

# **Observer-based Fault Detection and Identification for Rotating Machinery**

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**UNIST**

# Contents

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- Rotating Machinery
- Model-based FDI
- FDI for Rotating Machinery
- Case Study
- Conclusion

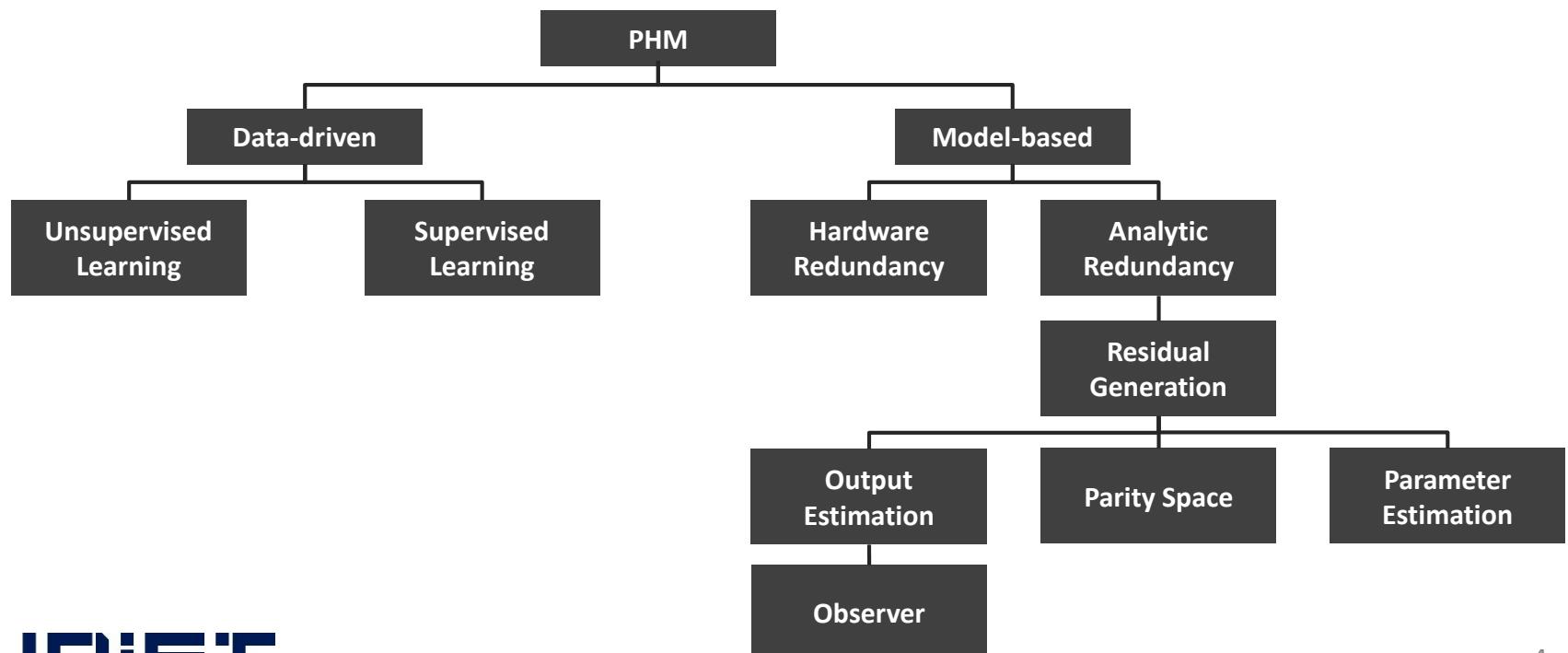
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# Rotating Machinery

- Key component
  - Safety and efficiency issues
- Prognostics and Health Management (PHM)
  - Data-driven Method (Machine Learning)
  - Model-based FDI (Fault detection, isolation and identification)



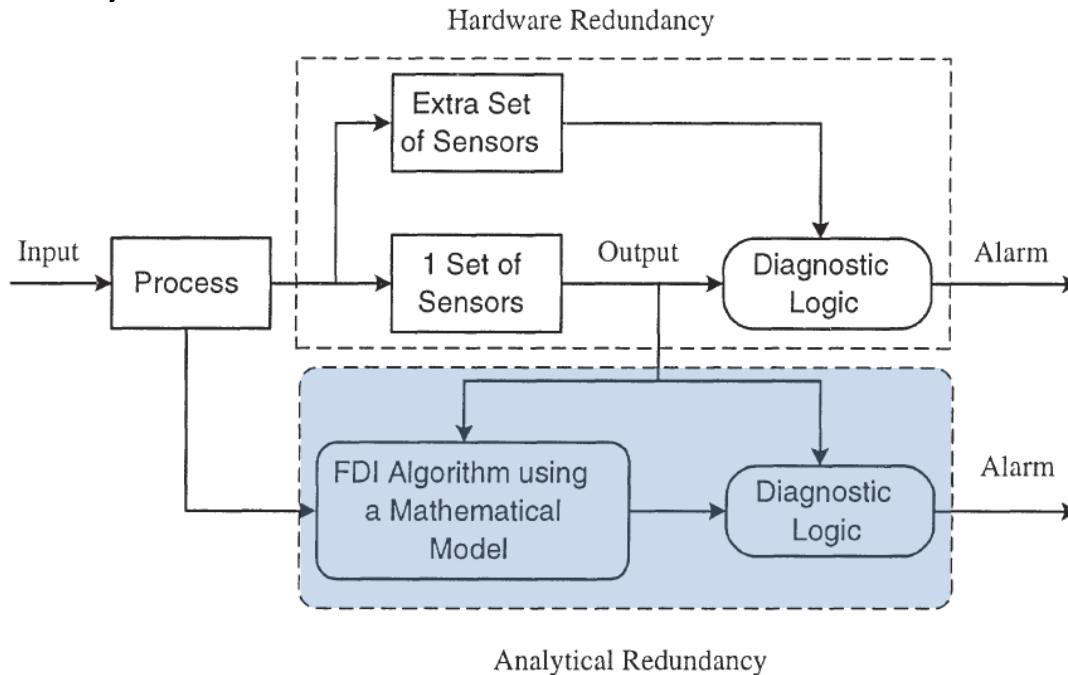
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# Model-based FDI

- Analytic Redundancy



- Residual

$$\text{residual} = \begin{cases} \text{nonzero} & \text{if fault exists} \\ 0 & \text{if no fault} \end{cases}$$

- A quantitative difference between target system and mathematical model
- **Observer-based residual estimation**

# Assumptions and System Representation

- Linear Time-Invariant system (LTI)
- Discrete state space for normal system

$$x[k+1] = Ax[k] + Bu[k]$$
$$y[k] = Cx[k]$$

$A$  : system matrix       $x$  : state  
 $B$  : control matrix       $y$  : output  
 $C$  : output matrix       $u$  : input

- Faulty system
  - Detailed equation or representation depends on the type of fault
    - The equations have similar forms
  - Enable to transform to generalized form

Fault Type	Equation	Generalization
Actuator Fault	$x[k+1] = Ax[k] + Bu[k] + B\tilde{u}[k]$ $y[k] = Cx[k]$	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$
Sensor Fault	$d[k+1] = Ad[k] + Bu[k] + D\mu[k+1] - AD\mu[k]$ $y[k] = Cd[k]$ $\left\{ \begin{array}{l} \mu[k] \text{ is sensor fault} \\ CD = I, d[k] = x[k] + D\mu[k] \end{array} \right.$	$A$ : system matrix $x$ : state $B$ : control matrix $y$ : output $C$ : output matrix $u$ : input $F$ : fault matrix $f$ : fault
System Fault	$x[k+1] = Ax[k] + Bu[k] + \Delta Ax[k]$ $y[k] = Cx[k]$	

# Observer-based Residual Generation

- Faulty system

$$x[k+1] = Ax[k] + Bu[k] + Ff[k]$$

$$y[k] = Cx[k]$$

- State estimation via Observer

$$\hat{x}[k+1] = A\hat{x}[k] + Bu[k] + L(y[k] - C\hat{x}[k])$$

$$\hat{y}[k] = C\hat{x}[k]$$

$\hat{x}$  : state estimation

$\hat{y}$  : output estimation

$L$  : observer gain

- Output estimation error (Residual)
  - Relation between residual and fault signal

Residual

$$\begin{aligned} e[k+1] &= \underline{y[k+1] - \hat{y}[k+1]} \\ &= C(x[k+1] - \hat{x}[k+1]) \\ &= C\varepsilon[k+1] \\ &= C((A - LC)\varepsilon[k] + \underline{Ff[k]}) \\ &= C \left( (A - LC)^{k+1}\varepsilon[0] + \sum_{q=0}^k (A - LC)^q Ff[k-q] \right) \end{aligned}$$

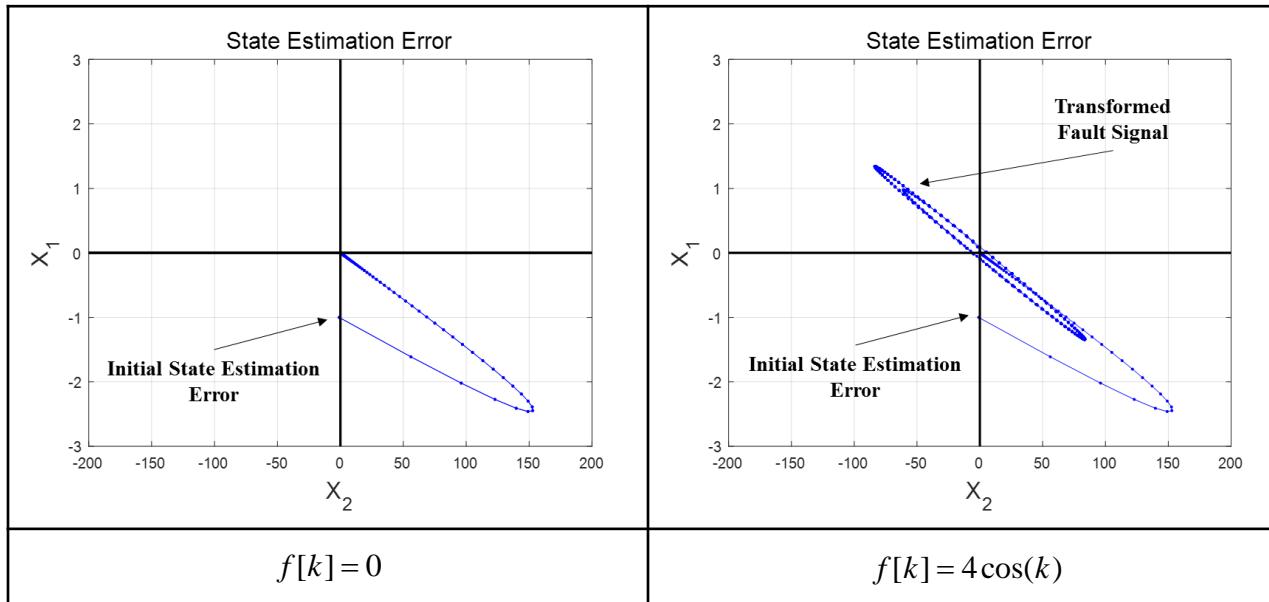
$$\begin{aligned} \text{When } e[k+1] &= \underline{x[k+1] - \hat{x}[k+1]} \\ &= (A - LC)\varepsilon[k] + Ff[k] \end{aligned}$$

# Observer-based Residual Generation

- Normal case
  - The residual value converges to zero-value
- Faulty case
  - The residual value converges to specific shape

$$e[k+1] = C \left( (A - LC)^{k+1} e[0] + \sum_{q=0}^k (A - LC)^q F f[k-q] \right)$$

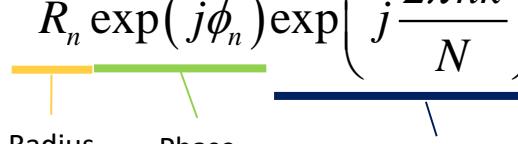
$$\begin{aligned} x[k+1] &= \begin{bmatrix} 0.099 & -0.011 \\ 0.009 & 0.999 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 0.009 \end{bmatrix} u[k] + \begin{bmatrix} 0 \\ 0.009 \end{bmatrix} f[k] \\ y[k] &= [0 \ 1] x[k] \\ L &= [56.139 \ -0.602]^T, \quad x[k] \in R^2, \quad u[k] = 5 \sin(k) \end{aligned}$$



# Fourier Decomposition for General Faults

- Fourier Series
  - Decompose a function of time into the frequency domain

$$f[k] = \sum_{n=-N/2}^{N/2-1} R_n \exp(j\phi_n) \exp\left(j\frac{2\pi nk}{N}\right)$$

  
Radius      Phase      Sinusoid per frequency



- General fault signal
  - Modeling as Fourier series with unknown coefficient  $\theta$

$$\begin{aligned} f[k] &= \sum_{n=-N/2}^{N/2-1} R_n \exp(j\phi_n) \exp\left(j\frac{2\pi nk}{N}\right) \\ &= \left[ \dots \exp\left(j\left(\frac{2\pi(-1)}{N}\right)k\right) \exp\left(j\left(\frac{2\pi(0)}{N}\right)k\right) \exp\left(j\left(\frac{2\pi(+1)}{N}\right)k\right) \dots \right] \theta \end{aligned}$$

$$\text{where } \theta = [\dots \ R_{2-} \exp(j\phi_{2-}) \ R_{1-} \exp(j\phi_{1-}) \ R_0 \exp(j\phi_0) \ R_{1+} \exp(j\phi_{1+}) \ R_{2+} \exp(j\phi_{2+}) \ \dots]^T$$

# Fault Projection Matrix

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$$\hat{\Phi}[k] = \begin{bmatrix} \dots & e^{\left(j\left(\frac{2\pi(-1)}{N}\right)k\right)}(I - T_{-1})^{-1}(I - T_{-1}^{k+1})F & (I - T_0)^{-1}(I - T_0^{k+1})F & e^{\left(j\left(\frac{2\pi(+1)}{N}\right)k\right)}(I - T_{+1})^{-1}(I - T_{+1}^{k+1})F & \dots \end{bmatrix}$$

$$\Phi[k] = \begin{bmatrix} \dots & \exp\left(j\left(\frac{2\pi(-1)}{N}\right)k\right)(I - T_{-1})^{-1}F & (I - T_0)^{-1}F & \exp\left(j\left(\frac{2\pi(+1)}{N}\right)k\right)(I - T_{+1})^{-1}F & \dots \end{bmatrix} \text{ when } k \rightarrow \infty$$

$\Phi[k]$  : fault projection matrix, where  $T_n = \exp\left(-j\left(\frac{2\pi n}{N}\right)\right)(A - LC)$

- State estimation error

$$\begin{aligned} e[k+1] &= x[k+1] - \hat{x}[k+1] \\ &= (A - LC)\varepsilon[k] + Ff[k] \\ &= (A - LC)^{k+1}\varepsilon[0] + \sum_{q=0}^k (A - LC)^q Ff[k-q] \\ &= (A - LC)^{k+1}\varepsilon[0] + \hat{\Phi}[k]\theta \\ &= \underline{\Phi[k]\theta} \end{aligned}$$

- Output estimation error

$$\begin{aligned} e[k+1] &= y[k+1] - \hat{y}[k+1] \\ &= C(x[k+1] - \hat{x}[k+1]) \\ &= C\varepsilon[k+1] \\ &= C(A - LC)^{k+1}\varepsilon[0] + C\hat{\Phi}[k]\theta \\ &= \underline{C\Phi[k]\theta} \quad \text{when } k \rightarrow \infty \end{aligned}$$

# Recursive Fault Parameter Estimation

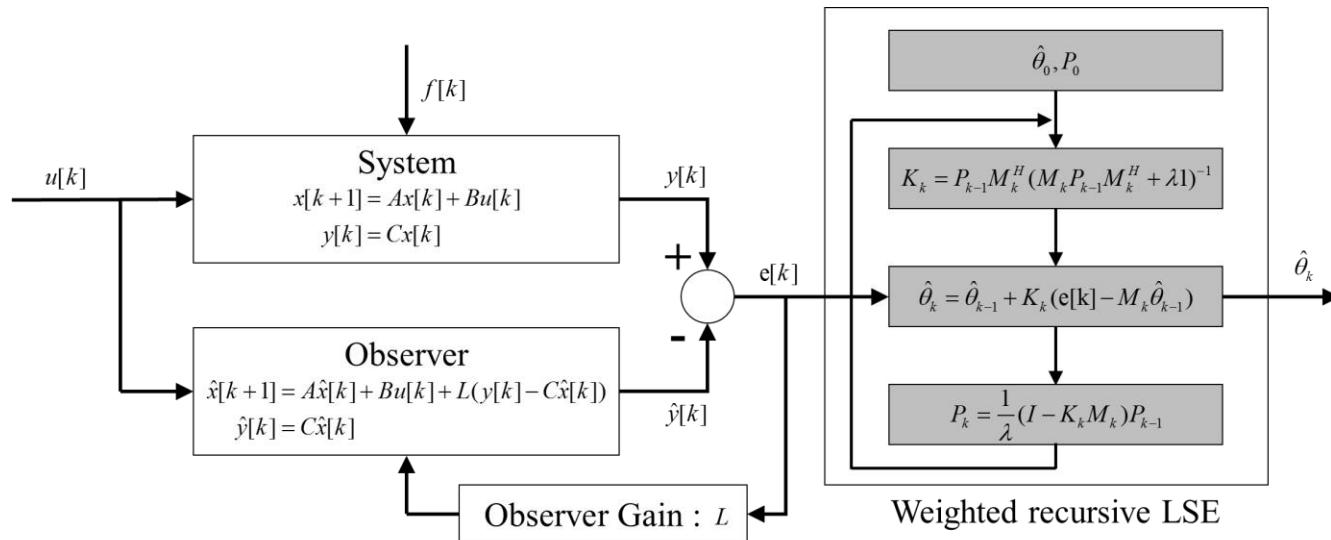
- The linear relationship
  - Least Square Estimation (LSE) can be used

$$\mathbf{e}[k+1] = M_k \theta \quad \text{where } M_k = C\Phi[k]$$

- The object function and solution

$$J = \min_{\theta} \sum_{k=1}^n \lambda^{n-k} (\mathbf{e}[k+1] - M_k \theta)^2$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k (\mathbf{e}[k] - M_k \hat{\theta}_{k-1})$$



# FDI for General Fault

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- Fault detection

$\hat{\theta}_k$  : Fault Parameter Vector

$\hat{\theta}_{ki}$  :  $i$ th Parameter in Fault Parameter Vector

No Fault is present:  $\sum_{ki} |\hat{\theta}_{ki}| = 0$

Fault is present:  $\sum_{ki} |\hat{\theta}_{ki}| \neq 0$

- Fault parameter

Magnitude of fault =  $|\hat{\theta}_{ki}|$

Phase of fault =  $\arctan \left[ \frac{\text{Im}(\hat{\theta}_{ki})}{\text{Re}(\hat{\theta}_{ki})} \right]$        $i \in \{-N/2, \dots, N/2\}$

- Fault identification

$$\hat{f}[k] = \left[ \cdots \exp \left( j \left( \frac{2\pi(-1)}{N} \right) k \right) \exp \left( j \left( \frac{2\pi(0)}{N} \right) k \right) \exp \left( j \left( \frac{2\pi(+1)}{N} \right) k \right) \cdots \right] \hat{\theta}_k$$

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# System Modeling

- Only consider the bearing housing (X, Y)
- Spring-mass-damper system

$$m\ddot{x}_1 + c\dot{x}_1 + Kx_1 = F_c \cos(\omega t)$$

$$m\ddot{x}_2 + c\dot{x}_2 + Kx_2 = F_c \sin(\omega t)$$

$m$  : shaft mass

$c$  : damping ratio

$K$  : spring constant

- In a complex form

$$\dot{z} = Az + Bu$$

$$A = \begin{bmatrix} -c/m & -K/m \\ 1 & 0 \end{bmatrix}$$

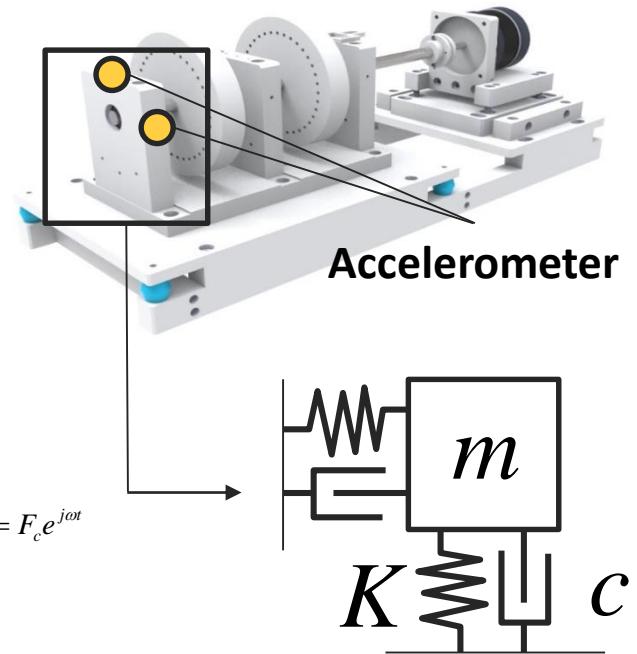
$$y = Cz$$

$$B = [0 \ 1]^T$$

$$C = [0 \ 1]$$

$$u = F_c (\cos(\omega t) + j \sin(\omega t)) = F_c e^{j\omega t}$$

$$z = \begin{bmatrix} \dot{x}_1 + j\dot{x}_2 \\ x_1 + jx_2 \end{bmatrix}$$



- Discrete state space form

$$z[k+1] = A_d z[k] + B_d u[k] + F_d f[k]$$

$$y[k] = C z[k]$$

$A_d$  : System Dynamics

$B_d$  : Input Direction

$C_d$  : Sensor Matrix

$F_d$  : Fault Direction

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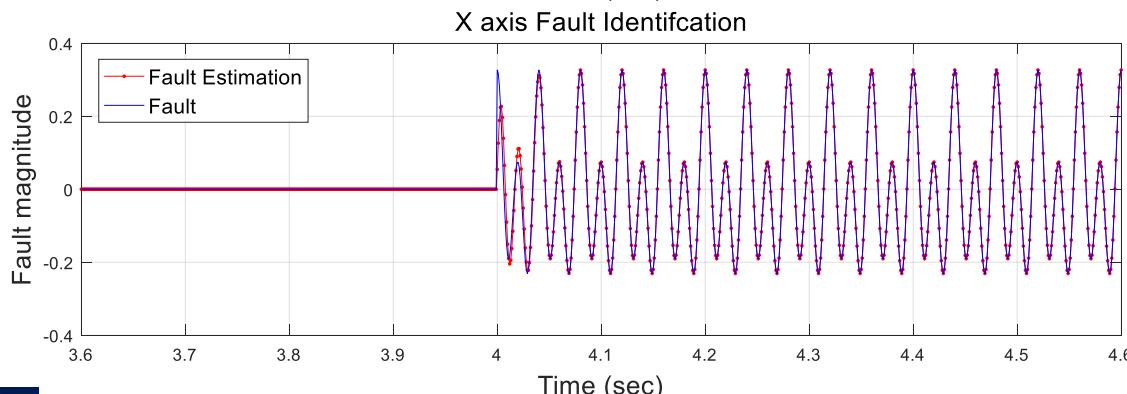
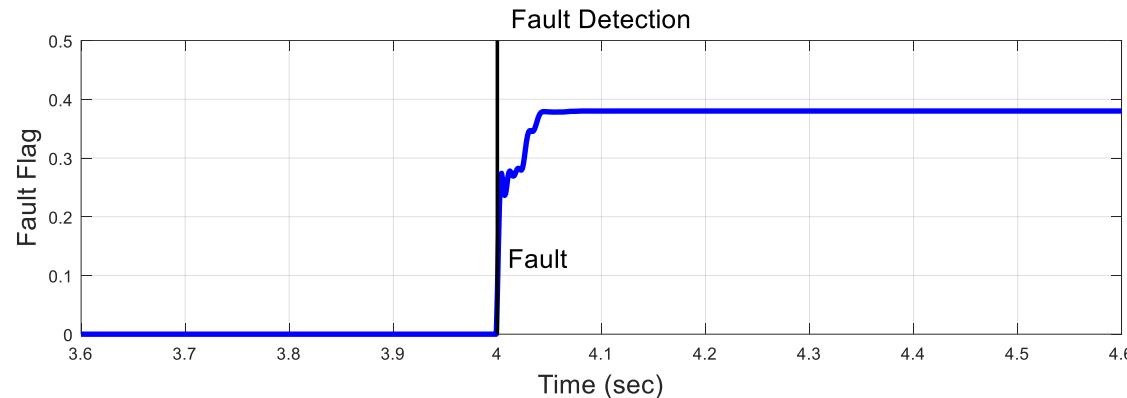
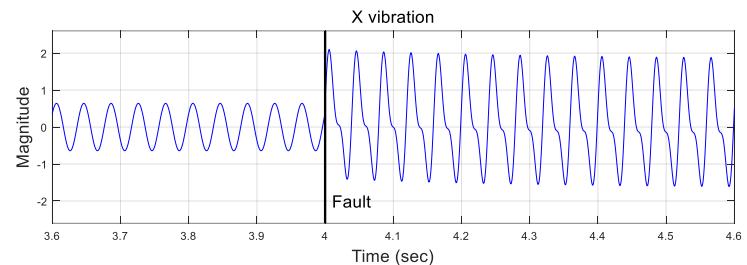
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# Case Study 1

- Fault detection and identification

$$u[k] = 100e^{\left(\frac{j\pi}{6}\right)} e^{(j\omega k)} \quad \text{where } \omega = 2\pi \frac{25}{1000}$$

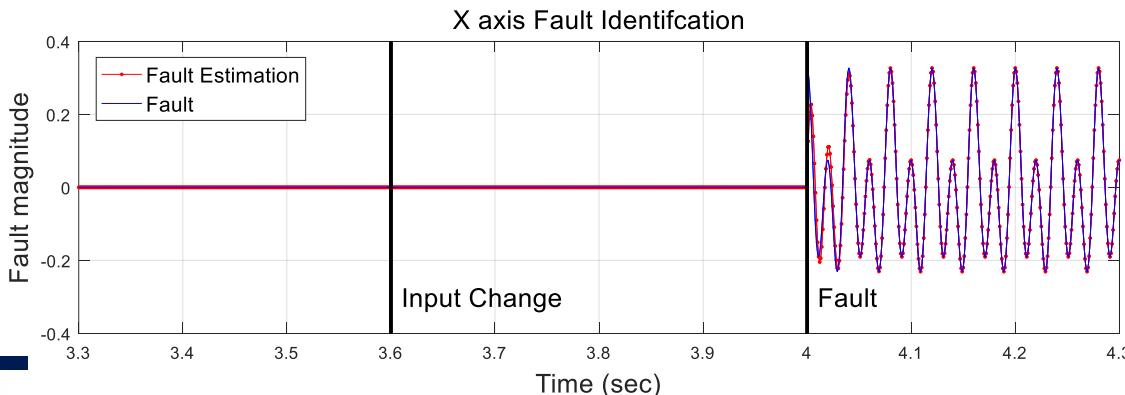
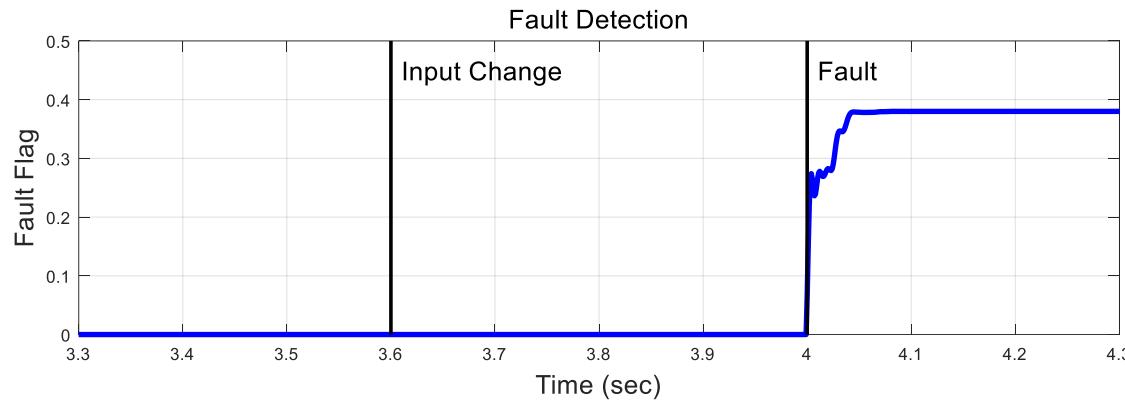
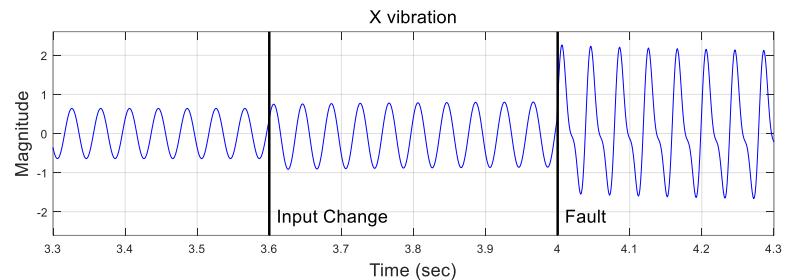
$$f[k] = 0.08\exp(j\omega k) + 0.05\exp(j\frac{\pi}{8})\exp(-j\omega k) + 0.15\exp(j\frac{\pi}{6})\exp(j2\omega k) + 0.1\exp(j\frac{\pi}{4})\exp(-j2\omega k)$$



# Case Study 2

- Different Operating Regime
  - Input signal is changed

$$u[k] = \begin{cases} u_1[k] = 100e^{\left(\frac{j\pi}{6}\right)} e^{(j\omega k)} & \text{when } k < 3.6 \\ u_2[k] = 130e^{\left(\frac{j\pi}{6}\right)} e^{(j\omega k)} & \text{when } k \geq 3.6 \end{cases}$$



# Case Study 3

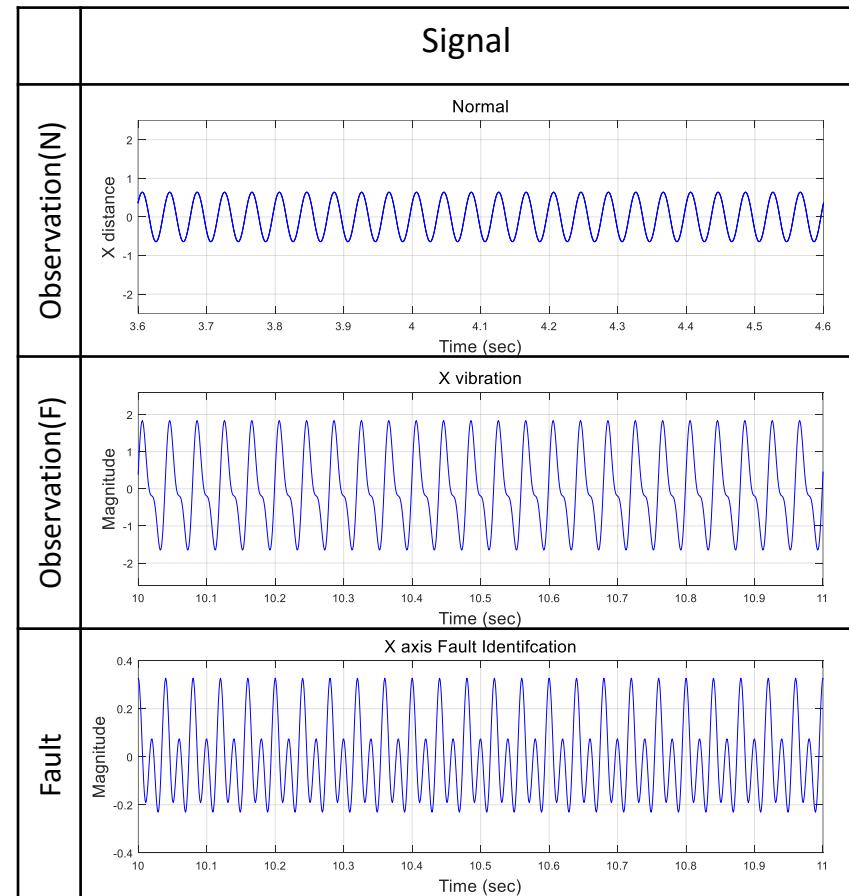
- Data-driven vs. Model-based
- Fault occurs
  - Output is changed
- Change of operation regime (input vector is changed)
  - Output is also changed → multi-regime operation (this is not a fault)

	Data-driven	Model-based
System	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$	
Operation		$u[k]$
Normal	$y[k+1] = C(Ax[k] + Bu[k])$	$y[k+1] - \hat{y}[k+1] = 0$
Change of Operation	$y[k+1] = C(Ax[k] + Bu[k] + \boxed{B\hat{u}[k]})$	$y[k+1] - \hat{y}[k+1] = 0$
Fault	$y[k+1] = C(Ax[k] + Bu[k] + \boxed{Ff[k]})$	$y[k+1] - \hat{y}[k+1] = \boxed{C\Phi[k]\theta}$

# Case Study 3

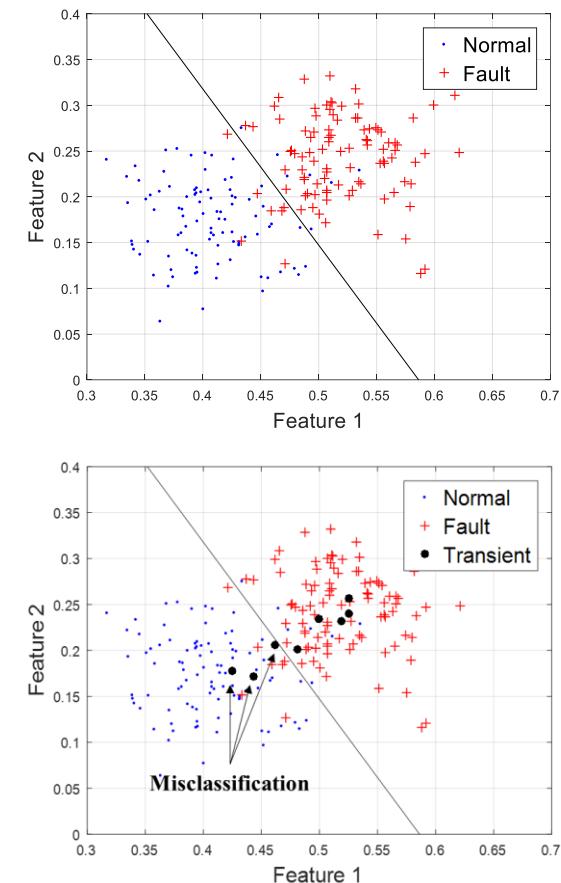
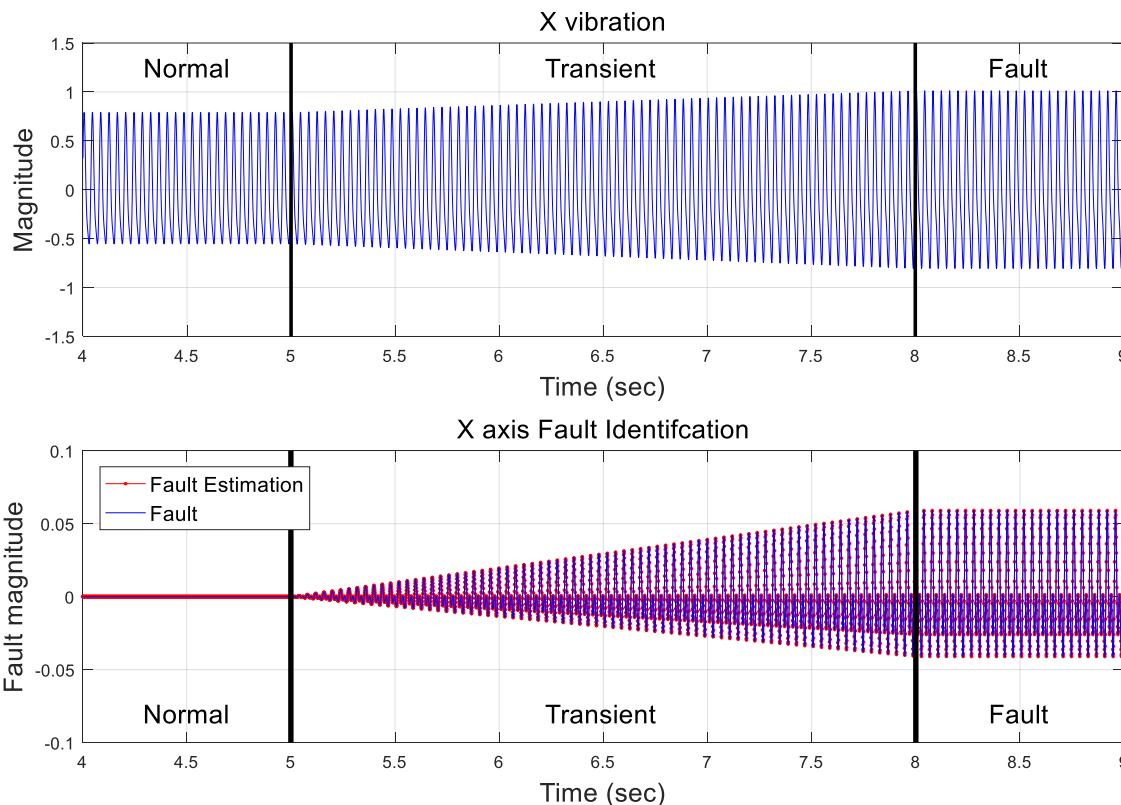
- Data-driven vs. Model-based
  - Under stationary condition
    - Observation (Normal) VS Observation (Fault)
    - Observation (Fault) VS Fault signal
  - Fault detection problem
    - Classification problem

Data-driven	$y[k+1] = Cx[k+1]$ $= C(A_d x[k] + B_d u[k] + F_d f[k])$
FDI	$\hat{f}[k+1]$



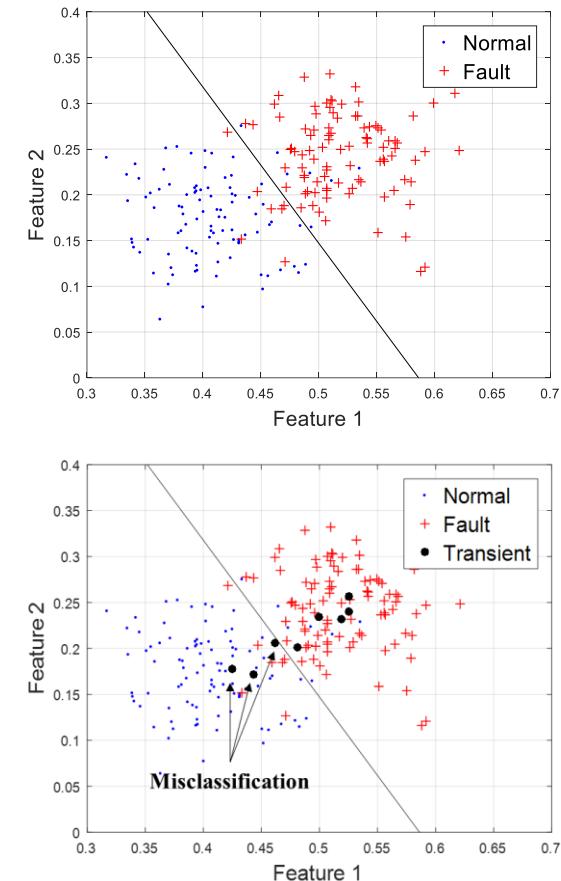
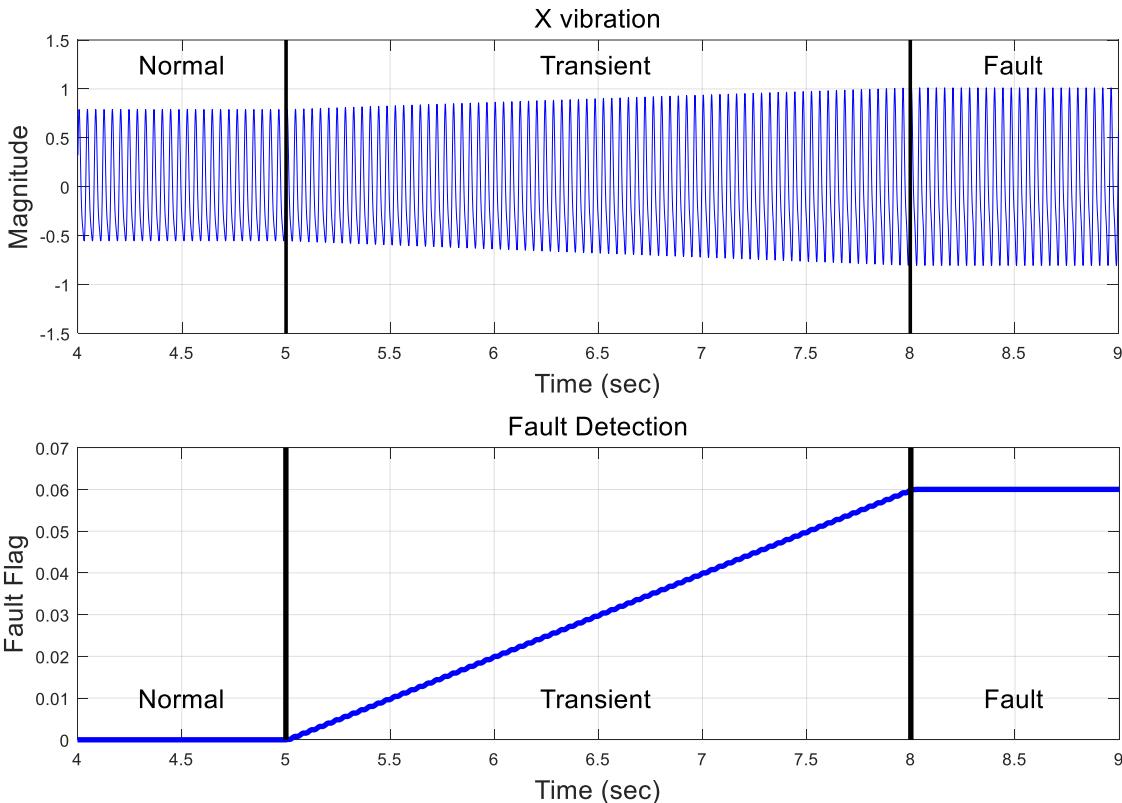
# Case Study 3

- Data-driven vs. Model-based
  - Under non-stationary condition (or Transient)
    - 1)  $k < 5$  : The normal state
    - 2)  $5 < k \leq 8$  : Although the fault occurs, the state is transient
    - 3)  $k \geq 8$  : The fault state



# Case Study 3

- Data-driven vs. Model-based
  - Under non-stationary condition (or Transient)
    - 1)  $k < 5$  : The normal state
    - 2)  $5 < k \leq 8$  : Although the fault occurs, the state is transient
    - 3)  $k \geq 8$  : The fault state



# Case Study 4

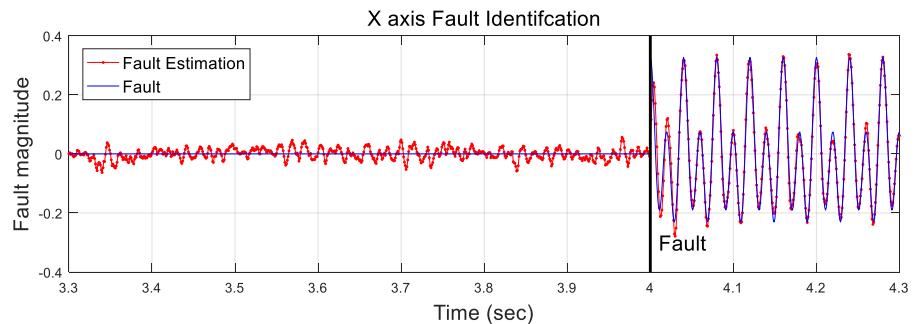
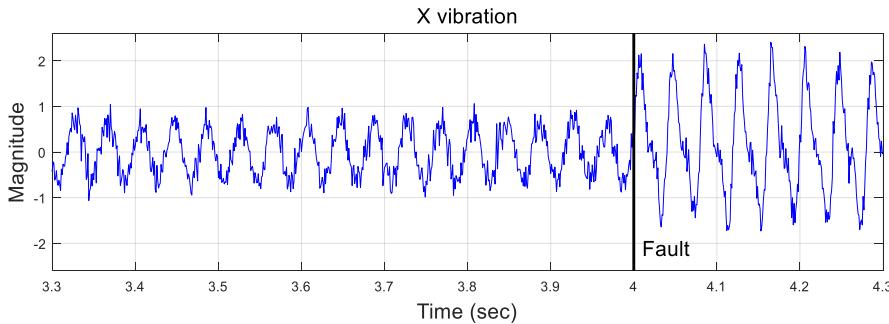
- Proposed method
  - Deterministic study
  - But the method can be applied for the stochastic case
    - Reduction of the number of basis in the fault projection matrix  $\Phi[k]$
    - Effect of low-pass filter

The number of basis is determined by sampling rate

$$\Phi[k] = \left[ \dots \exp\left(j\left(\frac{2\pi(-2)}{N}\right)k\right)(I - T_{-2})^{-1}F \quad \exp\left(j\left(\frac{2\pi(-1)}{N}\right)k\right)(I - T_{-1})^{-1}F \quad (I - T_0)^{-1}F \quad \exp\left(j\left(\frac{2\pi(+1)}{N}\right)k\right)(I - T_{+1})^{-1}F \quad \exp\left(j\left(\frac{2\pi(+2)}{N}\right)k\right)(I - T_{+2})^{-1}F \quad \dots \right]$$

Low frequency

$$\Phi[k] = \left[ e^{\left(j\left(\frac{2\pi(-1)}{N}\right)k\right)}(I - T_{-1})^{-1}F \quad (I - T_0)^{-1}F \quad e^{\left(j\left(\frac{2\pi(+1)}{N}\right)k\right)}(I - T_{+1})^{-1}F \right]$$



# Conclusion

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- Model-based FDI is introduced
  - Observer-based residual estimation
    - The fault projection matrix is defined for FDI
- Proposed FDI method is validated through simulation
  - 4 case studies
- Future work
  - Include uncertainty into model (Modeling error, disturbance)
    - Kalman filtered FDI or Unknown input observer-based FDI
  - Implementation on testbed

