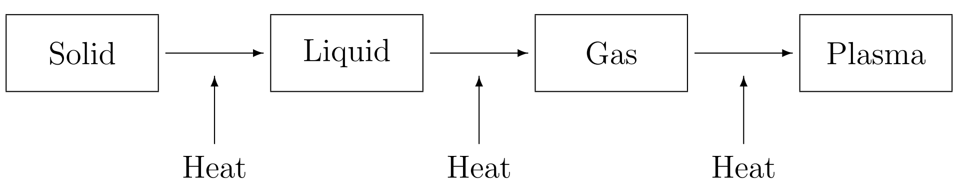
Chapter 1

INTRODUCTION

1.1 Plasma state

Plasma

- A plasma is a gas of charged particles.
  But not every gas of charged particles qualifies as a plasma.
- The 4th state of matter (coined by W. Crookes in 1879):



- Compare:
  - Ancient: Universe $\cdots$ earth, water, air, fire
  - Modern: Universe $\cdots$ solid, liquid, gas, plasma
- "Plasma" – introduced by Tonks and Langmuir in 1928.
- "Plasma" – something formed or molded in Greek.

Naturally occuring plasmas:

- Stellar interior
- Van Allen radiation belts
- Solar winds
- Flash of lightening bolts

- Rocket exhaust

Ionization rate (from the Saha equation in statistical mechanics):

$$\frac{n_i}{n_n} \simeq 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-\frac{U_i}{KT}} \tag{1.1}$$

For $T = 300$ K, $n_n \sim 3 \times 10^{25}$ m^{-3}, and $U_i = 14.5$ eV (nitrogen),

$$\frac{n_i}{n_n} \sim 10^{-122}\,.$$

Note that $K = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant.

1.2 Distribution function

- The most detailed description of a plasma gives the location and velocity of each plasma particle as a function of time.
- It is impossible to obtain such a description of a real plasma.
  Rather than require an exact knowledge of a system with many particles, the behavior of such a particle system can be studied statistically.
- It is customary to use the distribution function to describe a plasma. The distribution function is the number of particles per unit volume in phase space.
  Namely, $f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v}$ represents the expected number of particles at time t in $(\mathbf{r}, \mathbf{v})$ space with coordinates $\mathbf{r}$ and $\mathbf{r} + d\mathbf{r}$ and velocity $\mathbf{v}$ and $\mathbf{v} + d\mathbf{v}$.
- A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution.

1.3 Temperature

The one-dimensional Maxwellian distribution is given by

$$f(v) = A \exp\left[-\frac{mv^2}{2KT}\right] \tag{1.2}$$

where $f dv$ is the number of particles per unit volume with velocity between v and $v + dv$. The density is given by

$$n = \int_{-\infty}^{\infty} f(v) dv \tag{1.3}$$

so that the constant A is found to be

$$A = n \left(\frac{m}{2\pi KT}\right)^{1/2} \tag{1.4}$$

Here, we have used

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}. \tag{1.5}$$

The average kinetic energy is given as

$$E_{av} = \frac{1}{n}\int_{-\infty}^{\infty}\frac{1}{2}mv^2 f(v)dv\,. \tag{1.6}$$

Using

$$\int_0^{\infty} x^n e^{-ax^2}dx = \frac{\Gamma(\frac{n+1}{2})}{2a^{\frac{n+1}{2}}} \tag{1.7}$$

$$\Gamma(n+1) = n\Gamma(n) \tag{1.8}$$

$$\Gamma(\frac{1}{2}) = \sqrt{\pi} \tag{1.9}$$

$$E_{av} = \frac{1}{2}m\left(\frac{m}{2\pi KT}\right)^{1/2}\frac{\frac{1}{2}\sqrt{\pi}}{\left(\frac{m}{2KT}\right)^{\frac{3}{2}}} = \frac{1}{2}KT$$

In three dimension,

$$f(\mathbf{v}) = A_3 e^{-\frac{mv^2}{2KT}} = A_3\exp[-\frac{1}{2}m(v_x^2+v_y^2+v_z^2)/KT] \tag{1.10}$$

$$n = A_3\int_{-\infty}^{\infty}e^{-\frac{mv_x^2}{2KT}}dv_x\int_{-\infty}^{\infty}e^{-\frac{mv_y^2}{2KT}}dv_y\int_{-\infty}^{\infty}e^{-\frac{mv_z^2}{2KT}}dv_z = \left(\frac{2\pi KT}{m}\right)^{3/2}A_3\,,$$

we obtain

$$A_3 = n\left(\frac{m}{2\pi KT}\right)^{3/2}\,. \tag{1.11}$$

The average keinetic energy is

$$\begin{aligned}
E_{av} &= \frac{1}{n}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A_3\frac{1}{2}m(v_x^2+v_y^2+v_z^2)\exp[-\frac{1}{2}m(v_x^2+v_y^2+v_z^2)/KT]dv_xdv_ydv_z\,,\\
&= \left(\frac{m}{2\pi KT}\right)^{3/2}3\int_{-\infty}^{\infty}\frac{1}{2}mv_x^2\exp[-\frac{1}{2}mv_x^2/KT]dv_x\left[\int_{-\infty}^{\infty}\exp[-\frac{1}{2}mv_y^2/KT]dv_y\right]^2,
\end{aligned}$$

or

$$E_{av} = \frac{3}{2}KT\,. \tag{1.12}$$

NOTES:

- If a system described by classical statistical mechanics is in equilibrium at the absolute temperature T, the average kinetic energy is $\frac{1}{2}KT$ per degree of freedom (Equipartition theorem).

- It is customary in plasma physics to give temperatures in units of energy.
  To avoid confusion on the number of dimensions involved, we use the energy corresponding to KT to denote the temperature.
  For $KT = 1$ eV,

  $$T = \frac{1.6\times10^{-19}\mathrm{J}}{1.38\times10^{-23}\ \mathrm{J/K}} = 11,600\ \mathrm{K}\,.$$

  Thus the conversion factor is

  $$1\ \mathrm{eV} = 11,600\ \mathrm{K}\,.$$

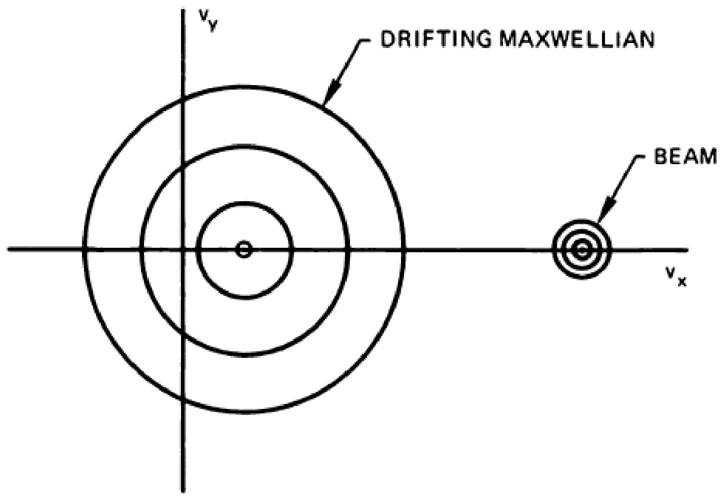


Figure 1.1: Temperature is determined by the spread (or width) of the distribution, not its shift.

- Thermal velocity defines the spread (or, equivalently standard deviation) of the distribution.

$$v_{th} = \sqrt{\frac{2KT}{m}} \tag{1.13}$$

In some other books, to emphasize it is indeed related with the standard deviation of the Gaussian distribution, thermal velocity is defined differently as

$$v_t = \sqrt{\frac{KT}{m}} \tag{1.14}$$

- In general, collision rate among the same species is larger than between others.

$$T_i \neq T_e \tag{1.15}$$

- In the presence of a strong magnetic field:

$$T_\parallel \neq T_\perp, \tag{1.16}$$

where

$$\left\langle \frac{1}{2} m v_\parallel^2 \right\rangle = \frac{1}{2} K T_\parallel, \ \left\langle \frac{1}{2} m v_\perp^2 \right\rangle = K T_\perp \tag{1.17}$$

1.4 Debye Shielding

Consider a test charge q and electron cloud around it in a singly charged plasma. Assume that

- the ions are fixed ($\frac{m_i}{m_e} \longrightarrow \infty$),
- the electrons obey Boltzmann relation.

$$\begin{aligned} n_i(r) &= n \\ n_e(r) &= n e^{e\phi(r)/KT} \qquad (1.18) \\ \nabla^2\phi(r) &= -\frac{e}{\epsilon_0}\left[n_i(r) - n_e(r)\right] \qquad (1.19) \end{aligned}$$

Eq. (1.18) $\longrightarrow$ Eq. (1.19):

$$\nabla^2\phi(r) = \frac{e}{\epsilon_0} n \left(e^{e\phi(r)/KT} - 1\right)$$

<u>Assume that $\left|\frac{e\phi}{KT}\right| \ll 1$, then</u>

$$e^{e\phi(r)/KT} = 1 + e\phi(r)/KT + \cdots$$

$$\nabla^2\phi(r) = \frac{e^2 n}{\epsilon_0 KT}\phi(r)$$

or

$$\nabla^2\phi(r) = \frac{\phi(r)}{\lambda_D^2}$$

$$\boxed{\lambda_D = \sqrt{\frac{\epsilon_0 KT}{ne^2}}} \quad \text{Debye length} \qquad (1.20)$$

In shperical coordinate,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{\phi}{\lambda_D^2} = 0$$

$$\phi'' + \frac{2}{r}\phi' - \frac{1}{\lambda_D^2}\phi = 0$$

$$r\phi'' + 2\phi' - \frac{1}{\lambda_D^2}r\phi = 0$$

Let $\psi(r) = r\phi$, then $\psi' = \phi + r\phi$ and $\psi'' = 2\phi' + r\phi''$ so that

$$\psi'' - \frac{1}{\lambda_D^2}\psi = 0$$

$$\psi(r) = C_1 e^{-r/\lambda_D} + C_2 e^{+r/\lambda_D}$$

$$\phi(r) = \frac{C_1}{r}e^{-r/\lambda_D} + \frac{C_2}{r}e^{+r/\lambda_D}$$

Applying boundary conditions,

$$\phi \longrightarrow 0 \quad \text{as} \quad r \longrightarrow \infty$$

$$\phi \longrightarrow \frac{e}{4\pi\epsilon_0 r} \quad \text{as} \quad r \longrightarrow 0\,,$$

we obtain

$$\begin{aligned} C_1 &= \frac{e}{4\pi\epsilon_0} \\ C_2 &= 0 \end{aligned} \,.$$

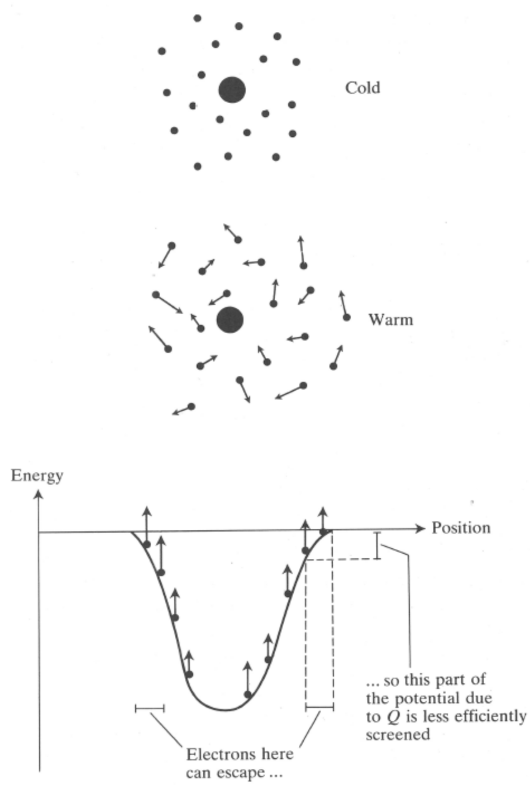


Figure 1.2: If temperature is absolutely 0, a test charge will be completely shielded from the rest of the plasma, and outside the cloud, there will be no electric field.

Hence, the solution is given by

$$\boxed{\phi(r) = \frac{e}{4\pi\epsilon_0 r} e^{-r/\lambda_D}} \tag{1.21}$$

We may say that λ_D is a measure of the shielding distance over which the influence of an individual charged particle is dominant.

1.5 Plasma Parameter

- The plasma parameter is defined as

$$\boxed{N_D = n\frac{4}{3}\pi\lambda_D^3}, \tag{1.22}$$

  which is the number of plasma particles in a Debye sphere.

- For Debye shielding to occur, and for the description of a plasma to be statistically meaningful, the number of particles in a Debye sphere must be large; that is $N_D \gg 1$.

- Since the thermal energy per particle is $\frac{3}{2}KT$ and the mean electrostatic energy per particle is $\frac{e^2}{4\pi\epsilon_0\lambda_D}$, the ratio of two energies is

$$\frac{3}{2}KT\frac{4\pi\epsilon_0\lambda_D}{e^2} = \frac{9}{2}N_D\,. \tag{1.23}$$

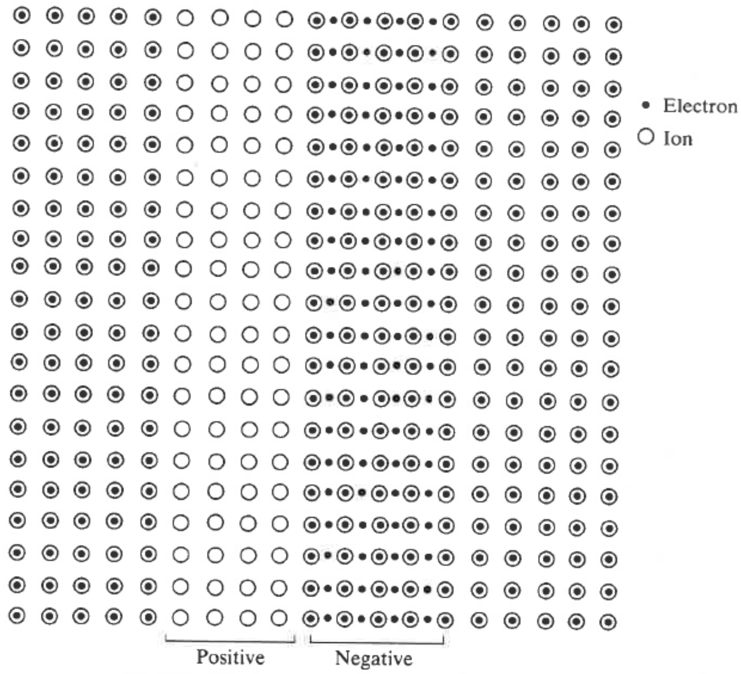


Figure 1.3: We assume the positive and negative layers are thin.

The plasma parameter is a measure of the ratio of the mean plasma kinetic energy to potential energy.

- Thus $N_D \gg 1$ means that the potential energy of a particle due to its nearest neighbor is much smaller than its kinetic energy.
  If this were not the case, there would be a strong tendency for electrons and ions to bind together into atoms, thus destroying the plasma.
- An ideal gas corresponds to zero potential energy between the particles.
  Since the plasma parameter is large, the plasma may be treated as an ideal gas of charged particles, that is, a gas that can have a charge density and electric field but in which no two discrete particles interact.
- In deriving the Debye potential, we assumed that the electrostatic energy was small compared to the thermal energy.
  The largeness of the plasma parameter guarantees the validity of the Debye potential.

1.6 Plasma Frequency

Consider a hypotehtical slab of plasma, where we assume that the ions have infinite mass (immobile) and the electrons can move freely through the ions.

1. Suppose the electron slab is displaced a distance x to the right of the ion slab and then allowed to move freely.

2. An electric field will be set up, causing the electron slab to be pulled back toward the ions.

3. When the electrons exactly overlap the ions (when $x = 0$), the net force is zero, but the electron slab overshoots.

4. The net result is harmonic oscillation.
   The frequency of the oscillation is called the electron plasma frequency.

From Gauss's law,

$$\oint \boldsymbol{D} \cdot d\boldsymbol{a} = Q\,,$$

we have

$$\epsilon_0 E A = neAx\,,$$

or

$$E = nex/\epsilon_0\,.$$

Since the force is given by

$$F = QE = (-neAx)(nex/\epsilon_0) = nAxm_e \frac{d^2x}{dt^2}\,,$$

we obtain the equation of motion

$$\frac{d^2x}{dt^2} = -\left(\frac{ne^2}{m_e\epsilon_0}\right)x\,, \tag{1.24}$$

or

$$\frac{d^2x}{dt^2} + \omega_{pe}^2 x = 0\,, \tag{1.25}$$

where

$$\boxed{\omega_{pe} = \sqrt{\frac{ne^2}{m_e\epsilon_0}}} \quad \text{electron plasma frequency} \tag{1.26}$$

NOTES:

- If there are no collisions, the disturbance will oscillate indefinitely.

- $v_{th}t \sim \frac{\sqrt{\frac{KT}{m}}}{\sqrt{\frac{ne^2}{m\epsilon_0}}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}} = \lambda_D$

1.7 Criteria for plasma

- $\lambda_D \ll L$: Quasinetrality
  - The plasma is neutral enough so that $n_e \simeq n_i$ but not so neutral that all the interesting electromagnetic forces vanish.
  - λ_D is also measure of the penetration depth of external electrostatic fields, i.e., of the thickness of the boundary sheath over which charge neutrality may not be maintained (Boyd & Sanderson).

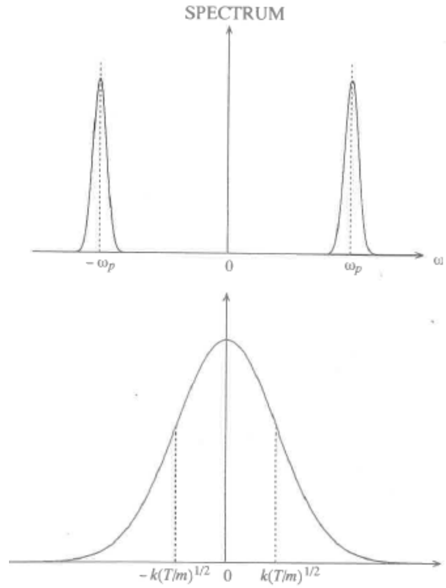


Figure 1.4: Frequency spectrum of density fluctuation (top) in the long-wavelength regime ($\lambda \gg \lambda_D$): collective motions, and (bottom) in the short-wavelength regime ($\lambda \ll \lambda_D$): individual particle-like behaviors (thermal spread).

- $N_D \gg 1$: Collective behavior
  - Motions that depend not only on local conditions but on the state of the plasma in remote regions as well.
  - Broadly speaking, the more particles there are in the Debye sphere, the less likely it is that there will be a significant resultant force on any give particle due to "local ordinary collisions" (Boyd & Sanderson).
- $\frac{\nu}{\omega_p} \ll 1$ (ν is the collision frequency)
  - The most fundamental of the collective interactions are the plasmas oscillations set up in response to a charge imbalance (Boyd & Sanderson).
  - For a plasma, the interactions of the charged particles are dominated electrostatic forces as opposed to collisions with neutral atoms or boundaries.

1.8 Definitions of plasma

- The ensemble of freely moving charged particles of both signs, i.e., ionized gas, can be considered as plasma if the Debye length is small compared with dimensions of the volume occupied by the gas. (Langmuir)
- A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior. (F.F. Chen)

Table 1.1. *Approximate values of parameters across the plasma universe.*

Plasma	n (m^{-3})	T (keV)	B (T)	ω_{pe} (s^{-1})	λ_{D} (m)	$n\lambda_{\mathrm{D}}^3$	ν_{ei} (Hz)
Interstellar	10^6	10^{-5}	10^{-9}	$6 \cdot 10^4$	0.7	$3 \cdot 10^5$	$4 \cdot 10^8$
Solar wind (1 AU)	10^7	10^{-2}	10^{-8}	$2 \cdot 10^5$	7	$4 \cdot 10^9$	10^{-4}
Ionosphere	10^{12}	10^{-4}	10^{-5}	$6 \cdot 10^7$	$2 \cdot 10^{-3}$	10^4	10^4
Solar corona	10^{12}	0.1	10^{-3}	$6 \cdot 10^7$	0.07	$4 \cdot 10^8$	0.5
Arc discharge	10^{20}	10^{-3}	0.1	$6 \cdot 10^{11}$	$7 \cdot 10^{-7}$	40	10^{10}
Tokamak	10^{20}	10	10	$6 \cdot 10^{11}$	$7 \cdot 10^{-5}$	$3 \cdot 10^7$	$4 \cdot 10^4$
ICF	10^{28}	10	—	$6 \cdot 10^{15}$	$7 \cdot 10^{-9}$	$4 \cdot 10^3$	$4 \cdot 10^{11}$

Figure 1.5: Here, ν_{ei} is the electron-ion collission frequency due to the cumulative effect of the much more frequent weak (long-range up to λ_D) Coulomb interactions.

- A plasma is a gas of charged particles, in which the potential energy of a typical particle due to its nearest neighbor is much smaller than its kinetic energy. (D. R. Nicholson)

- A plasma may be roughly defined as a system containing mobile charges, in which the electric and magnetic interactions between particles play a dominant role in the dynamics of the systems. (J. M. Dawson)

- A plasma is collection of charged particles, usually of opposite sign, that tends to be electrically neutral. We often describe a plasma as the fourth state of matter. Adding energy to a solid melts it and it becomes a liquid; adding energy to a liquid boils it and ib becomes a gas; adding energy to a gas ionizes it and it becomes a plasma. (J. L. Shohet)

- The plasma state is a characterization of matter where long range electromagnetic interactions dominate the short range interatomic or intermolecular forces among a large number of particles. (D. G. Swanson)

- Plasma physics is the study of charged particles collected in sufficient number so that the long-range Coulomb force is a factor in determining their statistical properties, yet low enough in density so that the force due to a near-neighbor particle is much less than the long-range Coulomb force exerted by the many distant particles. It is the study of low-density ionized gases. (N. A. Krall and A. W. Trivelpiece)

- Give your own definition.

1.9 Applications of plasma physics

- Gas discharges

- Controlled thermonuclear fusion
- Space Physics
- Astrophysics
- MHD energy conversion and ion propulsion
- Solid state plasmas
- Gas lasers
- Advanced accelerators

1.10 Industrial Applications of Plasma

Reference: J. L. Shohet, Plasma Science and Engineering in Encyclopedia of Modern Physics.

- **Plasma Polymerization**
  By ionizing a monomer gas, certain types of polymers can be made, which can be deposited as coatings on various materials. There is an important application of this work in the biotechnology field, since biocompatible polymers can be used to coat various implant materials that would otherwise be rejected by the body.

- **Plasma–assisted CVD (chemical vapor depostion)**
  Plasmas can be used to provide a mechanism to succesfully deposit various chemicals on surfaces, either treating the surface before deposition or by providing a chemical pathway for succeful deposition.

- **Sputter deposition**
  Plasmas are used to sputter from a target electrode particles that are then deposited on a particular material.

- **Plasma etching**
  The major application of this work is to the semiconductor industry. As the spacing between lines in integrated circuits shrinks 1 μm and below, conventional wet etching using chemicals begins to fail. Appropriately designed plasma etching (dry etching), perhaps combined with electric and magnetic fields or ion beams, offers a dramatic improvement in the etch process, and it is believed that the future of the entire semiconductor fabrication industry will rest with plasma processing.

- **Ion milling**
  Beams of ions can be used to cut or mill narrow regions of materials to great accuracy.

- **Surface modification**
  Plasmas can be used to modify the properties of materials by interacting on the surface of those materials in several ways. For example, tool steel can be hardened considerably by subjecting the tools to a nitrogen plasma. Turbine blades can be plasma coated for improved mechanical and thermal properties.

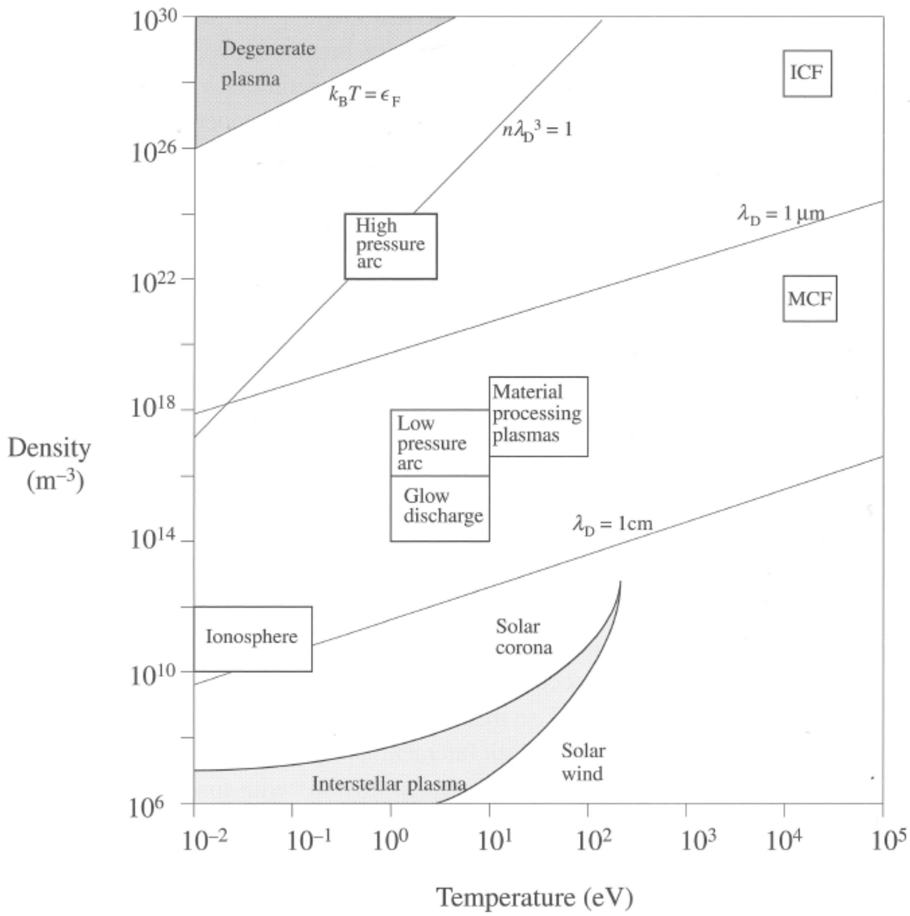


Figure 1.6: Here, ϵ_F denotes the Fermi energy.

- **Welding**
  The use of plasmas in welding, especially in arc welding, has been known for some time. However, many problems continues to exist with welding, and much or it is due to the lack of understanding of the plasma composition, the plasma temperature and density, and the electric field and current distribution in the welding arc.

- **Plasma displays**
  Many new uses for such displays are being found. A portable computer whose screen is a full plasma display has recently introduced.

- **Plasma spary**
  This is a coating process that sputters heavy particles form the cathode of an arc system and then directs the spray of these particles to a surface for coating.