

Rotating Machinery Diagnostics using Model-based Fault Detection and Isolation

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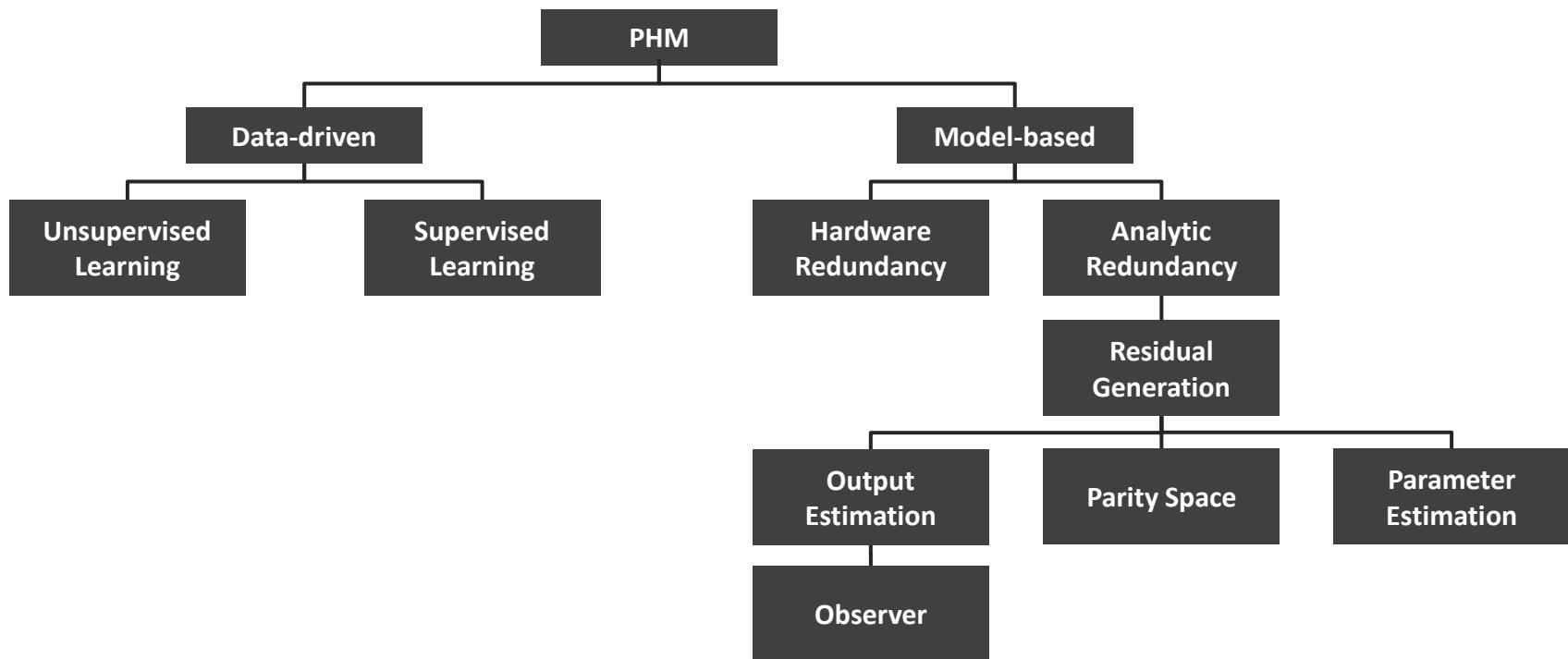
- Rotating Machinery
- Model-based Fault Detection and Isolation (FDI)
- FDI for Rotating Machinery
- Simulation Study
- Conclusion

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Rotating Machinery

- Key component
 - Safety and efficiency issues
- Prognostics and Health Management (PHM)
 - Data-driven Method (Machine Learning)
 - Model-based Fault Detection and Isolation (FDI)

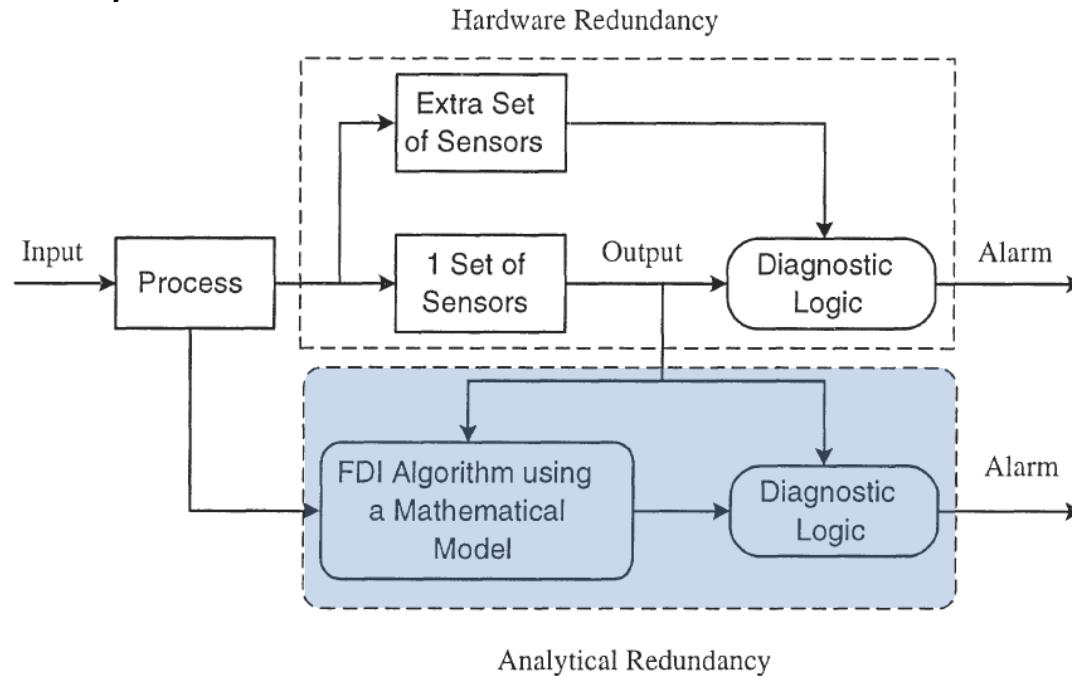


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Model-based Fault Detection and Isolation (FDI)

- Analytic Redundancy



- Residual

$$\text{residual} = \begin{cases} \text{nonzero} & \text{if fault exists} \\ 0 & \text{if no fault} \end{cases}$$

- A quantitative difference between target system and mathematical model
- **Observer-based residual estimation**

Assumptions and System Representation

- Linear Time-Invariant system (LTI)
- Discrete state space

$$x[k+1] = Ax[k] + Bu[k]$$

$$y[k] = Cx[k]$$

A : system matrix x : state
 B : control matrix y : output
 C : output matrix u : input

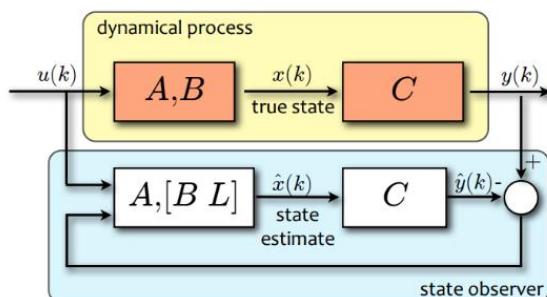
- State estimation via Observer

$$\hat{x}[k+1] = A\hat{x}[k] + Bu[k] + L(y[k] - C\hat{x}[k])$$

$$\hat{y}[k] = C\hat{x}[k]$$

\hat{x} : state estimation
 \hat{y} : output estimation
 L : observer gain

- State estimation error and output estimation error



$$\begin{aligned} \varepsilon[k+1] &= \underline{x[k+1] - \hat{x}[k+1]} \\ &= (A - LC)^{k+1} \varepsilon[0] \end{aligned}$$

$$\begin{aligned} e[k+1] &= \underline{y[k+1] - \hat{y}[k+1]} \\ &= C(x[k+1] - \hat{x}[k+1]) \\ &= C\varepsilon[k+1] \end{aligned}$$

Generalization of Fault Types

- Fault types
 - Detailed equation or representation depends on the type of fault
 - The equations have similar forms
 - Enable to transform to generalized form

Fault Type	Equation	Generalization
Actuator Fault	$x[k+1] = Ax[k] + Bu[k] + B\tilde{u}[k]$ $y[k] = Cx[k]$	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$
Sensor Fault	$d[k+1] = Ad[k] + Bu[k] + D\mu[k+1] - AD\mu[k]$ $y[k] = Cd[k]$ $\left\{ \begin{array}{l} \mu[k] \text{ is sensor fault} \\ CD = I, d[k] = x[k] + D\mu[k] \end{array} \right.$	A : system matrix x : state B : control matrix y : output C : output matrix u : input F : fault matrix f : fault
System Fault	$x[k+1] = Ax[k] + Bu[k] + \Delta Ax[k]$ $y[k] = Cx[k]$	

Fault Signal Modeling

- Now, fault signal modeling is needed
 - Constant (stationary) fault
 - General (non-stationary) fault

	General form for FDI	Fault Modeling
Equation	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$	<ul style="list-style-type: none">Constant fault (Stationary) $f[k] = c$
Observer	$\hat{x}[k+1] = A\hat{x}[k] + Bu[k] + L(y[k] - C\hat{x}[k])$ $\hat{y}[k] = C\hat{x}[k]$	<ul style="list-style-type: none">General fault (Non-stationary) $f[k]$

Fault Detection for Constant Fault

- Fault Detection
 - The residual is non-zero value for fault

$$f[k] = c = 1.2$$

	Normal Case	Constant Fault Case
System	$x[k+1] = Ax[k] + Bu[k]$ $y[k] = Cx[k]$	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$
Observer		$\hat{x}[k+1] = A\hat{x}[k] + Bu[k] + L(y[k] - C\hat{x}[k])$ $\hat{y}[k] = C\hat{x}[k]$
Residual		$y[k] - \hat{y}[k]$
	<p>Output Estimation Error</p>	
	<p>Output Estimation Error</p>	

Fault Isolation for Constant Fault

- Fault Detection

- The residual is non-zero value for fault

$$f[k] = c = 1.2$$

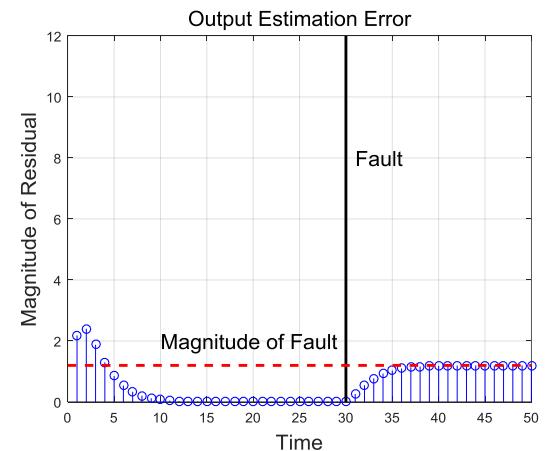
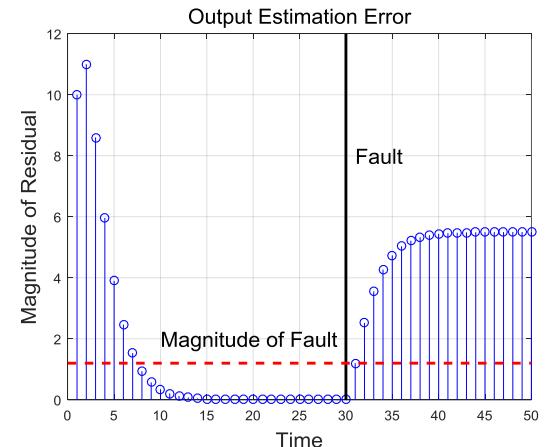
- Fault Isolation

- Estimation of magnitude of constant fault

$$\begin{aligned}\varepsilon[k+1] &= x[k+1] - \hat{x}[k+1] \\ &= (A - LC)\varepsilon[k] + Ff[k] \\ &= T^{k+1}\varepsilon[0] + c(I - T)^{-1}(I - T^{k+1})F \quad \text{where } T = (A - LC) \\ &= c(I - T)^{-1}F \quad \text{when } k \rightarrow \infty\end{aligned}$$

$$\begin{aligned}e[k+1] &= y[k+1] - \hat{y}[k+1] \\ &= C(x[k+1] - \hat{x}[k+1]) \\ &= C(c(I - T)^{-1}F) \\ &= C(I - T)^{-1}Fc\end{aligned}$$

$$c = \left(C(I - T)^{-1}F \right)^{-1} e[k+1]$$



Fourier Decomposition for General Faults

- Fourier Series
 - Decompose a function of time into the frequency domain

$$f[k] = \sum_n R_{n+} e^{j\alpha_{n+}} e^{j(n\omega)k} + R_{n-} e^{j\alpha_{n-}} e^{-j(n\omega)k}$$

$R_{1+} e^{j\alpha_{1+}}$ $e^{j\omega k}$
Radius Phase Sinusoid
 per frequency



- General fault signal
 - Modeling as Fourier series with unknown coefficient θ

$$f[k] = \sum_n R_{n+} e^{j\alpha_{n+}} e^{j(n\omega)k} + R_{n-} e^{j\alpha_{n-}} e^{-j(n\omega)k}$$

$$= \left[\quad \right]^T$$

$$\text{where } \theta = \left[\quad \right]^T$$

Fault Transition Matrix

$$\hat{\Phi}[k] = \begin{bmatrix} \cdots & e^{j\omega k} (I - T_{\omega_+})^{-1} F & (I - T_0)^{-1} F & e^{-j\omega k} (I - T_{\omega_-})^{-1} F & \cdots \end{bmatrix}$$

$$\Phi[k] = \begin{bmatrix} \cdots & e^{j\omega k} (I - T_{\omega_+})^{-1} F & (I - T_0)^{-1} F & e^{-j\omega k} (I - T_{\omega_-})^{-1} F & \cdots \end{bmatrix} \text{ when } k \rightarrow \infty$$

$\Phi[k]$: fault transition matrix

- State estimation error

$$\begin{aligned}\varepsilon[k+1] &= x[k+1] - \hat{x}[k+1] \\ &= (A - LC)\varepsilon[k] + Ff[k] \\ &= (A - LC)^{k+1}\varepsilon[0] + \sum_{q=0}^k (A - LC)^q Ff[k-q] \\ &= (A - LC)^{k+1}\varepsilon[0] + \hat{\Phi}[k]\theta \\ &= \Phi[k]\theta\end{aligned}$$

- Output estimation error

$$\begin{aligned}e[k+1] &= y[k+1] - \hat{y}[k+1] \\ &= C(x[k+1] - \hat{x}[k+1]) \\ &= C\varepsilon[k+1] \\ &= C(A - LC)^{k+1}\varepsilon[0] + C\hat{\Phi}[k]\theta \\ &= C\Phi[k]\theta \quad \text{when } k \rightarrow \infty\end{aligned}$$

FDI for General Fault

- Coefficient estimation

$\Phi[k]$: fault transition matrix

$$\begin{aligned} e[k+1] &= y[k+1] - \hat{y}[k+1] \\ &= C\Phi[k]\theta \quad \text{when } k \rightarrow \infty \end{aligned}$$

$$M = \begin{bmatrix} C\Phi[0] \\ C\Phi[1] \\ \vdots \end{bmatrix} \quad E = \begin{bmatrix} e[0] \\ e[1] \\ \vdots \end{bmatrix}$$

$$\begin{aligned} \hat{\theta} &= (M^H M)^{-1} M^H E \\ &= [\hat{\theta}_1 \quad \hat{\theta}_2 \quad \dots \quad \hat{\theta}_n]^T \end{aligned}$$

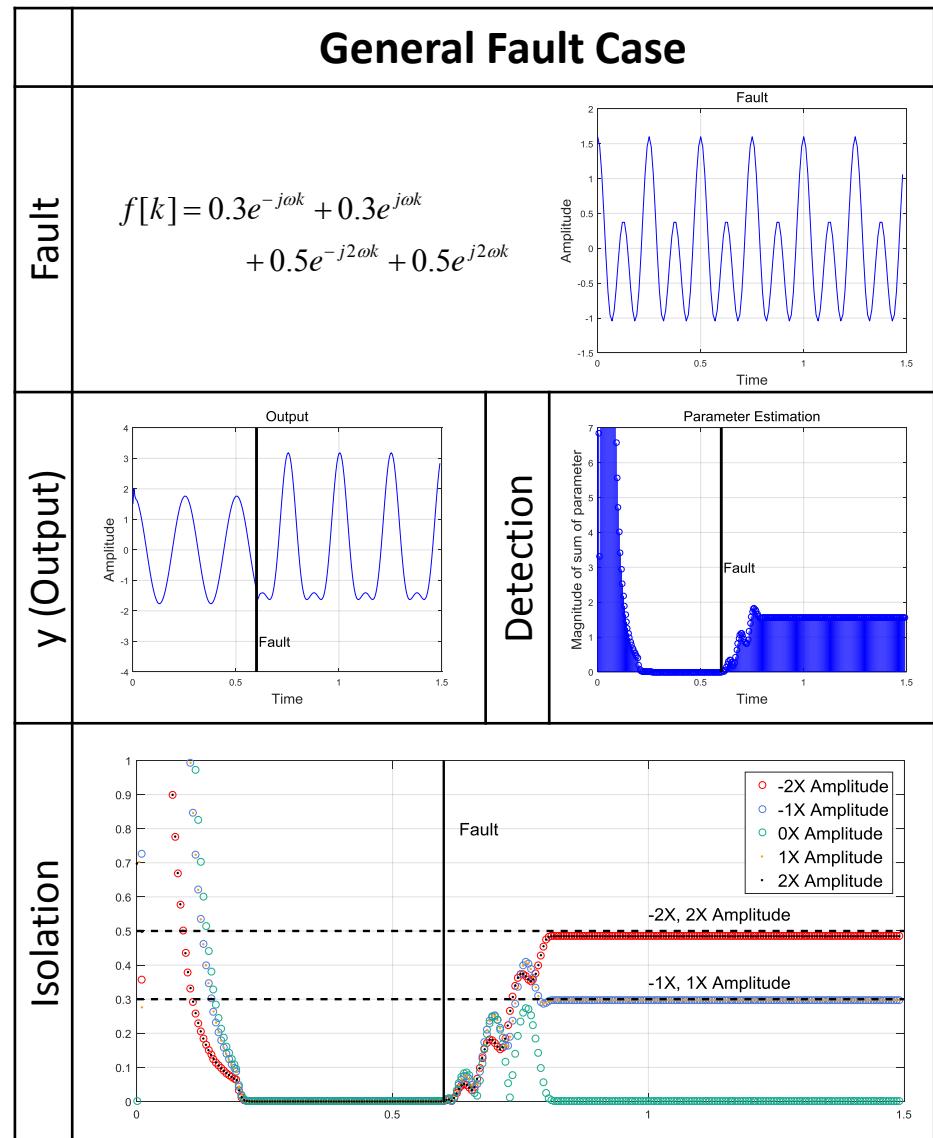
- Fault detection

$$\text{Fault Flag} = \sum_{m=1}^n |\hat{\theta}_m| = \begin{cases} \neq 0, & \text{if fault exists} \\ 0, & \text{if no fault} \end{cases}$$

- Fault isolation

$$\text{Magnitude of fault} = |\hat{\theta}_m|$$

$$\text{Phase of fault} = \arctan \left[\frac{\text{Im}(\hat{\theta}_m)}{\text{Re}(\hat{\theta}_m)} \right] \quad m \in \{1, 2, \dots, n\}$$



Data-driven vs. Model-based

- Fault occurs
 - Output is changed
- Change of operation regime (input vector is changed)
 - Output is also changed

	Data-driven	Model-based
System	$x[k+1] = Ax[k] + Bu[k] + Ff[k]$ $y[k] = Cx[k]$	
Operation		$u[k]$
Normal	$y[k+1] = C(Ax[k] + Bu[k])$	$y[k+1] - \hat{y}[k+1] = 0$
Fault	$y[k+1] = C(Ax[k] + Bu[k] + Ff[k])$	$y[k+1] - \hat{y}[k+1] = C\Phi[k]\theta$
Change of Operation	$y[k+1] = C(Ax[k] + Bu[k] + B\hat{u}[k])$	$y[k+1] - \hat{y}[k+1] = 0$

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System Modeling

- Only consider the bearing housing (X, Y)
- Spring-mass-damper system

$$m\ddot{x} + c\dot{x} + Kx = F_x$$

m : shaft mass

$$m\ddot{y} + c\dot{y} + Ky = F_y$$

c : damping ratio

K : spring constant

- In a complex form

$$m\ddot{z} + c\dot{z} + Kz = F_x + jF_y \quad \text{when } z = x + jy$$

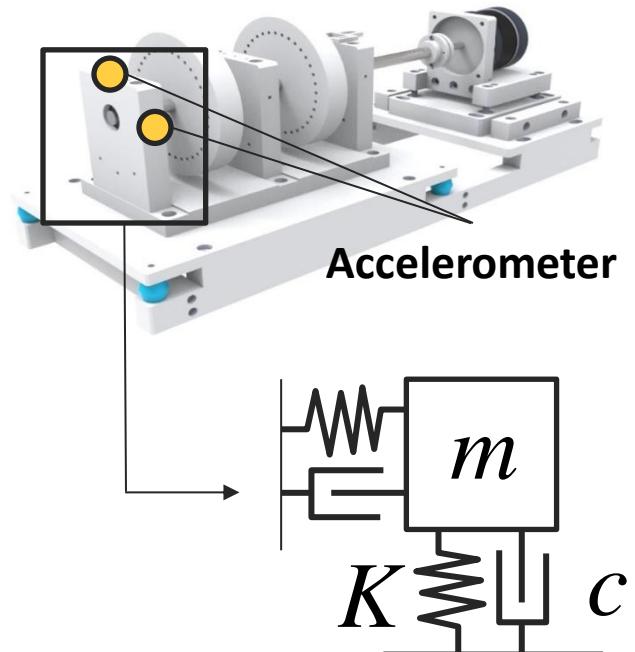
- Discrete state space form

$$\mathbf{z}[k+1] = A\mathbf{z}[k] + Bu[k] + Ff[k]$$

$$y[k] = Cz[k]$$

$$\mathbf{z}[k] = \begin{bmatrix} \dot{x}[k] + j\dot{y}[k] \\ x[k] + jy[k] \end{bmatrix} \quad A = \begin{bmatrix} -c/m & -K/m \\ 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -c/m & -K/m \end{bmatrix}$$

$$u[k] = F_x[k] + jF_y[k] \quad B = [0 \ 1]^T \quad F = [0 \ 1]^T$$



A : System Dynamics

B : Input Direction

C : Sensor Matrix

F : Fault Direction

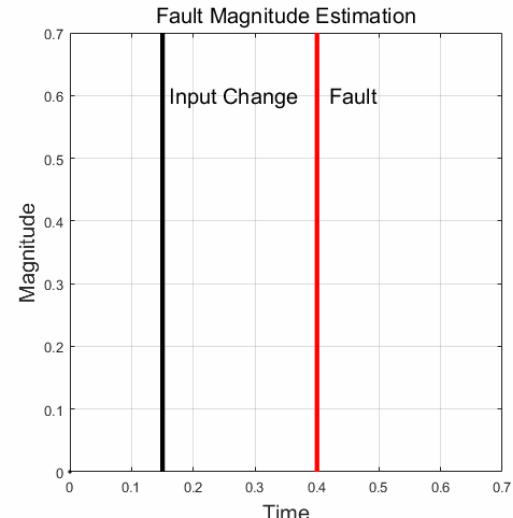
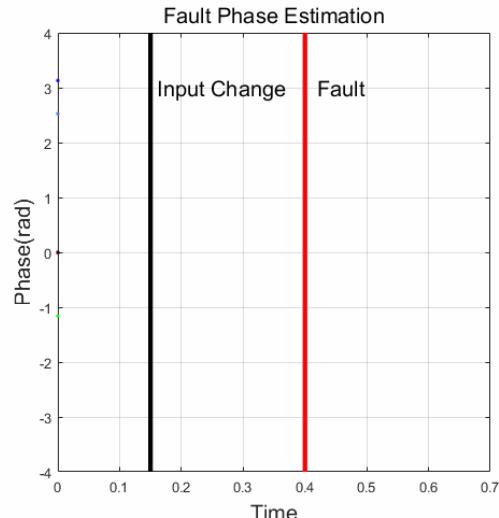
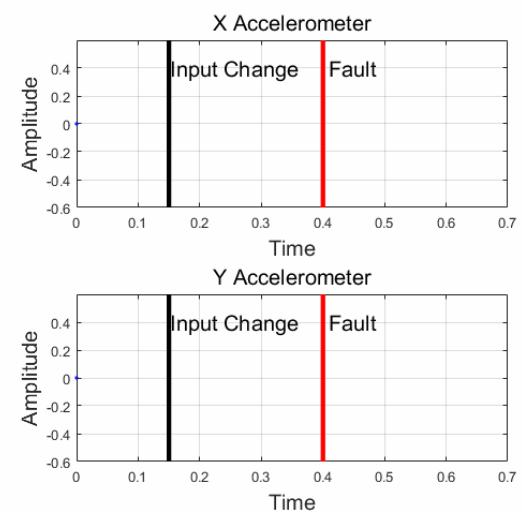
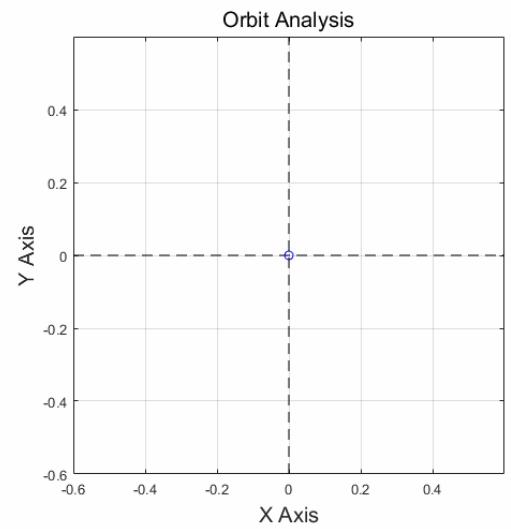
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Simulation Results

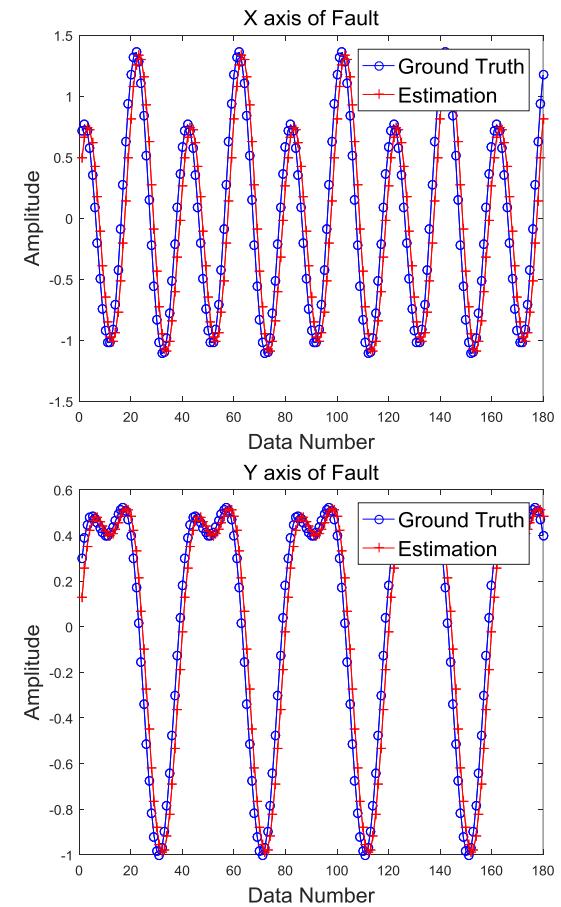
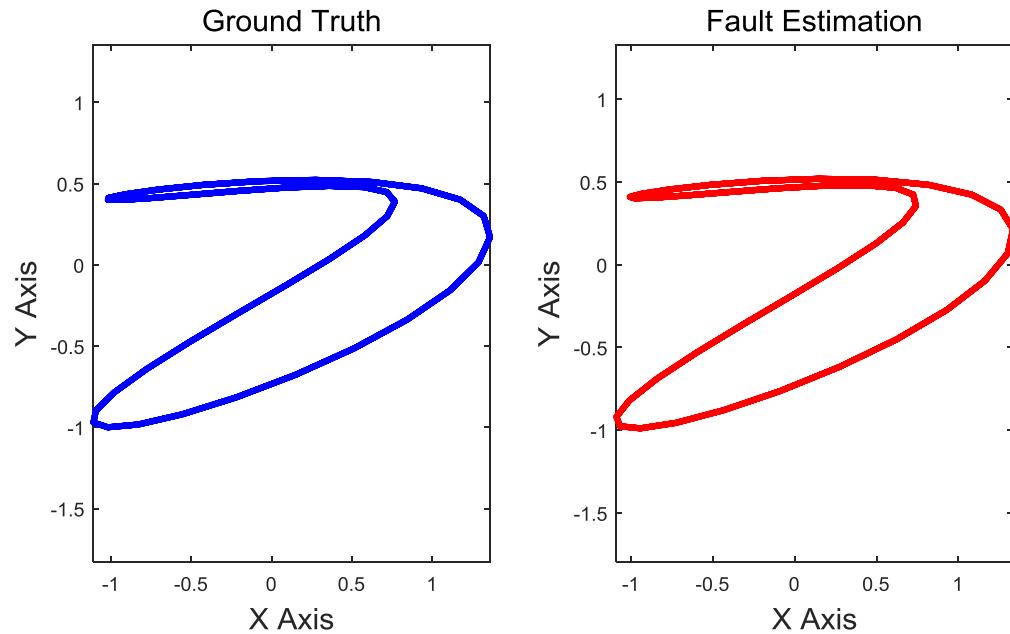
Environment
Sensor
X, Y Accelerometer
Sample Rate
1KHz
Operation RPM
1500 RPM
Input
$u[k] = e^{-j\pi/6}e^{j\omega k}$  $u[k] = 2e^{-j\pi/3}e^{j\omega k}$
Fault
$f[k] = 0.2e^{-j\omega k} + 0.5e^{-j\pi}e^{j\omega k}$ $+ 0.5e^{-j2\omega k} + 0.6e^{j\pi/6}e^{j2\omega k}$

- -2X
- -1X
- 0X
- 1X
- 2X



Simulation Results

- Fault estimation
 - Fault orbit shape
 - X and Y axis vibration signal



Conclusion

- Model-based FDI is introduced
 - Observer-based residual estimation
 - FDI for constant fault
 - FDI for general fault
- Fault detection and isolation are validated through simulation
 - Parameter estimation
- Future work
 - Include uncertainty into model
 - Kalman filtered FDI
 - Implementation on testbed

