Suppose we have an atom





The lattice consisting of this atom





The Bloch state

I. The simultaneous eigenstate of \hat{H} and $\hat{T}[\vec{R}]$

$$\left(\frac{1}{2}(-i\nabla)^2 + V(\vec{r})\right) \psi_{n,\vec{k}}(\vec{r}) = E_n(\vec{k})\psi_{n,\vec{k}}(\vec{r}) , \quad \psi_{n,\vec{k}}(\vec{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n,\vec{k}}(\vec{r})$$
$$\left(\frac{1}{2}(-i\nabla + \vec{k})^2 + V(\vec{r})\right) u_{n,\vec{k}}(\vec{r}) = E_n(\vec{k})u_{n,\vec{k}}(\vec{r}) , \quad u_{n,\vec{k}}(\vec{r} + \vec{R}) = u_{n,\vec{k}}(\vec{r})$$

II. Tight-binding limit, it is well described by the linear sum

$$\psi_{n,\vec{k}}(\vec{r}) = \sum_{\vec{R}} A_n e^{i\mathbf{k}\cdot\mathbf{R}} \varphi_n(\vec{r}-\vec{R})$$

The Bloch state

III. Show that it is indeed the translation eigenstate

$$\psi_{n,\vec{k}}(\vec{r}) = \sum_{\vec{R}} A_n e^{i\mathbf{k}\cdot\mathbf{R}} \varphi_n(\vec{r}-\vec{R})$$

$$\hat{T}(\vec{R})\psi_n(\vec{r}) = \psi_n(\vec{r}+\vec{R}) - e^{i\mathbf{k}\cdot\mathbf{R}}\psi_n(\vec{r}-\vec{R})$$

$$T(R)\psi_{n,\vec{k}}(\vec{r}) = \psi_{n,\vec{k}}(\vec{r}+R) = e^{i\mathbf{k}\cdot\mathbf{k}}\psi_{n,\vec{k}}(\vec{r})$$

IV. Tight-binding limit, it is well described by the linear sum

Think of one-dimensional energy band structure



Lattice constant



Suppose we have an atom



The shape of s-band wavefunction at each k



Suppose we have an atom

