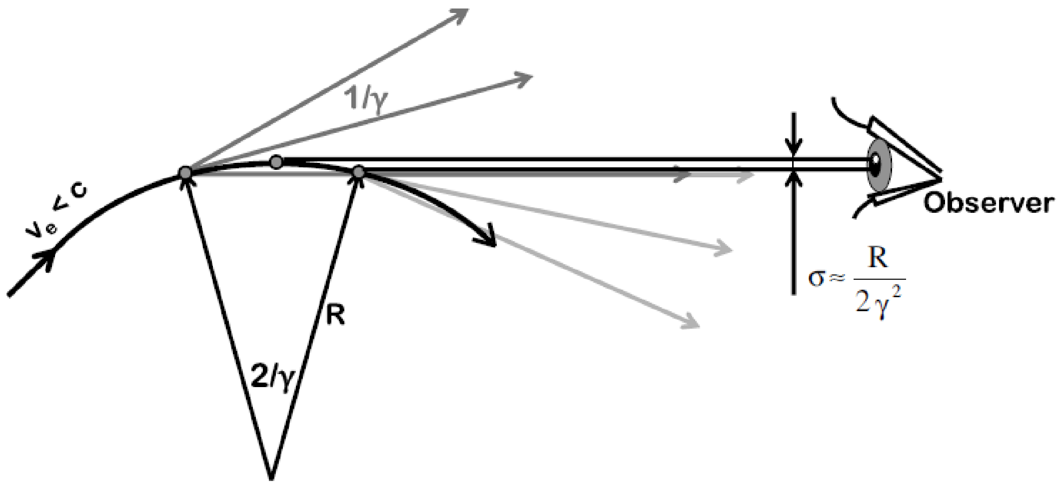
[Note] In the vertical plane, SR's contribution to emittance is only due to $1/\gamma$ angles of emitted photons, but usually the impact on highly relativistic beams is negligibly small.

[Note] The vertical equilibrium emittance is therefore usually defined not by SR directly, but by the coupling coefficient k (usually $\ll 1$) of the $x - y$ planes:

$$\varepsilon_{y0} \approx k\varepsilon_{x0}$$

[Note] In the above, we ignored partition numbers, but they can be taken into account in accurate calculations. The equilibrium energy spread of the beam can also be calculated in a similar manner.

3.3.1 Emittance of single radiated photon



- We first note that the angles of photons coming from the emitting regions are spread over

$$\sigma' \approx \frac{1}{\gamma}$$

- The size of the emitted region is given by the height of the arc segment:

$$\sigma \approx R[1 - \cos(1/\gamma)] \approx \frac{R}{2\gamma^2}$$

- The estimate from the emittance of an SR photon emitted by a single electron:

$$\varepsilon_{ph} = \sigma\sigma' \approx \frac{R}{2\gamma^3}$$

- Let's rewrite the photon emittance equation using the expression for photon wavelength:

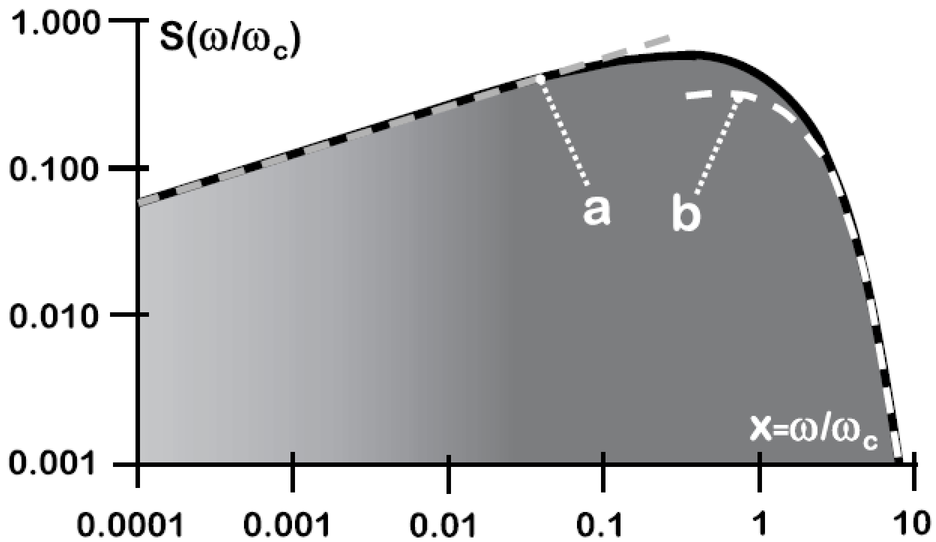
$$\omega_c = \frac{2\pi c}{\lambda_c} \approx \frac{c\gamma^3}{R} \longrightarrow \varepsilon_{ph} \approx \frac{\lambda_c}{4\pi} \longrightarrow \varepsilon_{ph} = \frac{\lambda}{4\pi} \text{ (in general)}$$

3.3.2 SR spectrum

- A large fraction of the photons will have energies close to ω_c. However, there is also a lower energy tail, as well as some fraction of higher energy photons.

- It is also natural to expect that the photons' angular distribution will deviate from the $1/\gamma$ rule, and indeed, the lower energy photons typically have larger angular spread.

$$\epsilon_c = \hbar\omega_c \approx \frac{\gamma^3 \hbar c}{R}$$



3.3.3 Brightness or Brilliance

- Flux is expressed as the number of photons emitted per units of time and per unit of bandwidth:

$$\mathrm{Flux} = \frac{\mathrm{Photons}}{\mathrm{s\ BW}}$$

- The bandwidth (interval of interest in the spectrum of photon frequencies) is denoted here BW and is expressed, typically, in %.

- Then brilliance (or brightness) is then defined as flux per unit of emitting area and product of angles:

$$\mathrm{Brilliance} = \frac{\mathrm{Flux}}{A_s \Delta\Phi \Delta\Psi} = \left[\frac{\mathrm{Photons}}{\mathrm{s\ mm^2\ mrad^2\ BW}}\right]$$

- In a typical case of Gaussian distributions, the definition of brilliance is based on total effective sizes and divergences

$$\mathrm{Brilliance} = \frac{\mathrm{Flux}}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

- Here, the total effective sizes include contributions from electrons as well as photons (similarily for y-plane):

$$\Sigma_x = \sqrt{\sigma_{x,e}^2 + \sigma_{ph}^2}, \quad \sigma_{x,e} = \sqrt{\varepsilon_x \beta_x + (D_x \sigma_\varepsilon)^2}$$

$$\Sigma_{x'} = \sqrt{\sigma_{x',e}^2 + \sigma_{ph}'^2}, \quad \sigma_{x',e} = \sqrt{\varepsilon_x \gamma_x + (D'_x \sigma_\varepsilon)^2}$$

3.3.4 Ultimate brightness

- Brilliance is defined by the overall effective emittance, which convolves electron and photon distributions:

$$\varepsilon_{eff} = \sqrt{\sigma_e^2 + \sigma_{ph}^2}\sqrt{\sigma_e'^2 + \sigma_{ph}'^2}$$

- Since the lowest photon emittance depends on the photon wavelength, the smallest overall emittance will be obtained when:

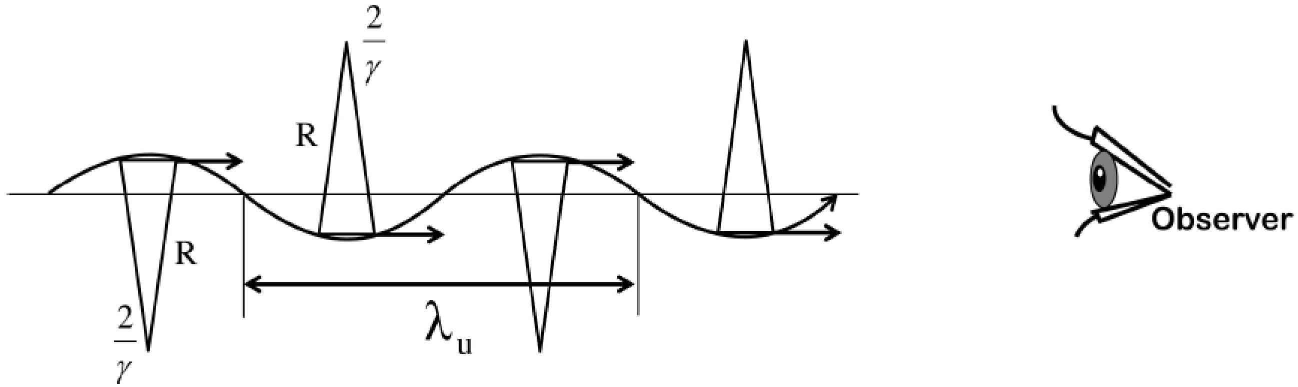
$$\varepsilon_e = \sigma_e \sigma_e' \leq \varepsilon_{ph} = \frac{\lambda}{4\pi}$$

- This corresponds to a diffraction-limited source.

[Example]

Let's take an example of 12.4 keV of photon energy which corresponds to $\varepsilon_{ph} \approx 8$ pm. For typical third-generation SR light source, the horizontal emittance ε_x is usually 1 and 5 nm and vertical emittance ε_y between 1 and 40 pm. Thus, such rings are close, in terms of its emittance, to the ultimate performance in the vertical plane. However, they are many orders of magnitude away from the diffraction-limited emittance in the horizontal plane.

3.3.5 Wiggler and Undulator radiation



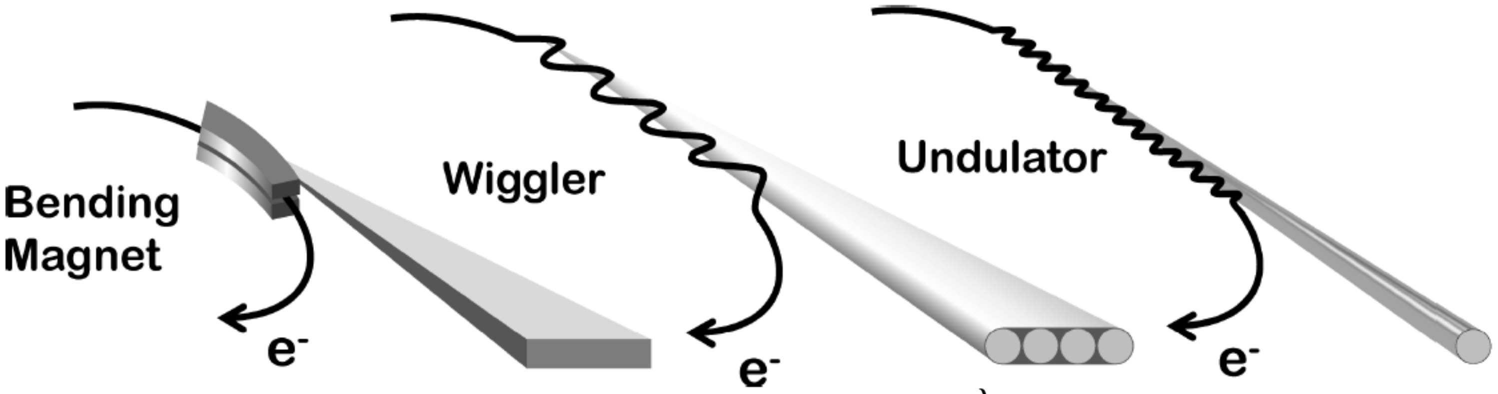
- An external observer will see photons emitted by the particle during its travel along the arc $2R/\gamma$.
- Let's define the parameter K as the ratio of the wiggling period to the length of this arc:

$$K \sim \frac{\lambda_u}{R/\gamma} \ (\text{more precisely} \equiv \frac{\lambda_u \gamma}{2\pi R})$$

  – If $2R/\gamma \ll \lambda_u$, then the radiation emitted at each wiggler is independent. Wiggler regime ($K \gg 1$)

  – If $2R/\gamma \gg \lambda_u$, then we are in a regime where the entire wiggling trajectory contributes to radiation (therefore interference leads to coherence radiation). Undulator regime ($K \ll 1$)

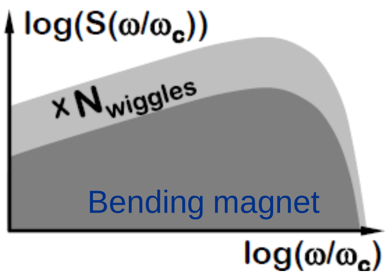
3.3.5 Wiggler and Undulator radiation



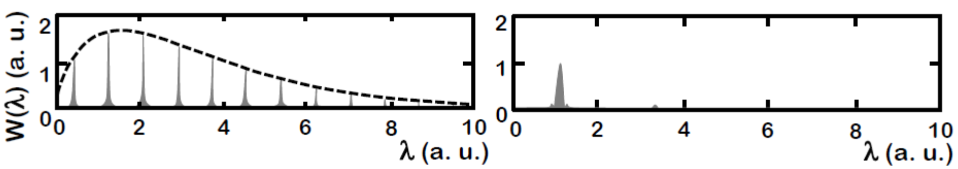
$$K \sim \frac{\lambda_u}{R/\gamma} \gg 1$$

$$K \sim \frac{\lambda_u}{R/\gamma} \ll 1$$

Bending magnet VS Wiggler



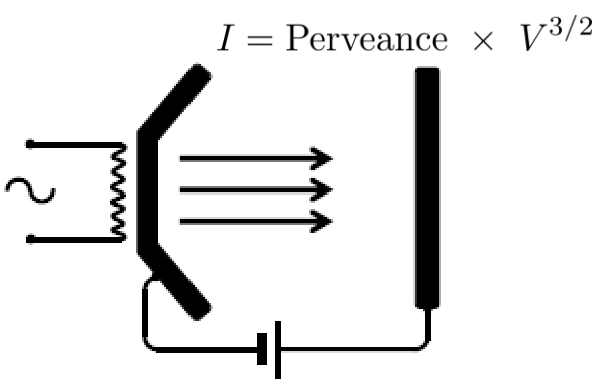
Wiggler VS Undulator



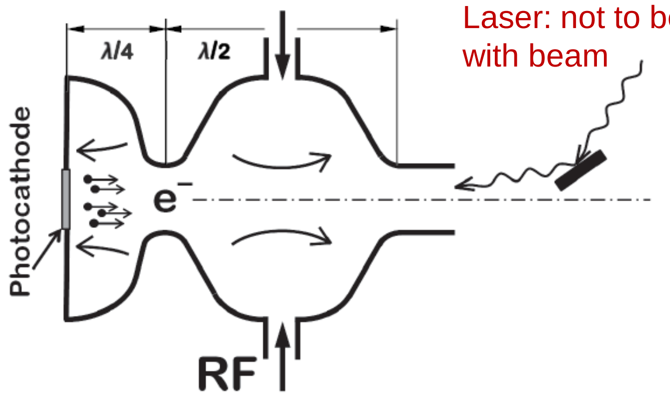
Bunch Compression

10.1.1 Bunch Compression

- Short electron bunches are typically produced by RF photo guns where the cathode is illuminated by a short laser pulse.



DC Thermal cathode e-gun



One-and-a-half cell RF photocathode e-gun

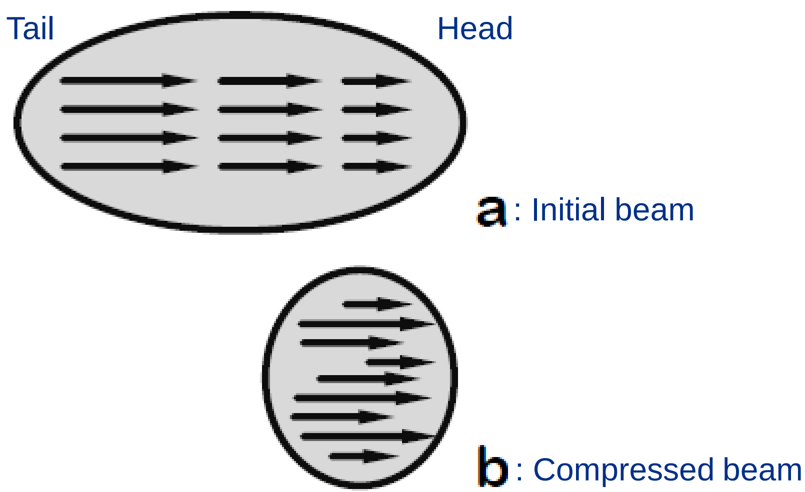
- While a pure metal photocathode is robust, its quantum efficiency (the number of electrons per number of incident photons) is usually quite low, of the order of 10^{-4}. Photocathodes with alkali metals such as cesium can have a quantum efficiency of 10% or higher.

- However, such photo injectors produce still relatively long pulses (a few ps), while for many applications, such bunches would be too long and fs bunches would be required instead.

Velocity Bunching

- By properly adjusting the timing when the laser illuminates the cathode with the RF field in the gun, one can create correlation between the energy of electrons and the longitudinal position within the bunch.

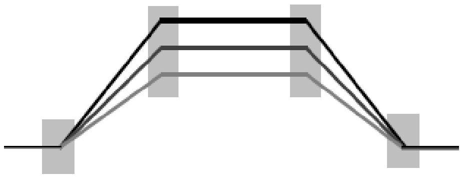
- The difference of energy for beams that are still weakly relativistic can create a difference in longitudinal velocities.



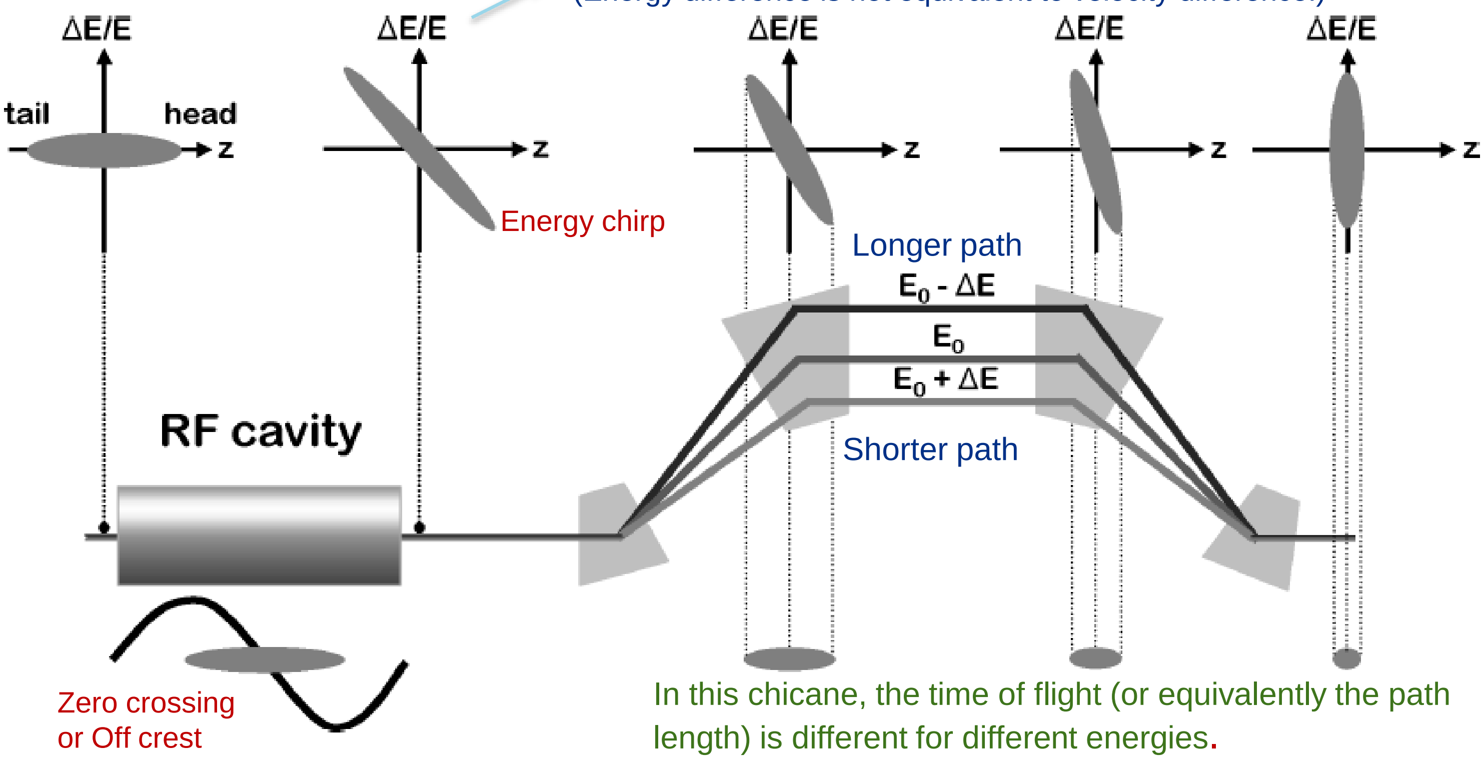
- Because velocity bunching is based on the velocity's dependence on energy, it can work only for weakly relativistic beams.
- Weakly relativistic electron beams can, in particular, suffer strongly from space charge effects, limiting the degree of compression

Chicane (자동차 속도를 줄이게 하기 위한 이중 급커브길)

- Achieving ultra-short electron bunches is usually done at higher energies, when the beam is relativistic and space charge effects are less severe.



Particle velocities are nearly constant at c
(Energy difference is not equivalent to velocity difference!)



In this chicane, the time of flight (or equivalently the path length) is different for different energies.

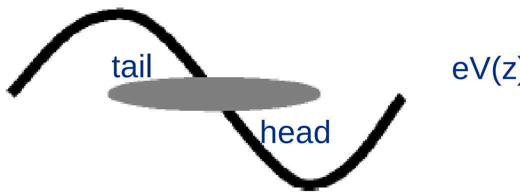
Analytical Description of Bunch Compressor

- Let's use the linear transfer matrices to evaluate the evolution of the longitudinal position and relative energy offset (z, δ).

- Passing through the RF cavity (assuming the cavity is thin), the longitudinal coordinate does not change, while the energy change depends on the initial position as follows:

$$\begin{aligned} z_1 &= z_0 \\ \delta_1 &= \delta_0 + \frac{eV_{RF}}{E_0}\sin(\pi + k_{RF} z_0) \approx \delta_0 - \frac{eV_{RF}}{E_0} k_{RF} z_0 \end{aligned}$$

Reference beam energy



Assume zero crossing for simplicity

$$\begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ R_{65} & 1 \end{pmatrix} \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}, \quad R_{65} = -\frac{eV_{RF}}{E_0} k_{RF}$$

- The next step is to take into account the bunch compressor itself.

∝ Momentum compaction

$$\begin{aligned} z_2 &= z_1 + R_{56}\delta_1 + \overbrace{T_{566}\delta_1^2 + U_{5666}\delta_1^3 + \cdots}^{\text{Nonlinear terms}} \\ \delta_2 &= \delta_1 \end{aligned}$$

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \begin{pmatrix} 1 & R_{56} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix}$$

Analytical Description of Bunch Compressor

- The full transformation is given by multiplying the matrices of each element:

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \mathbf{M} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} 1 + R_{65}R_{56} & R_{56} \\ R_{65} & 1 \end{pmatrix}$$

- Then the final bunch length is

$$\sigma_{z2} = \sqrt{(1 + R_{65}R_{56})^2 \sigma_{z0}^2 + R_{56}^2 \sigma_{\delta 0}^2}$$

- In order to achieve maximal compression, we need should adjust R_{65} (Energy chirp induced by the RF cavity) or RF voltage in such a way that

$$R_{65}R_{56} \approx -1$$

[Note]

- Taking second and higher-order terms into account will give increased final bunch lengths.
- Linear transformation preserves the longitudinal beam emittance:

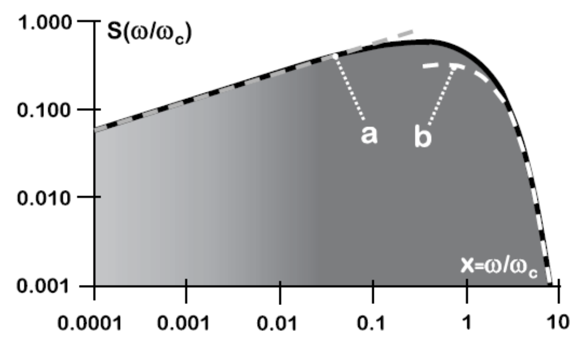
$$\varepsilon = \sqrt{\sigma_z^2 \sigma_\delta^2 - \sigma_{z\delta}^2}$$

- The achievable compression is limited due to the effects associated with beam's high peak current: longitudinal space-charge (LSC), wakefields and coherent synchrotron radiation (CSR).

10.1.2 Coherent Synchrotron Radiation (CSR)

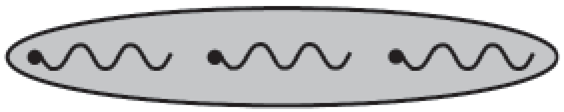
- Characteristic frequency of Synchrotron Radiation (SR):

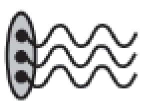
$$\omega \lesssim \omega_c = \frac{3}{2}\frac{c\gamma^3}{R}$$



- Wavelength of the radiation $\sim c/\omega$ and bunch length σ:

$$\frac{c}{\omega} < \sigma$$



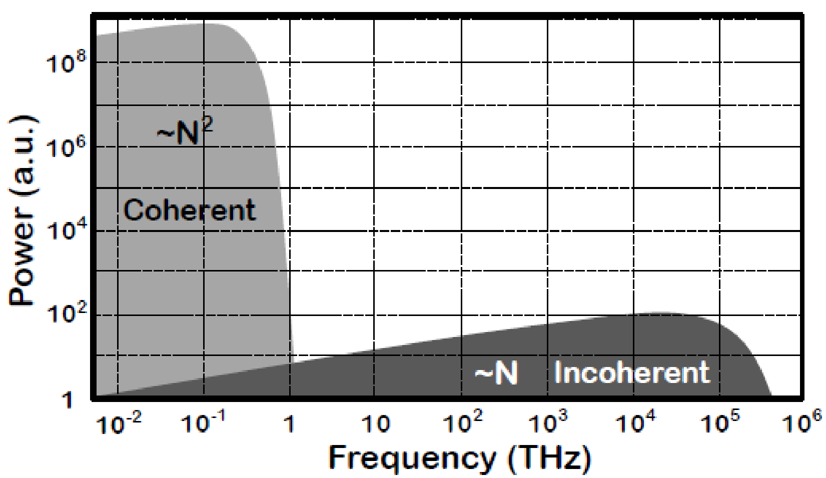


$$\sigma < \frac{c}{\omega}$$

Incoherent radiation

Coherent radiation

- Radiation power:



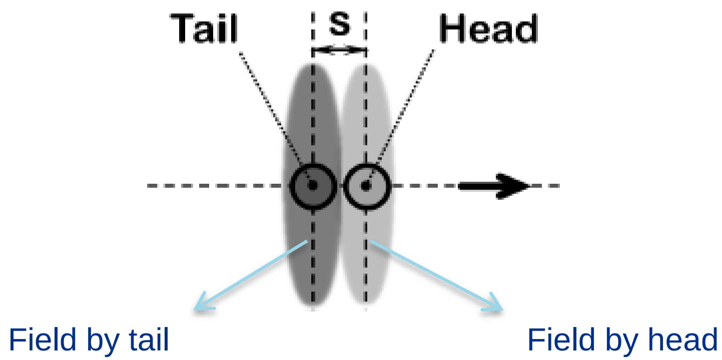
$$\text{Radiation power} \propto \left(\sum_{j=1}^{N} E_j e^{i\phi_j}\right)^2$$

$$= \sum_{j=1}^{N} E_j^2 + \sum_{j=1}^{N}\sum_{k\neq j}^{N} E_j E_k^* e^{i(\phi_j-\phi_k)}$$

If $\phi_j \approx \phi_k$, then the 2nd term >> 1st term as there are N^2 of them and results in coherent radiation

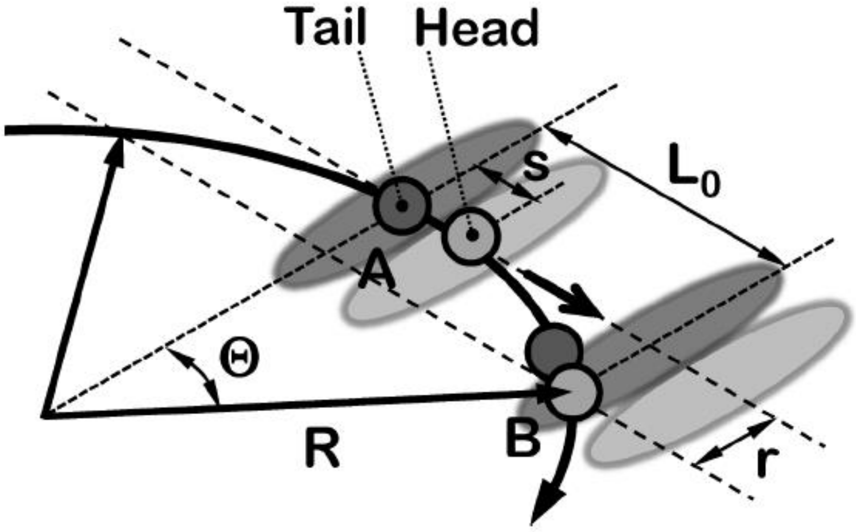
10.1.3 CSR Effects on the Longitudinal Motion

- Two particle model: the beam is represented just by two particles (at the head and in the tail of the bunch).



- The CSR effects are essentially caused by the possibility for the tail field to overtake the head particle while the beam is moving on a curved trajectory.

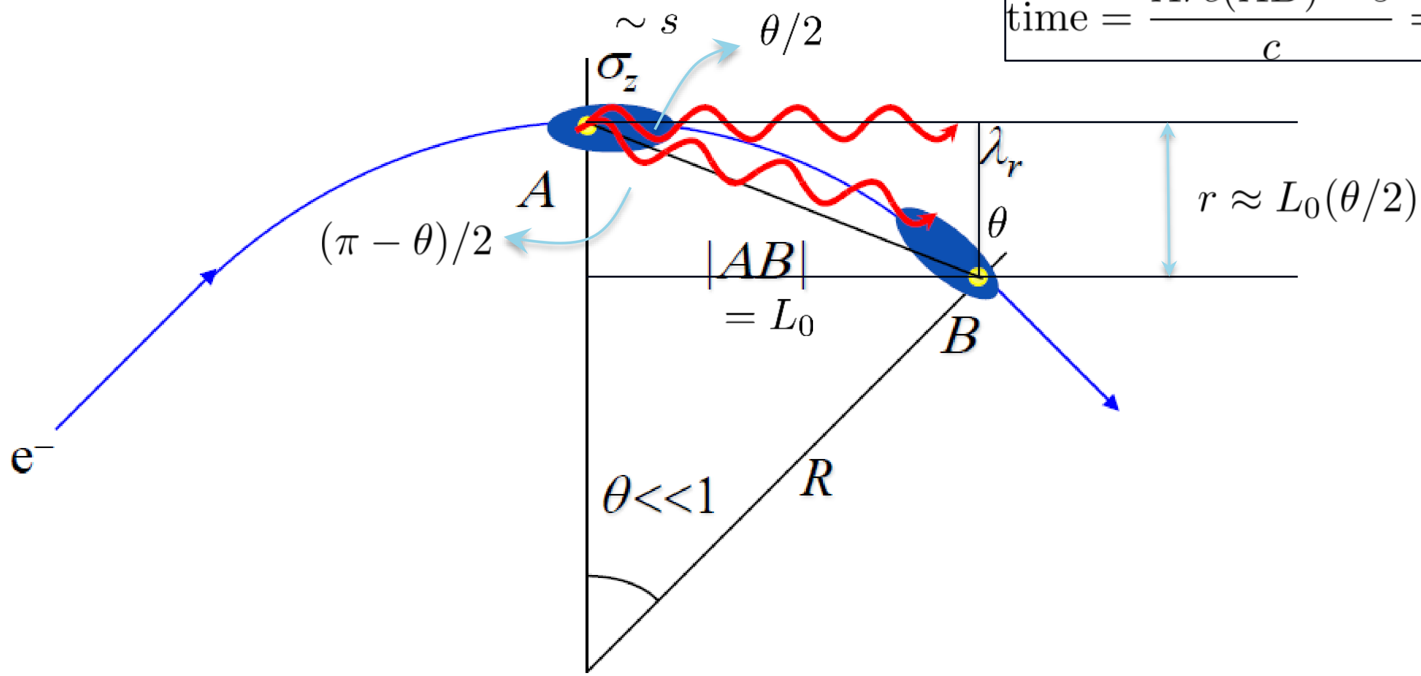
We assume both particles and fields are moving with c



10.1.3 CSR Effects on the Longitudinal Motion

- The condition for the catch-up (overtake) to happen in point B:

$$\text{time} = \frac{Arc(AB) - s}{c} = \frac{|AB|}{c}$$



$$Arc(AB) - s = |AB| \qquad R\theta - s = 2R\sin(\theta/2) \longrightarrow s \approx R\theta^3/24$$

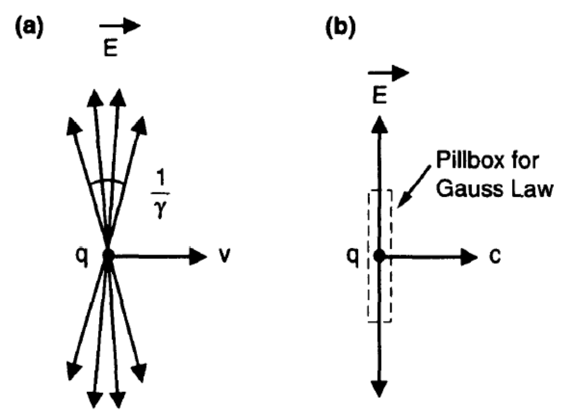
- Overtaking distance L_0 can be estimated as

$$L_0 = |AB| = R\theta \approx 2(3sR^2)^{1/3}$$

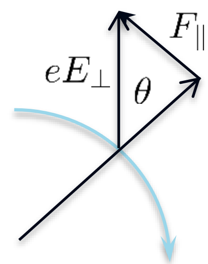
10.1.3 CSR Effects on the Longitudinal Motion

- Values of the transvers field (MKS unit):

$$E_\perp \approx \frac{\text{Line charge density}}{2\pi\epsilon_0 r} \approx \frac{eN\lambda}{2\pi\epsilon_0 r}$$



- Longitudinal force acting on the head:



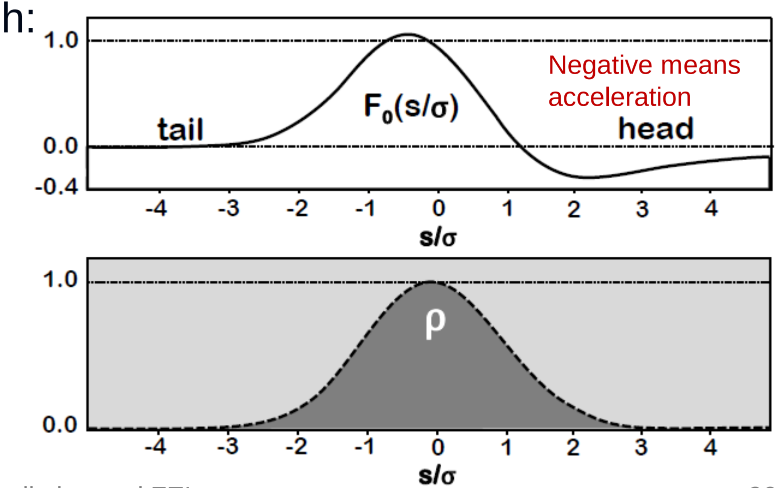
$$F_\parallel = eE_\perp \sin\theta \approx eE_\perp\theta = \frac{Ne^2\lambda\theta}{2\pi\epsilon_0 r} = \frac{Ne^2\lambda}{2\pi\epsilon_0(3sR^2)^{1/3}}$$

$$r \approx L_0(\theta/2) \approx 2(9s^2R)^{1/3}$$

- With more accurate derivation for a Gaussian bunch:

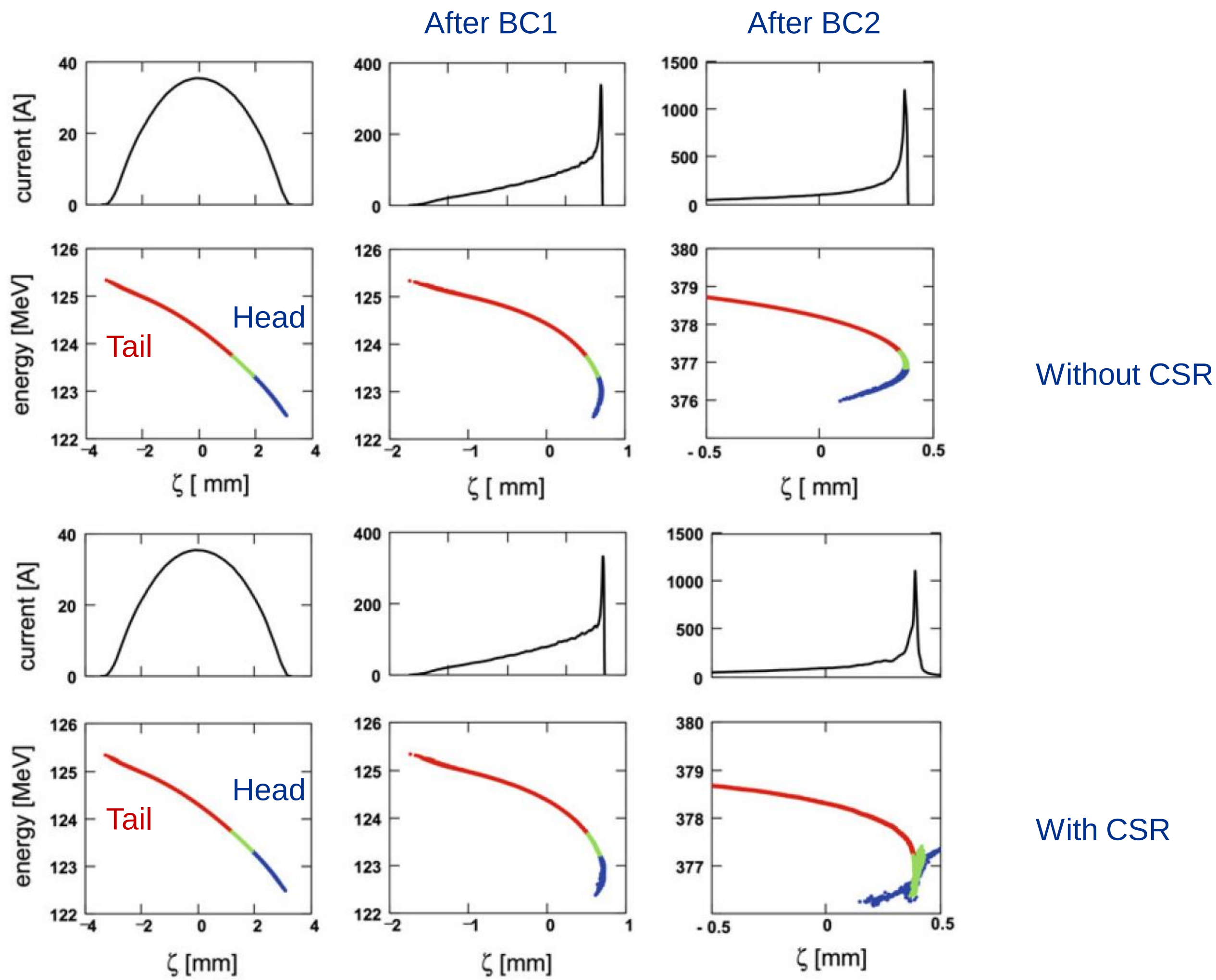
$$F_\parallel = const. \times F_0(\text{Shape function})$$

  - The shape function changes its sign.
  - The head of the bunch slightly accelerates.
  - The major part of the bunch decelerates.



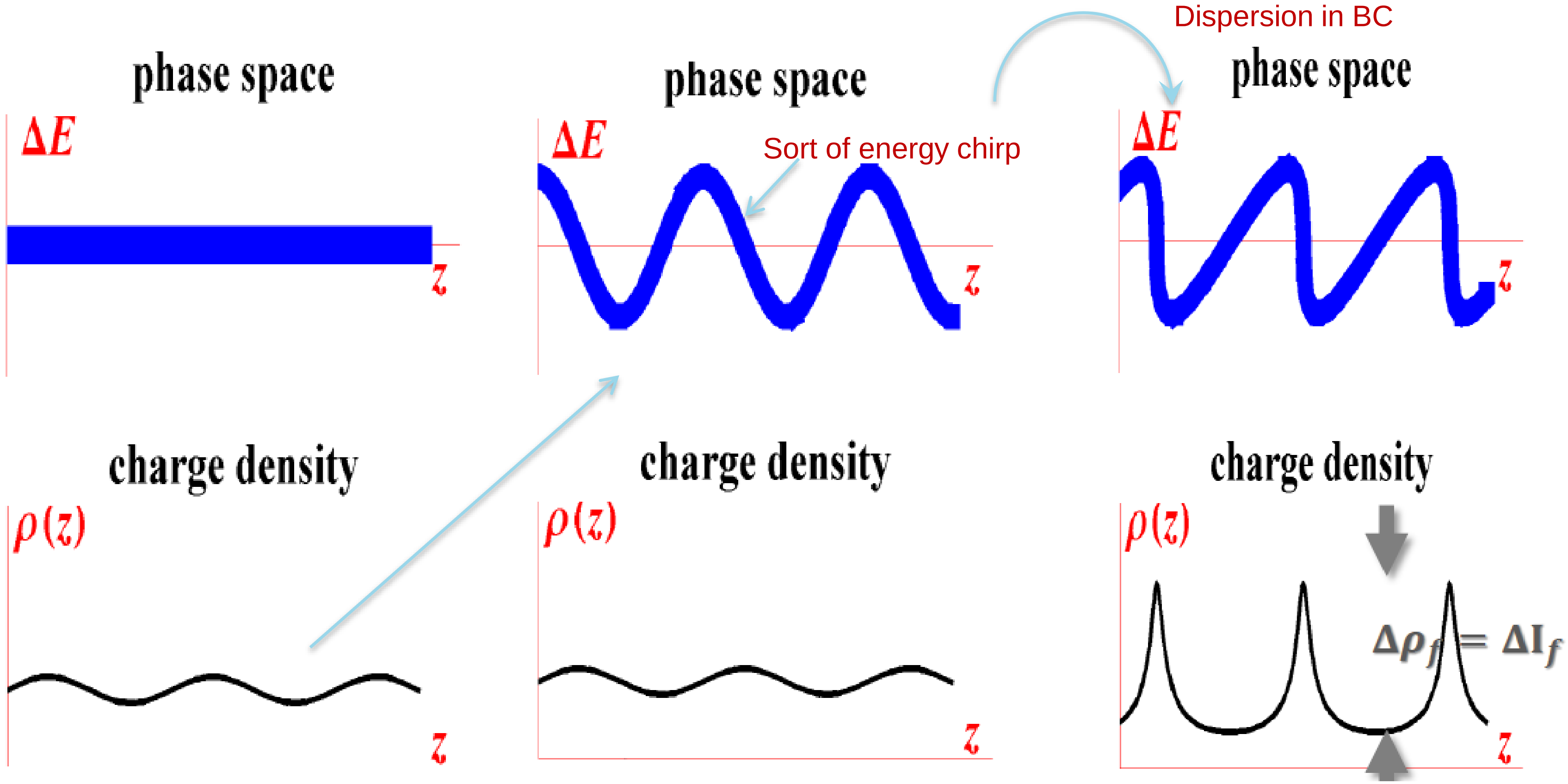
10.1.3 CSR Effects on the Longitudinal Motion

- The effects of CSR are particularly important in bunch compressors and can prevent achieving ultra-short bunches.
- CSR can cause bunch instability and microbunching, and can therefore deteriorate the longitudinal phase space of the beam.



Micro-bunching Instability

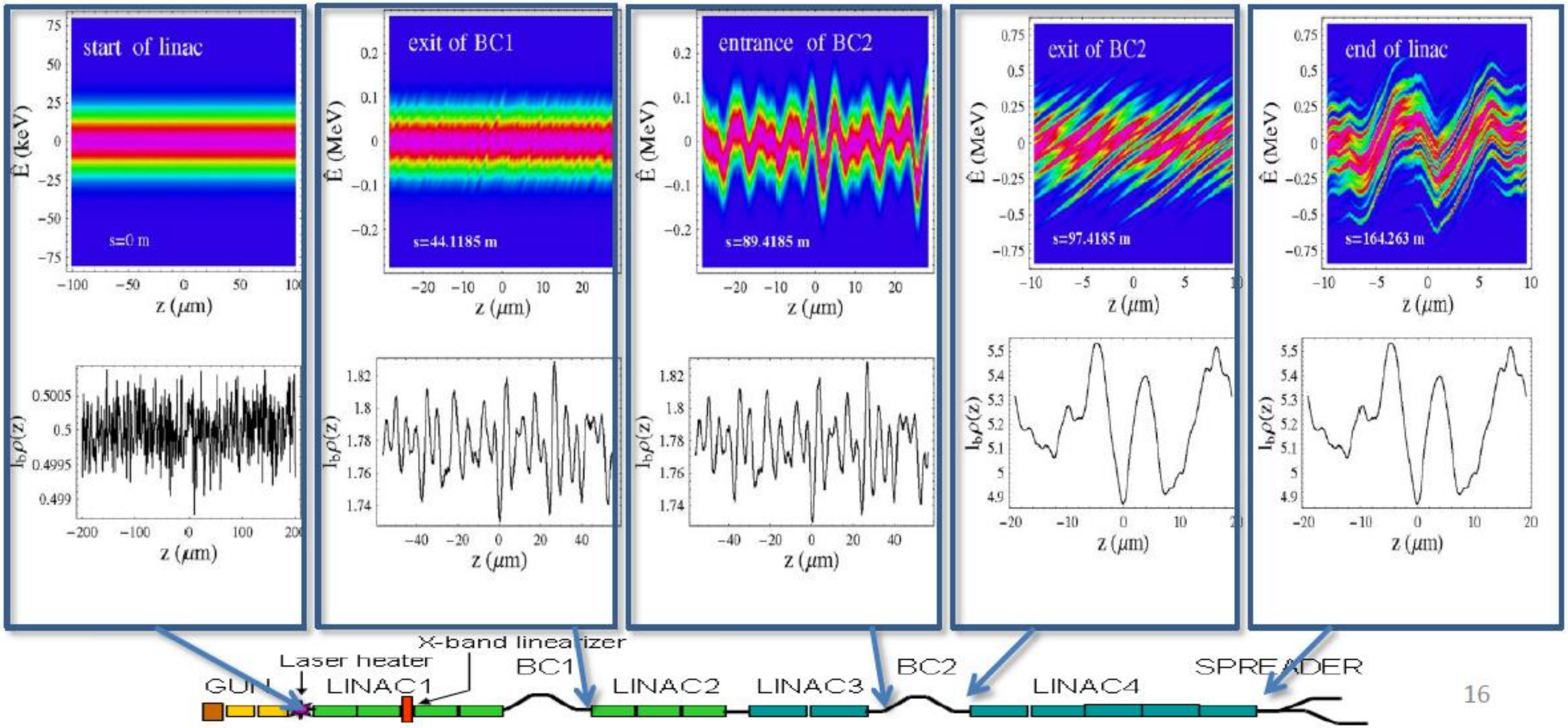
- Collective effects (Longitudinal Space Charge and CSR) turn ripples of charge-density into energy modulation.
- Dispersion turns energy modulation into larger charge-density ripples.
- Main adverse effect of micro-bunching instability is growth in energy spread.



Due to either LSC or CSR
(LSC is more relevant as dipoles are short compared to rest of machine)

Micro-bunching Instability

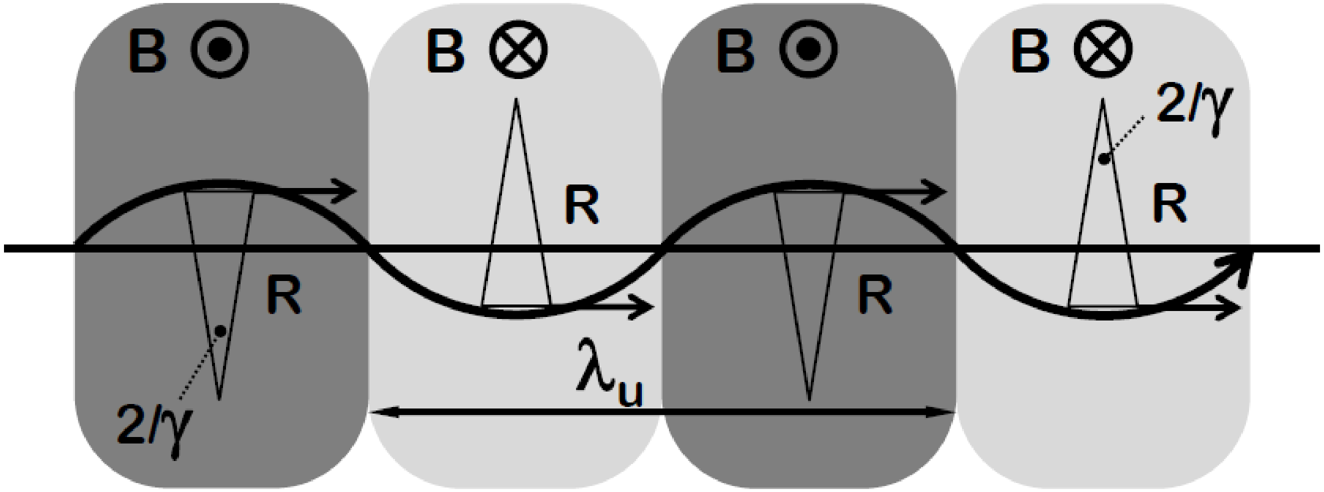
- Multiple-stage bunch compression enhances instability



Wiggeler VS Undulator

Wiggler VS Undulator

- Bending magnets arranged in a sequence with NSNS polarity with period λ_u:



- Wiggler regime: If the length of the emitting region is much less than the length of an individual.

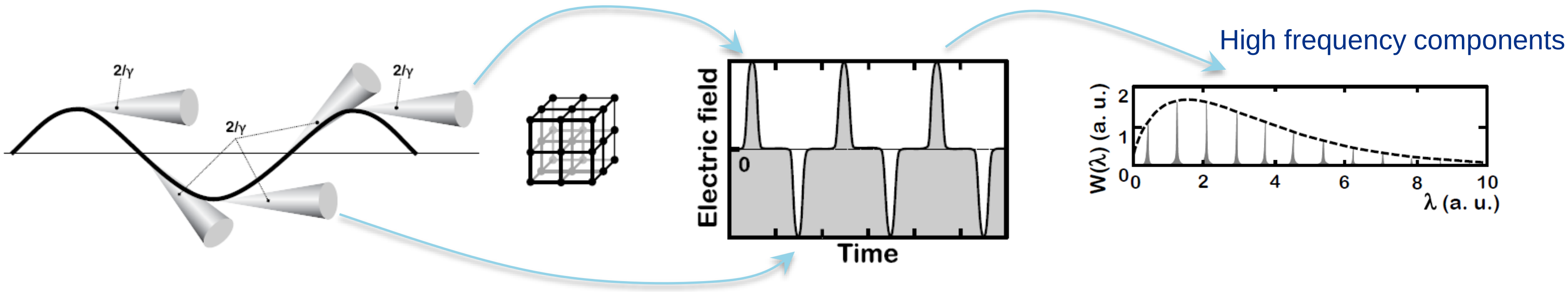
$$R\frac{2}{\gamma} \ll \frac{\lambda_u}{2} \rightarrow \text{the radiation emitted in each bend is independent}$$

- Undulator regime: The entire wiggling trajectory contributes to radiation, therefore interference leads coherence of radiation.

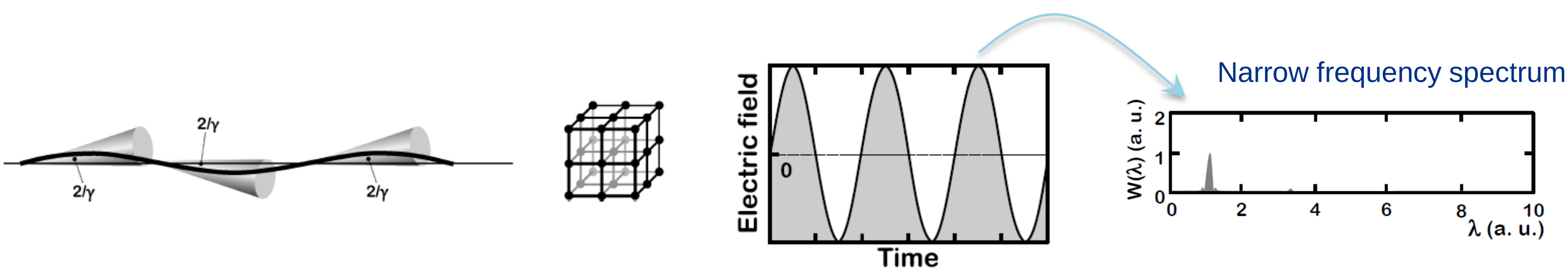
$$R\frac{2}{\gamma} \gg \frac{\lambda_u}{2} \rightarrow \text{the radiation is coherently build up}$$

Wiggler VS Undulator

- Wiggler: $R\dfrac{2}{\gamma} \ll \dfrac{\lambda_u}{2}$, or $K(\text{undulator parameter}) = \dfrac{\gamma\lambda_u}{2\pi R} \gg 1$



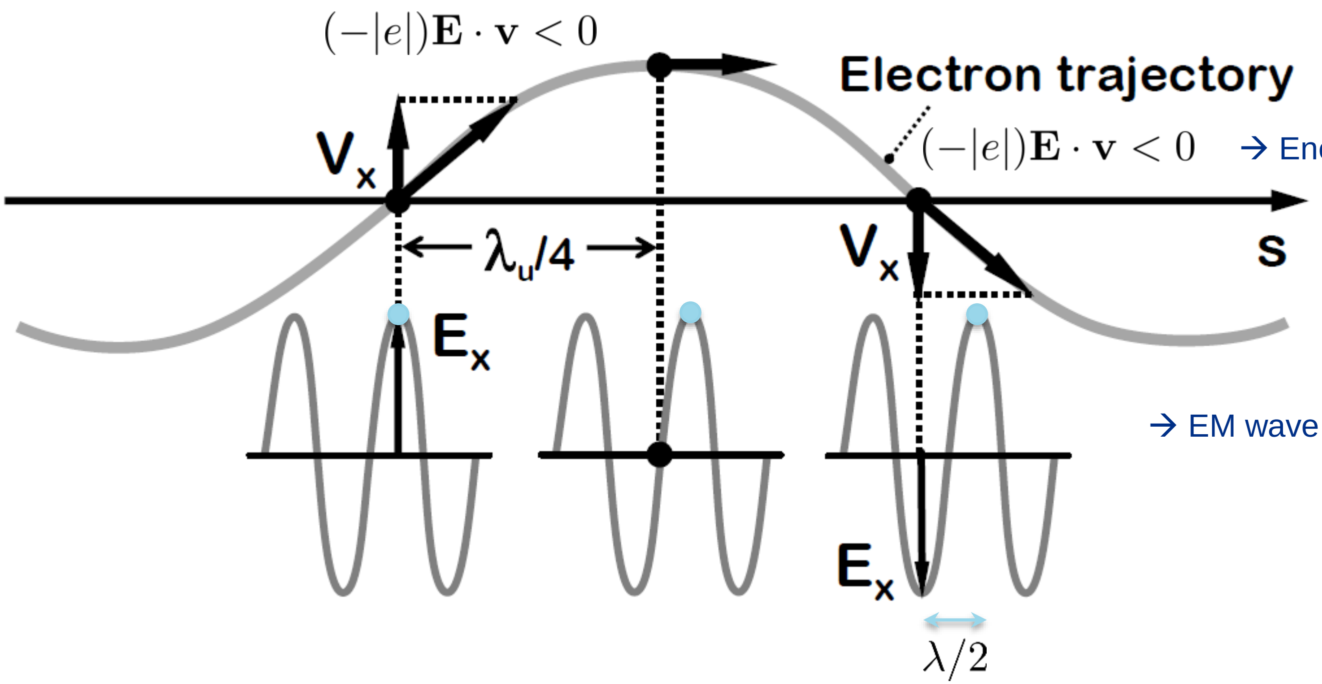
- Undulator: $R\dfrac{2}{\gamma} \gg \dfrac{\lambda_u}{2}$, or $K(\text{undulator parameter}) = \dfrac{\gamma\lambda_u}{2\pi R} \ll 1$



Particle and Field Energy Exchange

- There is longitudinal retardation of the particle in the undulator:

$$\langle v_z \rangle = v_{z0} + \text{sin-like trajectory contribution} = c\left(1 - \frac{1}{2\gamma^2}\right) - c\frac{K^2}{4\gamma^2} = c\left(1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right)$$



→ Energy transfer to wave

→ EM wave is moving faster

- The necessary condition for the resonant energy transfer is that the EM wave slips forward with respect to an electron by λ_u per half period of electron trajectory

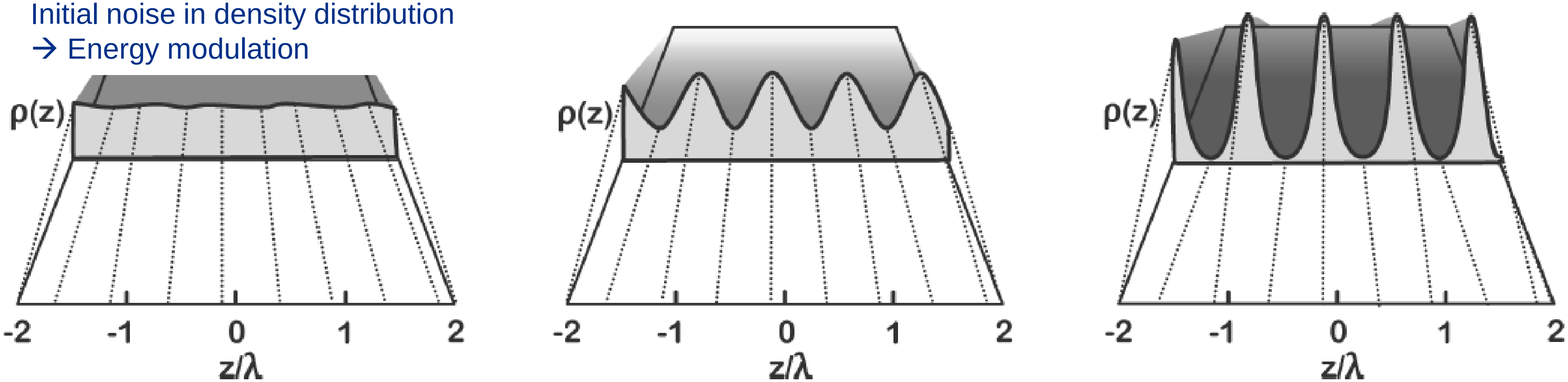
$$\frac{\lambda_u/2}{\langle v_z \rangle} = \frac{\lambda_u/2 + \lambda/2 + n\lambda}{c} \rightarrow \lambda = \frac{\lambda_u}{2n+1}\left(1 - \langle v_z \rangle / c\right) = \frac{\lambda_u}{m2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

harmonics

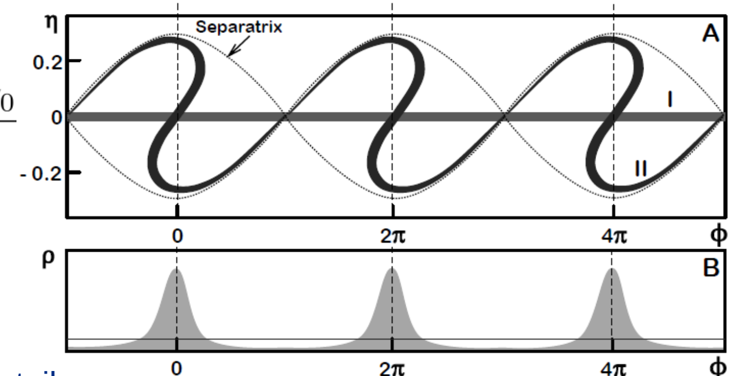
Micro-bunching in the Undulator

- The interaction of particles with the resonant EM wave can create energy modulation in the particle beam.
- As particles move along the curved sine-like trajectory, the different path lengths can in turn create density modulations along the beam.

Initial noise in density distribution
→ Energy modulation



$$\eta = \frac{\gamma - \gamma_0}{\gamma_0}$$

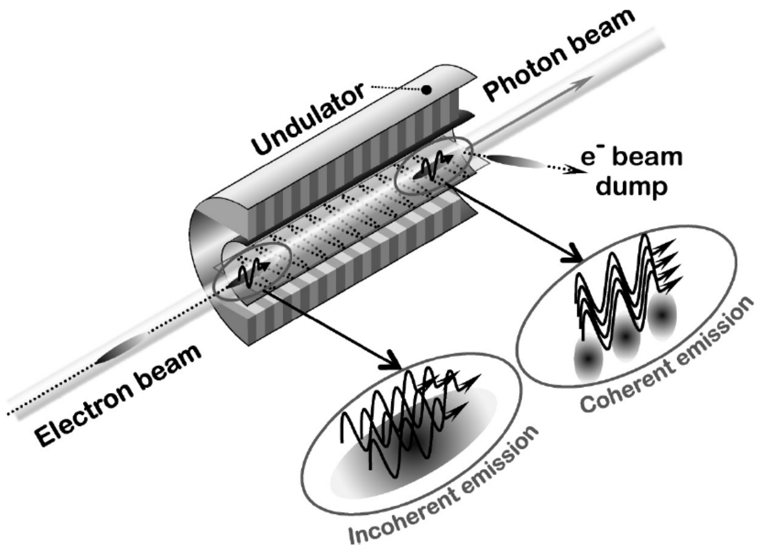


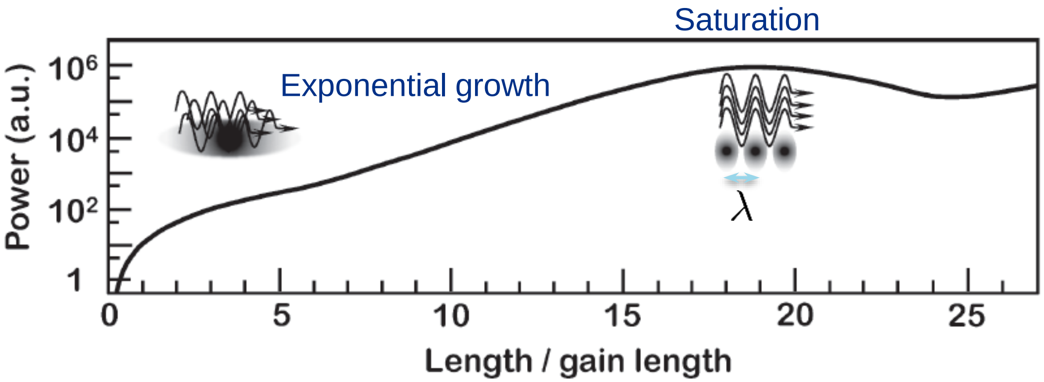
tail
head

- An initial EM wave of resonant wavelength can be external (Seeding) or can emerge from the noise that is always present in the beam (SASE: Self-Amplified Spontaneous Emission).

Single-pass FEL

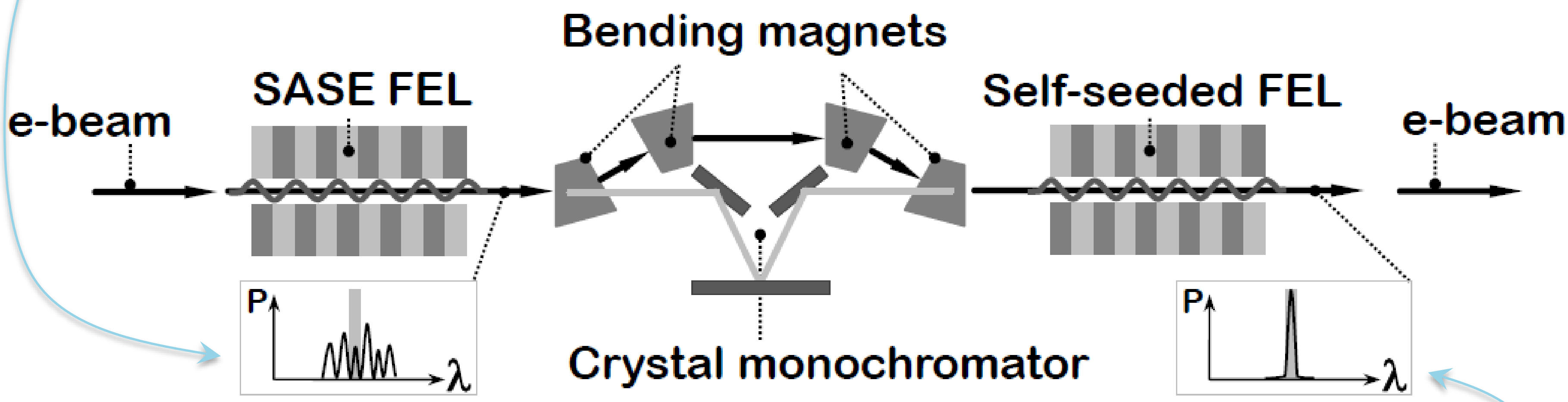
- In a single-pass FEL, the radiation has to grow within a single passage of the beam through the undulator (either seeded or SASE type).
- The need to use single-pass systems is actually a necessity dictated by an absence of good mirrors in the X-ray spectral region.
- The single-pass system has to be a high-gain system, which puts extreme constraints on the quality of the electron beam, as well as on the accuracy of the undulator.
- In the high-gain system, the gain is so large that the EM wave amplitude changes within a single pass in the undulator.





Self-seeded FEL

- In the SASE regime, the FEL lasing starts from noise → Each individual beam slice can generate radiation at slightly different wavelengths near the resonant wavelength, and, moreover, the amplitude of the radiation coming from each slice can be slightly different.



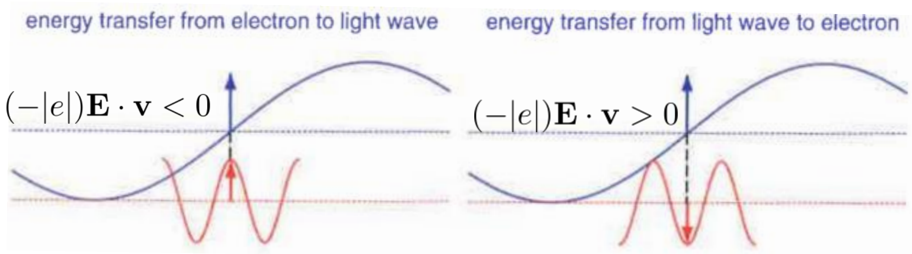
→ Filtering out other than the desired wavelength

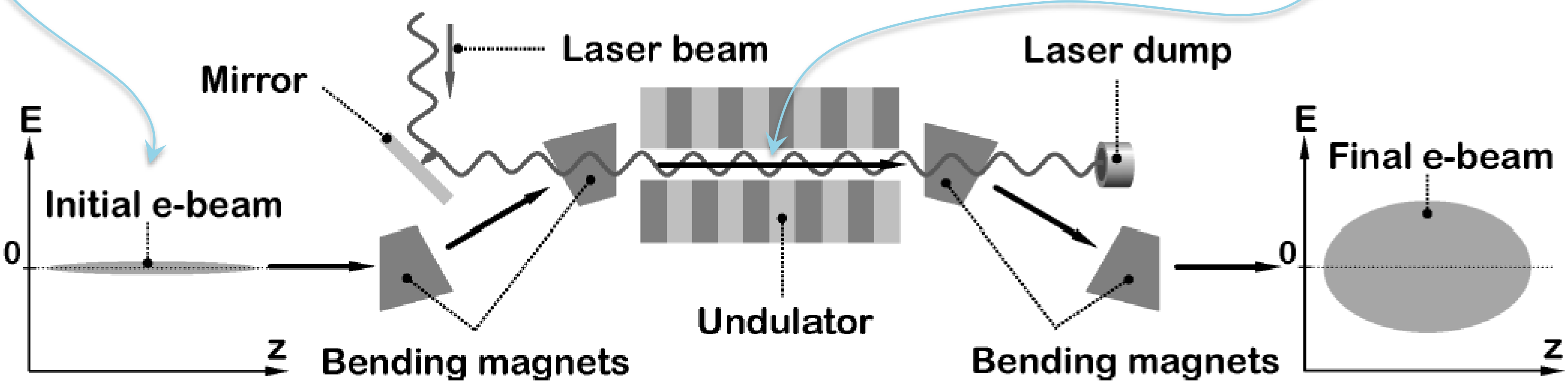
- In the self-seeded FEL, before entering the second part, radiation generated in the first part is passed through a crystal monochromator.

- The second part of the FEL is thus seeded with narrow spectrum radiation, which continues to be amplified along the way in the undulator; the resulting output spectrum of FEL then contains a narrow peak.

Beam laser heating

- The generation of femtosecond-short X-ray pulses requires fs-short electron bunches. However, the creation of short bunches in bunch compressors can be complicated due to CSR effects that can cause instability and microbunching.
- Instabilities can often be suppressed if the beam has sufficient spread of its relevant phase space coordinates (the energy spread in the case of CSR instability).
- The beam energy spread coming from a photocathode gun is extremely small, and insufficient for suppressing CSR instability.
- A method has been developed at SLAC to introduce additional uncorrelated energy spreads into the beam: Laser heater.

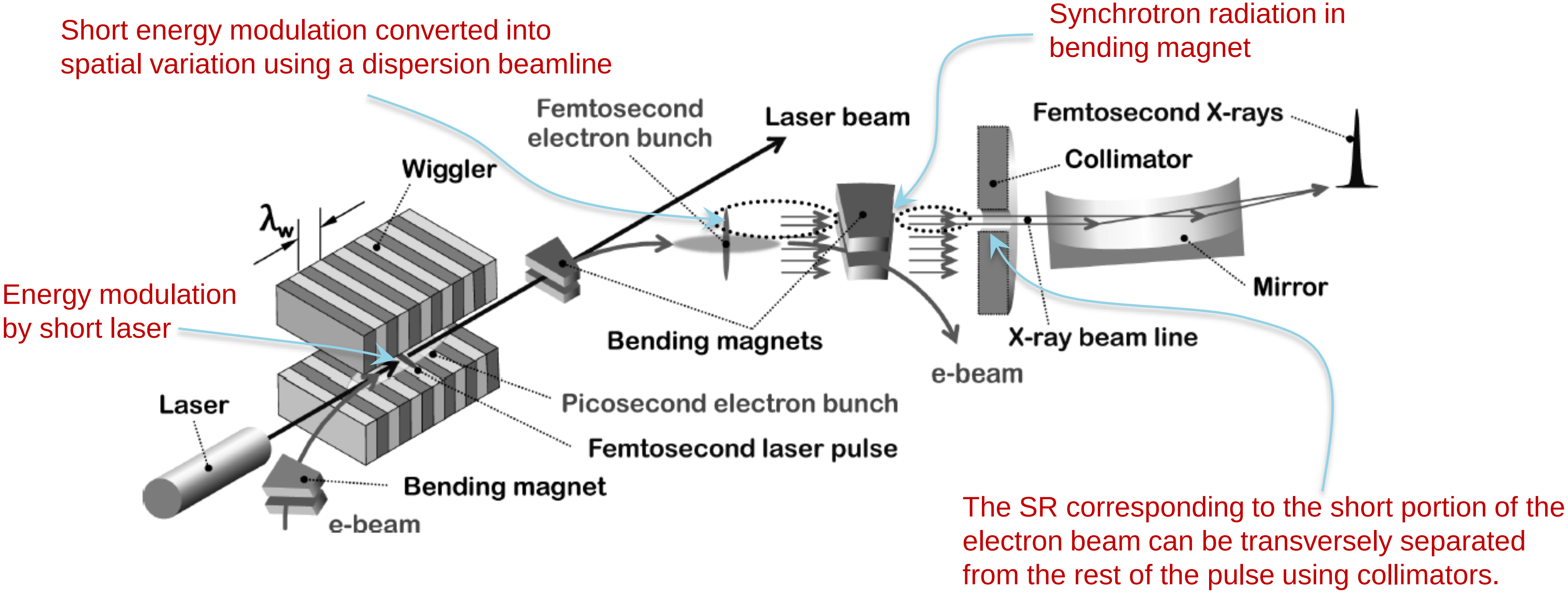




Beam laser slicing

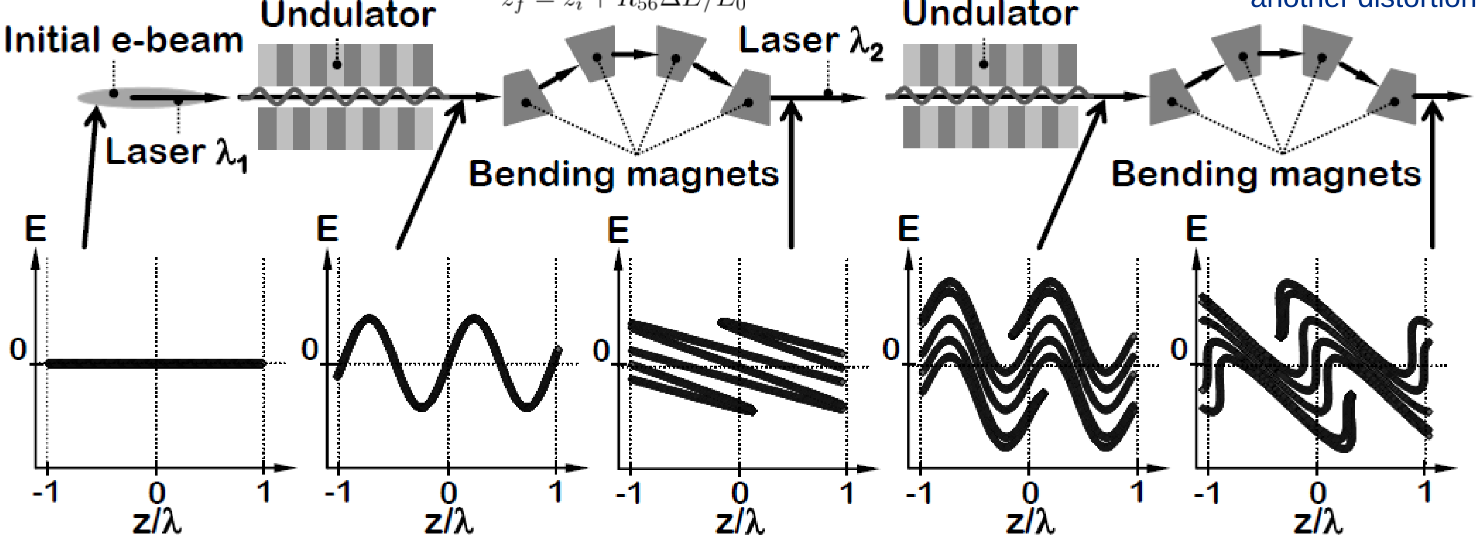
- A technique that selects a femtosecond short portion of radiation from a much longer initial electron pulse. → Suitable for generating femtosecond synchrotron radiation pulses even for ring-based 3rd generation SR sources.
- A very short laser pulse overlaps with the center of a longer bunch in the undulator or wiggler. The laser wavelength matches the undulator resonance condition and produces modulation of energy in the short beam slice.

$$\lambda_L = \frac{\lambda_W}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

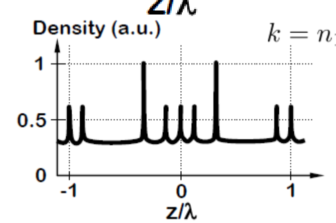


Beam laser harmonic generation

- Harmonic generation is the technique that can help to produce X-ray photons of much higher energies.
- An FEL would normally generate primarily the first harmonic. In order to primarily generate a higher-order harmonic, external seeding is required. There are, however, no conventional lasers of appropriately short wavelengths that can be used for such seeding.
- A technique invented by G. Stupakov is currently solving the problem: echo-enabled harmonic generation (EEHG).



Shorter bunching
→ Shorter radiation wavelength
→ Higher photon energy



Laser1 Laser2

$$k = n_1 k_1 + n_2 k_2 = \frac{2\pi}{\lambda}$$